CCGPS Frameworks

Mathematics

CCGPS Coordinate Algebra

Unit 3: Linear and Exponential Functions

.

Dr. John D. Barge, State School Superintendent
"Making Education Work for All Georgians"

Unit 3
Linear and Exponential Functions
Table of Contents

* New task added to the July 2014 edition
** Existing task modified in the July 2014 edition

OVERVIEW

In this unit students will:

- Represent and solve linear equations and inequalities graphically using real-world contexts.
- Use function notation.
- Interpret linear and exponential functions that arise in applications in terms of the context.
- Analyze linear and exponential functions and model how different representations may be used based on the situation presented.
- Build a function to model a relationship between two quantities.
- Create new functions from existing functions.
- Construct and compare linear and exponential models and solve problems.
- Interpret expressions for functions in terms of the situation they model.

Because we continually make theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city;" by an algebraic expression like $f(x) = a + bx$; or by a recursive rule. The graph of a function is often a useful way of visualizing the relationship of the function models, and manipulating a mathematical expression for a function can throw light on the function's properties.

Although the units in this instructional framework emphasize key standards and big ideas at specific times of the year, routine topics such as estimation, mental computation, and basic computation facts should be addressed on an ongoing basis. Ideas related to the eight practice standards should be addressed constantly as well. This unit provides much needed content information and excellent learning activities. However, the intent of the framework is not to provide a comprehensive resource for the implementation of all standards in the unit. A variety of resources should be utilized to supplement this unit. The tasks in this unit framework illustrate the types of learning activities that should be utilized from a variety of sources. To assure that this unit is taught with the appropriate emphasis, depth, and rigor, it is important that the **"Strategies for Teaching and Learning"** and the tasks listed under **"Evidence of Learning"** be reviewed early in the planning process.

Webinar Information

A two-hour course overview webinar may be accessed at
http://www.gpb.org/education/common-core/2012/02/28/mathematics-9th-grade

The unit-by-unit webinars may be accessed at
https://www.georgiastandards.org/Common-Core/Pages/Math-PL-Sessions.aspx

STANDARDS ADDRESSED IN THIS UNIT

Mathematical standards are interwoven and should be addressed throughout the year in as many different units and activities as possible in order to emphasize the natural connections that exist among mathematical topics.

KEY STANDARDS ADDRESSED

Represent and solve equations and inequalities graphically

MCC9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). *(Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.)*

MCC9-12.A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, ~~polynomial, rational, absolute value~~, exponential, and ~~logarithmic~~ functions.

Understand the concept of a function and use function notation

MCC9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *(Draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences.)*

Interpret functions that arise in applications in terms of the context

MCC9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~. *(Focus on linear and exponential functions.)*

MCC9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *(Focus on linear and exponential functions.)*

MCC9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *(Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers.)*

Analyze functions using different representations

MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

MCC9-12.F.IF.7a Graph linear ~~and quadratic~~ functions and show intercepts, maxima, and minima.

MCC9-12.F.IF.7e Graph exponential ~~and logarithmic~~ functions, showing intercepts and end behavior, and ~~trigonometric functions, showing period, midline, and amplitude.~~

MCC9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

Build a function that models a relationship between two quantities

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1b Combine standard function types using arithmetic operations. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Build new functions from existing functions

MCC9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. *(Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.)*

MCC9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

MCC9-12.F.LE.1a Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

MCC9-12.F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

MCC9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

MCC9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MCC9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, ~~quadratically, or (more generally) as a polynomial function.~~

MCC9-12.F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. *(Limit exponential functions to those of the form $f(x) = b^x + k$.)*

STANDARDS FOR MATHEMATICAL PRACTICE

Refer to the Comprehensive Course Overview for more detailed information about the Standards for Mathematical Practice.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

ENDURING UNDERSTANDINGS

- Linear equations and inequalities can be represented graphically and solved using real-world context.
- Understand the concept of a function and be able to use function notation.
- Understand how to interpret linear and exponential functions that arise in applications in terms of the context.
- When analyzing linear and exponential functions, different representations may be used based on the situation presented.
- A function may be built to model a relationship between two quantities.
- New functions can be created from existing functions.
- Understand how to construct and compare linear and exponential models and solve problems.
- Understand how to interpret expressions for functions in terms of the situation they model.

ESSENTIAL QUESTIONS

- How do I use graphs to represent and solve real-world equations and inequalities?
- Why is the concept of a function important and how do I use function notation to show a variety of situations modeled by functions?
- How do I interpret functions that arise in applications in terms of context?
- How do I use different representations to analyze linear and exponential functions?
- How do I build a linear or exponential function that models a relationship between two quantities?
- How do I build new functions from existing functions?
- How can we use real-world situations to construct and compare linear and exponential models and solve problems?
- How do I interpret expressions for functions in terms of the situation they model?

CONCEPTS AND SKILLS TO MAINTAIN

In order for students to be successful, the following skills and concepts need to be maintained:

- Know how to solve equations, using the distributive property, combining like terms and equations with variables on both sides.
- Know how to solve systems of linear equations.
- Understand and be able to explain what a function is.
- Determine if a table, graph or set of ordered pairs is a function.
- Distinguish between linear and non-linear functions.
- Write linear equations and use them to model real-world situations.

SELECTED TERMS AND SYMBOLS

The following terms and symbols are often misunderstood. These concepts are not an inclusive list and should not be taught in isolation. However, due to evidence of frequent difficulty and misunderstanding associated with these concepts, instructors should pay particular attention to them and how their students are able to explain and apply them.

The definitions below are for teacher reference only and are not to be memorized by the students. Students should explore these concepts using models and real life examples. Students should understand the concepts involved and be able to recognize and/or demonstrate them with words, models, pictures, or numbers.

The websites below are interactive and include a math glossary suitable for high school children.
Note – At the high school level, different sources use different definitions. Please preview any website for alignment to the definitions given in the frameworks
http://www.amathsdictionaryforkids.com/
This web site has activities to help students more fully understand and retain new vocabulary.
http://intermath.coe.uga.edu/dictnary/homepg.asp

Definitions and activities for these and other terms can be found on the Intermath website. Intermath is geared towards middle and high school students.

- **Arithmetic Sequence.** A sequence of numbers in which the difference between any two consecutive terms is the same.

- **Average Rate of Change.** The change in the value of a quantity by the elapsed time. For a function, this is the change in the *y*-value divided by the change in the *x*-value for two distinct points on the graph.

- **Coefficient.** A number multiplied by a variable in an algebraic expression.

- **Constant Rate of Change.** With respect to the variable x of a linear function $y = f(x)$, the constant rate of change is the slope of its graph.

- **Continuous.** Describes a connected set of numbers, such as an interval.

- **Discrete.** A set with elements that are disconnected.

- **Domain.** The set of *x*-coordinates of the set of points on a graph; the set of *x*-coordinates of a given set of ordered pairs. The value that is the input in a function or relation.

- **End Behaviors.** The appearance of a graph as it is followed farther and farther in either direction.

- **Explicit Expression.** A formula that allows direct computation of any term for a sequence $a_1, a_2, a_3, \ldots, a_n, \ldots$.

- **Exponential Function.** A nonlinear function in which the independent value is an exponent in the function, as in $y = ab^x$.

- **Exponential Model.** An exponential function representing real-world phenomena. The model also represents patterns found in graphs and/or data.
- **Expression.** Any mathematical calculation or formula combining numbers and/or variables using sums, differences, products, quotients including fractions, exponents, roots, logarithms, functions, or other mathematical operations.

- **Even Function.** A function with a graph that is symmetric with respect to the *y*-axis. A function is only even if and only if $f(–x) = f(x)$.

- **Factor.** For any number x, the numbers that can be evenly divided into x are called factors of x. For example, the number 20 has the factors 1, 2, 4, 5, 10, and 20.

- **Geometric Sequence.** A sequence of numbers in which the ratio between any two consecutive terms is the same. In other words, you multiply by the same number each time to get the next term in the sequence. This fixed number is called the common ratio for the sequence.

- **Interval Notation.** A notation representing an interval as a pair of numbers. The numbers are the endpoints of the interval. Parentheses and/or brackets are used to show whether the endpoints are excluded or included.

- **Linear Function.** A function with a constant rate of change and a straight line graph.

- **Linear Model.** A linear function representing real-world phenomena. The model also represents patterns found in graphs and/or data.

- **Odd Function.** A function with a graph that is symmetric with respect to the origin. A function is odd if and only if $f(-x) = -f(x)$.

- **Parameter.** The independent variable or variables in a system of equations with more than one dependent variable.

- **Range.** The set of all possible outputs of a function.

- **Recursive Formula.** A formula that requires the computation of all previous terms to find the value of a_n.

- **Slope.** The ratio of the vertical and horizontal changes between two points on a surface or a line.

- **Term.** A value in a sequence--the first value in a sequence is the 1st term, the second value is the 2nd term, and so on; a term is also any of the monomials that make up a polynomial.

- **Vertical Translation.** A shift in which a plane figure moves vertically.

- ***X*-intercept.** The point where a line meets or crosses the *x*-axis

- ***Y*-intercept.** The point where a line meets or crosses the *y*-axis

EVIDENCE OF LEARNING

By the conclusion of this unit, students should be able to demonstrate the following competencies:

- Explain what it means when two curves $\{y = f(x)$ and $y = g(x)\}$ intersect.

- Define and use function notation, evaluate functions at any point in the domain, give general statements about how $f(x)$ behaves at different regions in the domain (as x gets very large or very negative, close to 0 etc.), and interpret statements that use function notation.

- Explain the difference and relationship between domain and range and find the domain and range of a function from a function equation, table or graph.

- Examine data (from a table, graph, or set of points) and determine if the data is a function and explain any conclusions that can be drawn.

- Write a function from a sequence or a sequence from a function.

- Explain how an arithmetic or geometric sequence is related to its algebraic function notation.

- Interpret x and y intercepts, where the function is increasing or decreasing, where it is positive or negative, its end behaviors, given the graph, table or algebraic representation of a linear or exponential function in terms of the context of the function.

- Find and/or interpret appropriate domains and ranges for authentic linear or exponential functions.

- Calculate and interpret the average rate of change over a given interval of a function from a function equation, graph or table, and explain what that means in terms of the context of the function.

- Estimate the rate of change of a function from its graph at any point in its domain.

- Explain the relationship between the domain of a function and its graph in general and/or to the context of the function.

- Accurately graph a linear function by hand by identifying key features of the function such as the x- and y-intercepts and slope.

- Graph a linear or exponential function using technology.

- Sketch the graph of an exponential function accurately identifying x- and y-intercepts and asymptotes.

- Describe the end behavior of an exponential function (what happens as x goes to positive or negative infinity).

- Discuss and compare two different functions (linear and/or exponential) represented in different ways (tables, graphs or equations). Discussion and comparisons should include: identifying differences in rates of change, intercepts, and/or where each function is greater or less than the other.

- Write a function that describes a linear or exponential relationship between two quantities and combine different functions using addition, subtraction, multiplication, division and composition of functions to create a new function.

- Write recursively and an explicit formula for arithmetic and geometric sequences.

- Construct and compare linear and exponential models and solve problems.
  Recognize situations with a constant rate of change as well as those in which a quantity either grows or decays by a constant percent rate.

TEACHER RESOURCES

The following pages include teacher resources that teachers may wish to use to supplement instruction.

- Web Resources
- Compare / Contrast: Linear and Exponential Functions
- Graphic Organizer: Graphing Transformations

Web Resources

The following list is provided as a sample of available resources and is for informational purposes only. It is your responsibility to investigate them to determine their value and appropriateness for your district. GaDOE does not endorse or recommend the purchase of or use of any particular resource.

- Rate of Change Task
  http://www.nms.org/Portals/0/Docs/FreeLessons/Fill%20It%20Up,%20Please%20-%20Part%20III.pdf
  This task includes an extensive lesson plan with alignment to the standards

- Linear & Exponential Growth
  http://learnzillion.com/lessonsets/40-proving-how-linear-functions-grow
  This webpage includes short videos comparing linear and exponential functions.

- Distinguishing between Linear & Exponential
  http://learnzillion.com/lessonsets/35-distinguishing-between-linear-functions-and-exponential-functions
  Further video resources for exponential & linear functions.

Compare / Contrast: Linear and Exponential Functions

Show similarities and differences between linear and exponent functions:
What things are being compared? How are they similar? How are they different?

Attribute	Linear functions	Exponential functions
Rate of change		
Domain & Range		
Intercepts		
Asymptotes		
End Behavior		

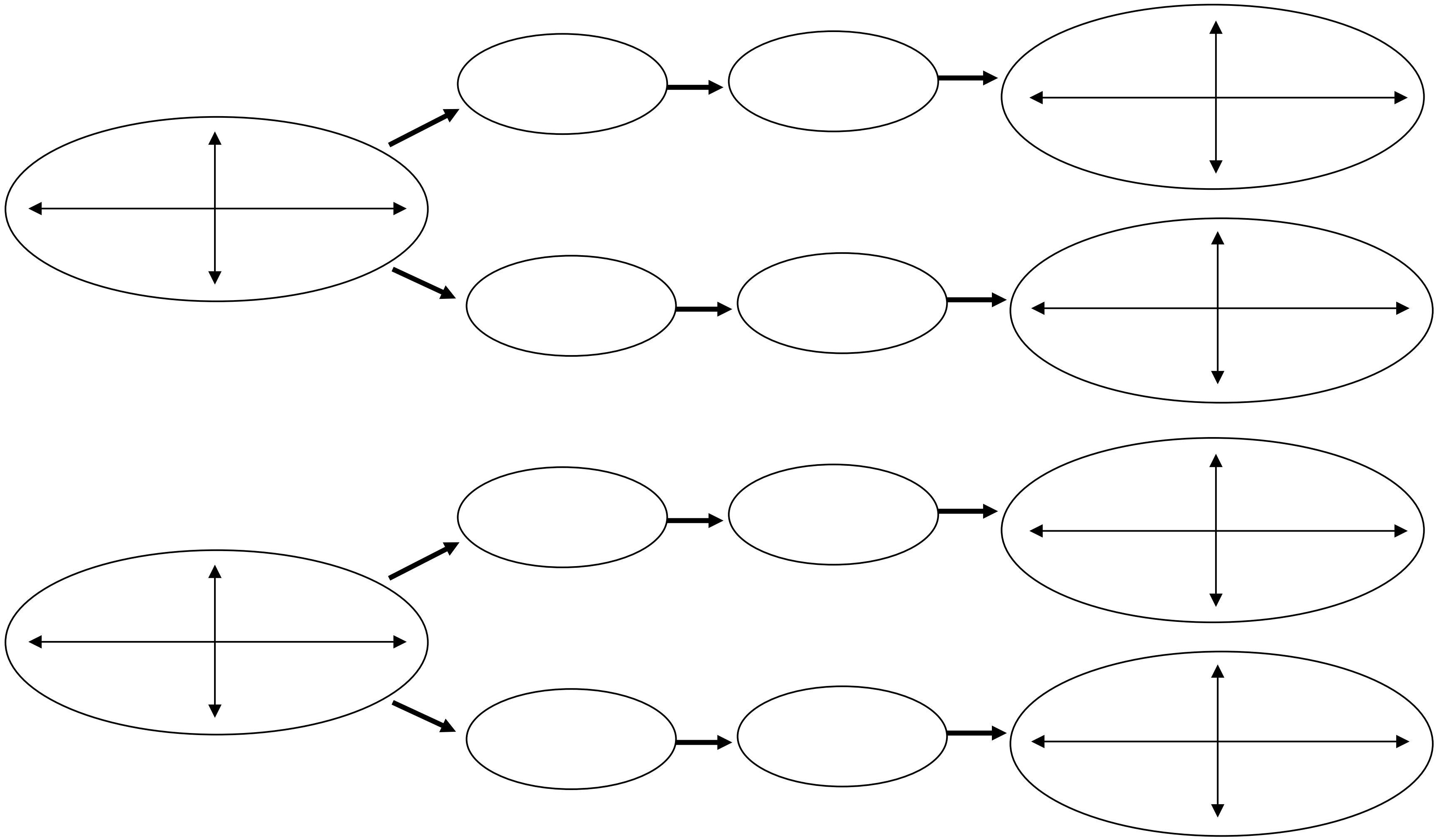
Example Functions to Graph and Discuss:

$f(x) = 2x + 3$ $\qquad$ $f(x) = 2^x + 3$

This page intentionally left blank

Graphic Organizer: Graphing Transformations

Parent Function and Graph | **Equation** | **Transformation** | **Graph**



Graphic Organizer by Dale Graham and Linda Meyer Thomas County Central High School; Thomasville GA

FORMATIVE ASSESSMENT LESSONS (FAL)

Formative Assessment Lessons are intended to support teachers in formative assessment. They reveal and develop students' understanding of key mathematical ideas and applications. These lessons enable teachers and students to monitor in more detail their progress towards the targets of the standards. They assess students' understanding of important concepts and problem solving performance, and help teachers and their students to work effectively together to move each student's mathematical reasoning forward.

More information on Formative Assessment Lessons may be found in the Comprehensive Course Overview.

SPOTLIGHT TASKS

A Spotlight Task has been added to each CCGPS mathematics unit in the Georgia resources for middle and high school. The Spotlight Tasks serve as exemplars for the use of the Standards for Mathematical Practice, appropriate unit-level Common Core Georgia Performance Standards, and research-based pedagogical strategies for instruction and engagement. Each task includes teacher commentary and support for classroom implementation. Some of the Spotlight Tasks are revisions of existing Georgia tasks and some are newly created. Additionally, some of the Spotlight Tasks are 3-Act Tasks based on 3-Act Problems from Dan Meyer and Problem-Based Learning from Robert Kaplinsky.

3-ACT TASKS

A Three-Act Task is a whole group mathematics task consisting of 3 distinct parts: an engaging and perplexing Act One, an information and solution seeking Act Two, and a solution discussion and solution revealing Act Three.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Overview.

TASKS

Task Name ***Suggested Time***	**Task Type** ***Grouping Strategy***	**Content Addressed**	**Standards**
**Talk is Cheap! (Spotlight Task) *45 min* *This task has been revised as a Spotlight Task*	Scaffolding Task *Individual/Partner*	• Graph linear functions • Use the graphing calculator to find the intersection of two linear functions • Interpret the intersection in terms of the problem situation • Compare functions represented algebraically, graphically, and in tables	• A.REI.10 • A.REI.11 • F.IF.2 • F.IF.7 • F.IF.9 • F.BF.1
Functioning Well *30 min*	Practice Task *Individual/Partner*	• Use function notation • Interpret statements that use function notation in terms of context • Recognize that sequences are functions	• F.IF.1 • F.IF.2
Skeleton Tower *20-30 minutes* PDF	Short Cycle Task *Partner / Small Group*	• Find, extend and describe mathematical patterns	• F.BF.1a, b • F.BF.2
*Detention Hall Buy Out (Spotlight Task) *60 minutes*	Constructive Task *Small Group*	• Model and solve problems involving the intersection of two straight lines. • Interpret the intersection in terms of the problem situation • Compare functions represented algebraically, graphically, and in tables	• A.REI.10 • A.REI.11 • F.IF.2 • F.IF.7 • F.IF.9 • F.BF.1
Comparing Investments (FAL) *≈ 2 hours* PDF	Formative Assessment Lesson *Partner / Small Group*	• Translate between descriptive, algebraic, and tabular data, and graphical representation of a function. • Recognizing how, and why, a quantity changes per unit interval.	• F.IF.1a, b, c • F.IF.2 • F.IF.3 • F.IF.5
Best Buy Tickets *20-30 minutes* PDF	Short Cycle Task *Partner / Small Group*	• Students use linear models to compare two purchasing options	• F.BF.1a, b • F.BF.2
Multiplying Cells *30-45 minutes* PDF	Short Cycle Task *Partner / Small Group*	• Use exponential functions to model real-life situations	• F.BF.1a, b • F.BF.2 • F.LE.1a, b, c • F.LE.2 • F.LE.3 • F.LE.5

Task Name *Suggested Time*	**Task Type** *Grouping Strategy*	**Content Addressed**	**Standards**
You're Toast, Dude! *30-45 minutes*	Scaffolding Task *Partner/Small Group*	• Use function notation • Interpret functions that arise in applications in terms of context • Analyze functions using different representations • Build a function that models a relationship between two quantities	• F.IF.2 • F.IF.4 • F.IF.7 • F.BF.1 • A.REI.11
Community Service, Sequences, and Functions *90 minutes*	Performance Task *Individual or Partner*	• Recognize that sequences are functions sometimes defined recursively • Use technology to graph and analyze functions • Convert a recursive relationship into an explicit function • Construct linear and exponential function (including reading these from a table) • Observe the difference between linear and exponential functions	• F.IF.3 • F.IF.5 • F.LE.1b, c • F.LE.2 • F.LE.3 • F.BF.2
Having Kittens (FAL) *≈ 2 hours* PDF	Formative Assessment Lesson *Partner / Small Group*	• Interpret a situation and represent the constraints and variables mathematically. • Select appropriate mathematical methods to use. • Make sensible estimates and assumptions. • Investigate an exponentially increasing sequence.	• F.LE.1a, b, c • F.LE.2 • F.LE.3
Building and Combining Functions *90 minutes*	Learning Task *Partner/Small group*	• Calculate and interpret rate of change • Combine functions • Write explicit function rules	• F.IF.6 • F.LE.1b • F.BF.1
Interpreting Functions *20-30 minutes* PDF	Short Cycle Task *Partner / Small Group*	• Interpret graphs of functions in context	• F.IF.1 • F.IF.2 • F.IF.3 • F.IF.4 • F.IF.5 • F.IF.6 • F.IF.7a, e • F.IF.9
*Birthday Gifts and Turtle Problem (FAL)	Formative Assessment Lesson *Individual / Small Group*	• Understand the rates of change of linear functions are constant, while the rates of change of exponential functions are not constant	• F.LE.1 • F.LE.2 • F.LE.3
*Exploring Paths	Formative Assessment Lesson *Individual / Small Group*	• Reasoning qualitatively, compare linear and exponential models verbally, numerically, algebraically, and graphically.	• F.BF.1 • F.LE.1

Task Name ***Suggested Time***	**Task Type** ***Grouping Strategy***	**Content Addressed**	**Standards**
High Functioning! *90-120 minutes*	Practice Task *Individual*	• Find the value of k given the graphs • Identify even and odd function • Relate vertical translations of a linear function to its y-intercept	• F.BF.3
Summing it up: Putting the “Fun” in Functions *3-4 days*	Culminating Task *Individual/Partner*	• Understand the concept of a function and use function notation • Interpret functions that arise in context • Analyze functions using different representations • Building new functions from existing functions • Construct and compare linear and exponential models and solve problems	• All

**Talk Is Cheap! (Spotlight Task)

The original Talk is Cheap task was featured in the CCGPS Coordinate Algebra Unit 3 Frameworks. This Spotlight version is designed to open the task to multiple levels of learners by allowing students to formulate their own questions based on a given bit of information. This task will allow easy access and high scalability for differentiation.

Introduction

This task can be used to introduce students to functions in a realistic setting—choosing a cell phone plan given certain conditions. Students gain experience working with decimals and translating among different representations of linear functions. They use the graphing calculator to find the intersection of two linear functions graphically and interpret the intersection in terms of the problem situation.

Essential Questions

- Why is the concept of a function important?
- How do I use function notation to show a variety of situations modeled by functions?
- How do I interpret expressions for functions in terms of the situation they model?

Common Core Georgia Performance Standards

MCC9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). ***(Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.)***

MCC9-12.A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, ~~polynomial, rational, absolute value~~, exponential, and ~~logarithmic~~ functions.

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. ***(Draw examples from linear and exponential functions.)***

MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. ***(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)***

MCC9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). ***(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)***

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. ***(Limit to linear and exponential functions.)***

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
   Students will have to make multiple tables and graphs to demonstrate the relationship. Students must make sense of the problem by identifying what information they need to solve it.
2. **Reason abstractly and quantitatively**. Students were asked to make an estimate (high and low).

3. **Construct viable arguments and critique the reasoning of others.** After writing down their own question, students discussed their question with tablemates, creating the opportunity to construct the argument of why they chose their question, as well as critiquing the questions that others came up with.

4. Model with mathematics. Students will create linear functions representing payment plans for cell phones.

5. Use appropriate tools strategically. Students may find points of intersection using graphing calculators

MATERIALS REQUIRED Graph paper and/or graphing calculator

TIME NEEDED 30 minutes to one hour depending on the depth of questioning

TEACHER NOTES

In this task, students will read the brief amount of information provided, and then discuss what they noticed. They will then be asked to formulate questions about what they wonder or are curious about. These questions will be recorded on the board and on student recording sheet. Students will then use mathematics to answer their own questions.

More information along with guidelines for 3-Act Tasks may be found in the Comprehensive Course Guide.

Act 1: The Information:

- Talk Fast cellular phone service charges $0.10 for each minute the phone is used.
- Talk Easy cellular phone service charges a basic monthly fee of $18.00 plus $0.04 for each minute the phone is used.
- Both plans charge $5.00 per month for unlimited texting service.

  NOTE: Students might ask if the $5.00 per month charge for unlimited texting is included in the Talk Easy cellular basic monthly fee of $18.00. This question opens the door for more mathematical discussion: What difference will that (including the $5.00) make on lining up the prices? For the purposes of the solution listed at the end of this task, the $5.00 fee was in addition to the $18.00 fee.

Act 2: The Investigation:

During Act 2 Students will come up with questions that interest them relative to the two cell phone carriers. They will then use various methods of solving systems of linear equations to answer the questions they posed. Recall in 8^{th} grade an in Unit 2 of CCGPS Coordinate Algebra students used graphing, substitution, and linear combination (elimination) methods to solve systems of linear equations. Students should be encouraged to develop the function notation for each of the cellular carriers (**MCC9-12.F.IF.2)** and follow with the graphing of these functions (**MCC9-12.F.IF.7).** This task lends well to discussion of parameters or feasible solutions and to compare the properties of two functions (**MCC9-12.F.IF.9)** in a "real world" setting. Such questions as: "Does it make sense to consider values below zero for the number of minutes? And "How much would you (the student) think is "reasonable" to spend on a cell phone plan for a month?" should be part of small group or class discussions as deemed appropriate.

Act 3: The Reveal:

During Act 3 Students will discuss, present, or defend their solutions to the problem(s) they investigated in Act 2.
Note: The solution will depend on the question asked by the student. If they decided to find out when the plans cost the same, they would find that for 300 minutes each plan will cost $35.00. Prior to 300 minutes of talk, the plan with $0.10 per minute charge (Talk Fast) will be cheaper but after the 300 minute mark, the plan for $0.04 per minute (Talk Easy) will be the better buy.
A template for working through the task is presented on the next page.

Task Title:________________________ Name:________________________

Adapted from Andrew Stadel

ACT 1

What did/do you notice?

What questions come to your mind?

Main Question:____

Estimate how

	Your estimate	Estimate (too low)	Estimate (too high)

ACT 2

What information would you like to know or do you need to solve the MAIN question?

Record the given information (measurements, materials, etc…)

If possible, give a better estimate using this information:_______________________________

Act 2 (con't)

Use this area for your work, tables, calculations, sketches, and final solution.

ACT 3

What was the result?

Which Standards for Mathematical Practice did you use?	
□ Make sense of problems & persevere in solving them	□ Use appropriate tools strategically.
□ Reason abstractly & quantitatively	□ Attend to precision.
□ Construct viable arguments & critique the reasoning of others.	□ Look for and make use of structure.
□ Model with mathematics.	□ Look for and express regularity in repeated reasoning.

Functioning Well! (Practice Task)

Introduction

This task is designed to allow students to practice working with functions prior to completing rigorous integrated tasks that require them to interpret, analyze, build, construct, and compare linear and exponential functions to solve problems and model real-world situations.

Mathematical Goals

- Understand the domain and range, notation, and graph of a function
- Use function notation
- Interpret statements that use function notation in terms of context
- Recognize that sequences are functions

Essential Questions

- How do I represent real life situations using function notation?

Common Core Georgia Performance Standards

MCC9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
Students must describe real life situations using function notation.

3. Construct viable arguments and critique the reasoning of others.
Students must defend why different representations of relations are functions.

Background Knowledge

- Students understand the difference between a relation and a function.
- Students can read and interpret graphs.
- Students can use function notation to relate inputs and outputs.

Common Misconceptions

- Students may confuse inputs and outputs. Students need to know how a function in defined in terms of inputs and outputs.
- Students may not see the distinction between adding a constant to the input or to the output of a function. They may not understand how that changes the application in a real-life situations.

Materials
- None

Grouping
- Partner / Individual

Differentiation

Extension:
- Create your own situations showing relations that are or are NOT functions. Show a verbal, graphical and numeric example.

Intervention:
- Use strategic grouping.
- Provide manipulatives.

Formative Assessment Questions
- What is a function and what are the different ways it can be expressed?
- How are arithmetic operations of functions similar/different to operations on real numbers?

<u>Functioning Well – Teacher Notes</u>

Consider the definition of a function (A function is a *rule* that assigns each element of set A to a *unique* element of set B. It may be represented as a set of ordered pairs such that no two ordered pairs have the same first member, i.e. each element of a set of inputs (the domain) is associated with a unique element of another set of outputs (the range)).

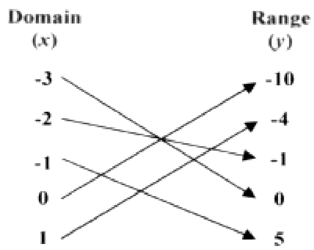
Part I – Function or Not

Determine whether or not each of the following is a function or not. Write "function" or "not a function" and explain why or why not.

<u>Comment:</u>
Make sure students explain their reasoning.

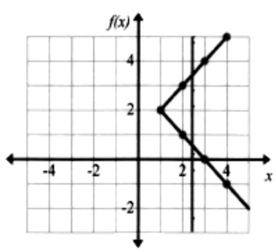
Relation | **Answer and Explanation**

1.



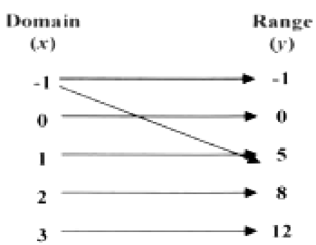
<u>Solution:</u>
Function. By following each arrow from an x (domain) to each y (range), you can see this is a function. Each x has only one y it connects to, which by definition is a function.

2.



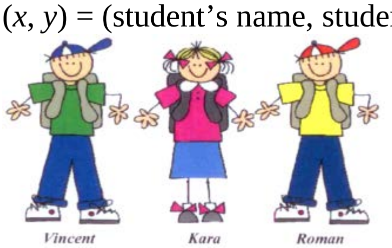
<u>Solution:</u>
Not a function. This graph fails the vertical line test. Also, some of the x's (2, 3, 4) are connected to more than one y each. The graph demonstrates one input is generating more than one output.

3.



<u>Solution:</u>
Not a function. An input of –1 sometimes gives an output of -1 and other times gives an output of 5. Therefore, there is no consistent rule and cannot be a function.

4. (x, y) = (student's name, student's shirt color)



<u>Solution:</u>
(x, y) = (student's name, shirt color) Function. Students may explain that each name is paired with a unique shirt color.

Part II – Function Notation

Suppose a restaurant has to figure the number of pounds of fresh fish to buy given the number of customers expected for the day. Let $p = f(E)$ where p is the pounds of fish needed and E is the expected number of customers.

5. What would the expressions $f(E + 15)$ and $f(E) + 15$ mean?

Solution:

These two expressions are similar in that they both involve adding 15. However, for f(E + 15), the 15 is added on the inside, so 15 is added to the number of customers expected. Therefore, f(E + 15) gives the number of pounds of fish needed for 15 extra customers. The expression f(E) + 15 represents an outside change. We are adding 15 to f(E), which represents pounds of fish, not expected number of customers. Therefore, f(E) + 15 means that we have 15 more pounds of fish than we need for E expected customers.

6. The restaurant figured out how many pounds of fish needed and bought 2 extra pounds just in case. Use function notation to show the relationship between domain and range in this context.

Solution:

$p = f(E) + 2$

7. On the day before a holiday when the fish markets are closed, the restaurant bought enough fish for two nights. Using function notation, illustrate how the relationship changed.

Solution:

$p = 2f(E)$

8. The owner of the restaurant planned to host his 2 fish-loving parents for dinner at the restaurant. Illustrate using function notation

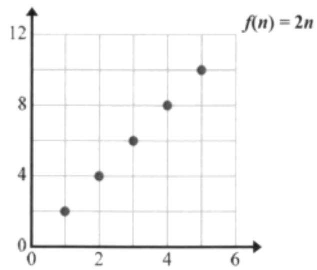
Solution:

$p = f(E + 2)$

Part III – Graphs are Functions

Write each of the points using function notation.

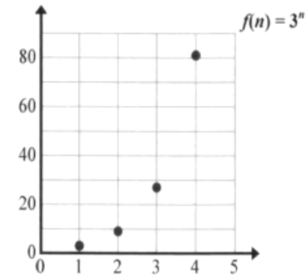
9.



Solution

$f(1) = 1$; $f(2) = 2$; $f(3) = 3$;
$f(4) = 4$; $f(5) = 5$

10.



Solution

$f(1) = 3$; $f(2) = 9$; $f(3) = 27$;
$f(4) = 81$; $f(5) = 243$

References: Jordan-Granite Consortium (2012). http://secmathccss.wordpress.com

Practice Task: Functioning Well

Name________________________________ **Date_________________**

Mathematical Goals
- Understand the domain and range, notation, and graph of a function
- Use function notation
- Interpret statements that use function notation in terms of context
- Recognize that sequences are functions

Essential Questions
- How do I represent real life situations using function notation?

Common Core Georgia Performance Standards

MCC9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

Standards for Mathematical Practice
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

Practice Task: Functioning Well

Name_________________________________ **Date__________________**

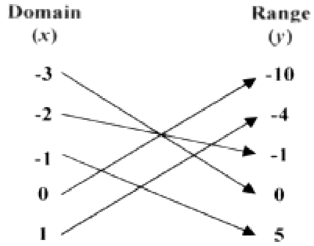
Consider the definition of a function (A function is a *rule* that assigns each element of set A to a *unique* element of set B. It may be represented as a set of ordered pairs such that no two ordered pairs have the same first member, i.e. each element of a set of inputs (the domain) is associated with a unique element of another set of outputs (the range)).

Part I – Function or Not

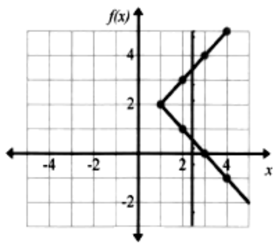
Determine whether or not each of the following is a function or not. Write "function" or "not a function" and explain why or why not.

Relation	Answer and Explanation

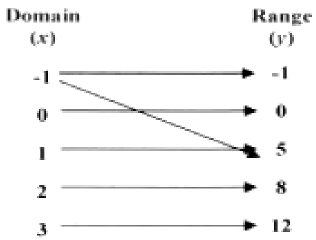
1.



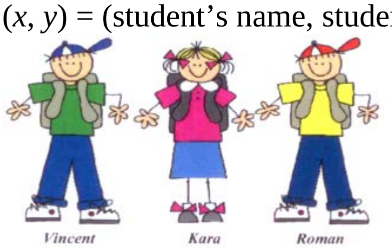
2.



3.



4. (x, y) = (student's name, student's shirt color)



Part II – Function Notation

Suppose a restaurant has to figure the number of pounds of fresh fish to buy given the number of customers expected for the day. Let $p = f(E)$ where p is the pounds of fish needed and E is the expected number of customers.

5. What would the expressions $f(E+15)$ and $f(E) + 15$ mean?

6. The restaurant figured out how many pounds of fish needed and bought 2 extra pounds just in case. Use function notation to show the relationship between domain and range in this context.

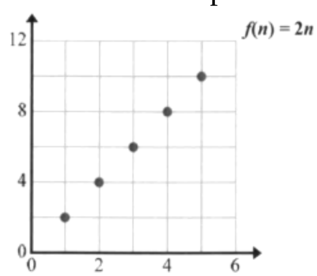
7. On the day before a holiday when the fish markets are closed, the restaurant bought enough fish for two nights. Using function notation, illustrate how the relationship changed.

8. The owner of the restaurant planned to host his 2 fish-loving parents in addition to his expected customers for dinner at the restaurant. Illustrate using function notation

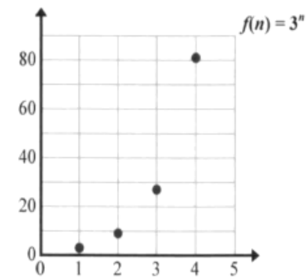
Part III – Graphs are Functions

Write each of the points using function notation.

9.



10.



Skeleton Tower (Short Cycle Task)

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=810

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Skeleton Tower*, is a Mathematics Assessment Project Assessment Task that can be found at the website: http://www.map.mathshell.org/materials/tasks.php?taskid=279&subpage=expert

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=810

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=811

Mathematical Goals

- Find, extend, and describe mathematical patterns.

Essential Questions

- How do I find, extend, and describe mathematical patterns?

Common Core Georgia Performance Standards

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1b Combine standard function types using arithmetic operations. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Background Knowledge

- Students can represent sequences algebraically.

Common Misconceptions

- Students may confuse geometric and arithmetic sequences.
- Students may think about sequences recursively but incorrectly write their pattern as explicit formulas.

Materials

- see FAL website

Grouping

- Individual / small group

*The Detention Buy-Out (Spotlight Task)

"Detention Hall Buy Out" originally accessed at http://tapintoteenminds.com/real-world-math/exploring-linear-relationships-and-patterning/ involves setting up and solving a linear system in an engaging context. This task includes video support along with a handout for students support.

Estimated Task Time: One Hour

Key Standards Addressed:

MCC9-12.A.REI.10 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). (Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.)

MCC9-12.A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

MCC9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities.

Standards for Mathematical Practice
1. Make sense of problems and persevere in solving them: analyze a real-world situation and make a connection to prior knowledge.
4. Model with mathematics: deconstruct the problem and use prior knowledge to create a solution to the problem.
5. Use appropriate tools strategically: show adequate steps to clearly demonstrate understanding using a variety of methods.

The Task is listed below and has been adapted from http://tapintoteenminds.com/real-world-math/exploring-linear-relationships-and-patterning/.

Part 1: Students will watch a video called The Detention Buy-Out. In the video, three administrators from Tecumseh Vista Academy K-12 School are interviewed and propose individual options for students to avoid serving detentions by paying the administrators according to their buy-out offers.

Part 2: After watching the video Students will then be split into groups of 2 or 3, to determine which administrator should each student buy-out from.

Encourage students to show their solution in any way they would like or you can assign certain methods to particular groups.
The exploring linear relationships problem can be solved in a number of ways which increase the scalability of this task and provide opportunities for multiple methods including:

- Trial and error / guess and check
- Table of values
- Graphing to find point of intersection
- Creating equations and substitute different values of ***x***
- Solving a system of equations using elimination
- Solving the system of equations using substitution

Part 3: Students should report the results of their exploration with supporting evidence from their method of solving the problem.

Advertisement Picture for Detention Hall Buy Out
http://i0.wp.com/tapintoteenminds.com/wp-content/uploads/2014/05/The-Detention-Buy-Out-Which-administrator-should-they-buy-out-from.png

Link to actual PDF https://dl.dropboxusercontent.com/u/7108182/TVAMathletes/mfm1p/TVADetentionBuy-Out.pdf

Comparing Investments (Formative Assessment Lesson)

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1250

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Comparing Investments*, is a Formative Assessment Lesson (FAL) that can be found at the website:
http://map.mathshell.org/materials/lessons.php?taskid=426&subpage=concept

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1250

Mathematical Goals

- Translate between descriptive, algebraic, and tabular data, and graphical representation of a function.
- Recognize how, and why, a quantity changes per unit interval.

Essential Questions

- How do you relate real-life problems to linear or exponential models?

Common Core Georgia Performance Standards

MCC9-12.F.LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.
MCC9-12.F.LE.1a	Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
MCC9-12.F.LE.1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MCC9-12.F.LE.1c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MCC9-12.F.LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MCC9-12.F.LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

MCC9-12.F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. *(Limit exponential functions to those of the form $f(x) = b^x + k$.)*

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
4. Model with mathematics.
7. Look for and make use of structure.

Background Knowledge

- Students can compute simple and compound interest.
- Students understand linear and exponential models.

Common Misconceptions

- Students may confuse the formulas and meanings of simple and compound interest.

Essential Questions

- How can I use linear models to decide which of the two payment models is cheaper?

Materials

- see FAL website

Grouping

- Individual / partners

Best Buy Tickets (Short Cycle Task)

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=824

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Best Buy Tickets*, is a Mathematics Assessment Project Assessment Task that can be found at the website: http://www.map.mathshell.org/materials/tasks.php?taskid=286&subpage=expert

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=824

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=825

Mathematical Goals

- Students can use linear models to compare two purchasing options.

Essential Questions

- How can I use linear models to decide which of the two payment models is cheaper?

Common Core Georgia Performance Standards

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1b Combine standard function types using arithmetic operations. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.

Background Knowledge

- Students understand the meaning of slope and *y*-intercept in context when writing linear equations.
- Students may need to know how to solve a system of linear equations, depending on the solution path they follow.

Common Misconceptions

- Students may confuse the slope and the *y*-intercept of a linear equation.
- Students may fail to realize that the answer to the “which is the better buy” question depends on the number of people who attend.

Materials

- see FAL website

Grouping

- Individual / partners

Multiplying Cells (Short Cycle Task)

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=788

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Multiplying Cells*, is a Mathematics Assessment Project Assessment Task that can be found at the website: http://www.map.mathshell.org/materials/tasks.php?taskid=268&subpage=apprentice

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=788

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=789

Mathematical Goals

- Use exponential functions to model real-world situations.

Essential Questions

- How can I use exponential functions to model real-world situations?

Common Core Georgia Performance Standards

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1b Combine standard function types using arithmetic operations. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

MCC9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

MCC9-12.F.LE.1a Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

MCC9-12.F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

MCC9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

MCC9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MCC9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, ~~quadratically, or (more generally) as a polynomial function.~~

MCC9-12.F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. *(Limit exponential functions to those of the form $f(x) = b^x + k$.)*

Standards for Mathematical Practice

3. Construct viable arguments and critique the reasoning of others.
7. Look for and make use of structure.

Background Knowledge

- Students can work with exponents.
- Students recognize exponential relationships.

Common Misconceptions

- Students may think about sequences recursively but incorrectly write their pattern as explicit formulas.
- Students may interpret 2^3 as $2 \cdot 3$, or they may believe the growth is linear.

Materials

- see FAL website

Grouping

- Individual / partner

You're Toast, Dude! (Scaffolding Task)

Introduction

Students extend their understanding of functions. Students will gain experience in moving between a problem context and its mathematical model in order to solve problems and make decisions.

Mathematical Goals

- Use function notation
- Interpret functions that arise in applications in terms of context
- Analyze functions using different representations
- Build a function that models a relationship between two quantities

Essential Questions

- How do I interpret functions that arise in applications in terms of context?

Common Core Georgia Performance Standards

MCC9-12.A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, ~~polynomial, rational, absolute value~~, exponential, and ~~logarithmic~~ functions.

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~. *(Focus on linear and exponential functions.)*

MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. *(Limit to linear and exponential functions.)*

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.

4. Model with mathematics.
Students will express real life situations about toaster cost and production algebraically.

5. Use appropriate tools strategically.
Students will be aided by graphing calculators.

Background Knowledge

- understanding slope and y-intercept in building a linear function
- using function notation to answer contextual problems
- basic graphing calculator skills (if available)

Common Misconceptions

- For #5, students may neglect dividing the whole expression by x.
- Students may have difficulty finding the correct parameters for their graphing calculator window.

Materials

- Graphing Calculator (optional)

Grouping

- Partner / Individual

Differentiation

Extension:

- See teacher comments for #6c.

Intervention:

- Demonstrate graph for #6 as a whole group.

Formative Assessment Questions

- How can we use function notation to describe real-life situations?
- How can we use operations on functions to solve problems?

You're Toast, Dude! – Teacher Notes

Comment
#6 asks students to graph a rational function. If graphing calculators are not available, teachers can demonstrate the graph to number 6 as a whole group. For students with stronger graphing skills, they make a table and look for the pattern as number of toasters increases.

At the You're Toast, Dude! toaster company, the weekly cost to run the factory is $1400 and the cost of producing each toaster is an additional $4 per toaster.

1. Write a function rule representing the weekly cost in dollars, $C(x)$, of producing x toasters.

   Solution:
   C(x) = 4x + 1400

2. What is the total cost of producing 100 toasters in one week?

   Solution:
   C(100) = 4(100) + 1400 = 1800. It will cost $1800 to produce 100 toasters in one week.

3. If you produce 100 toasters in one week, what is the total production cost per toaster?

   Solution:
   1800 / 100 = 18. If 100 toasters are produced the total production cost per toaster is $18.

4. Will the total production cost per toaster always be the same? Justify your answer.

   Solution:
   No. Justifications may vary. If 200 toasters are produced in one week, the cost is C(200) = 4(200) + 1400 = 2200. The total production cost is $2200 / 200 = $11 per toaster. Since $11 does not equal $18, the total production cost per toaster is not the same when the number of toasters produced varies.

5. Write a function rule representing the total production cost per toaster $P(x)$ for producing x toasters.

   Solution:
   P(x) = (4x + 1400) / x or P(x) = 4 + (1400/x) or P(x) = C(x)/x

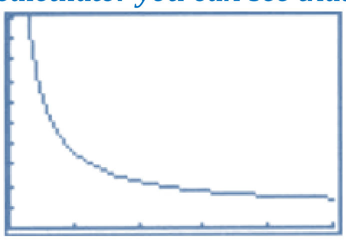
6. Using your graphing calculator, create a graph of your function rule from question 5. Use either the graph or algebraic methods to answer the following questions:

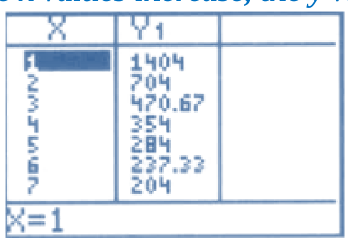
a. What is the production cost per toaster if 300 toasters are produced in one week? If 500 toasters are produced in one week?

Solution:
If 300 toasters are produced, the total production cost per toaster is $8.67.
If 500 toasters are produced, the total production cost per toaster is $6.80.

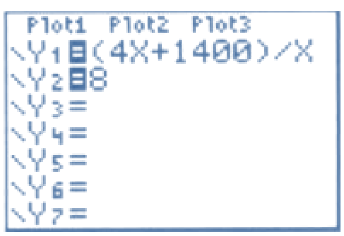
b. What happens to the total production cost per toaster as the number of toasters produced increases? Explain your answer.

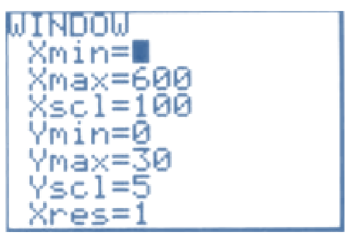
Solution:
As the number of toasters produced increases, the total production cost per toaster decreases. Looking at the graph will show that as x increases, y decreases. In addition, looking at the table of values in the graphing calculator you can see that as the x values increase, the y values decrease.

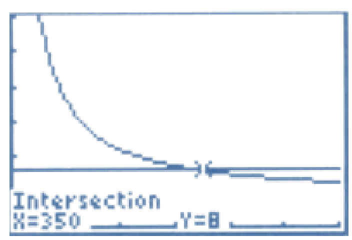




c. How many toasters must be produced to have a total production cost per toaster of $8?







Solution:
The viewing window above shows that the intersection of y = 4 + (1400/x) and y = 8 is the point (350, 8). Therefore, when 350 toasters are produced in one week, the total production cost per toaster is $8.

Possible Extension:

Have students discuss the mathematical parameters/limitations of x, C(x), and P(x) compared to real-life limitations of the mathematical model for the particular situation. Ask students to consider what happens at extremely large or small values of x, extending the mathematical discussion to asymptotes. Ask students to find the asymptotes of the production cost per toaster. As the number of toasters, x, nears 0, the total production cost per toaster gets larger. For example, when x = 10 toasters, the total production cost per toaster is $144. When x = 5, the total production cost per toaster is $284. When x = 0 the function cannot be solved for P(x).

As the number of toasters, x, becomes larger and larger, the average cost per toaster nears $4. For example, when x = 5,000 toasters, the average cost per toaster is $4.28. When P(x) = 4 the function cannot be solved for x. The horizontal asymptote of the function is y = 4, and the vertical asymptote is x = 0.

References

Charles A. Dana Center (2012). http://www.utdanacenter.org/mathtoolkit

Scaffolding Task: You're Toast, Dude!

Name__________________________________ **Date**___________________

Mathematical Goals
- Use function notation
- Interpret functions that arise in applications in terms of context
- Analyze functions using different representations
- Build a function that models a relationship between two quantities

Essential Questions
- How do I interpret functions that arise in applications in terms of context?

Common Core Georgia Performance Standards

MCC9-12.A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, ~~polynomial, rational, absolute value~~, exponential, and ~~logarithmic~~ functions.

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~. *(Focus on linear and exponential functions.)*

MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. *(Limit to linear and exponential functions.)*

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
4. Model with mathematics.
5. Use appropriate tools strategically.

Scaffolding Task: You're Toast, Dude!

Name________________________________ **Date**_________________

At the You're Toast, Dude! toaster company, the weekly cost to run the factory is $1400 and the cost of producing each toaster is an additional $4 per toaster.

1. Write a function rule representing the weekly cost in dollars, $C(x)$, of producing x toasters.

2. What is the total cost of producing 100 toasters in one week?

3. If you produce 100 toasters in one week, what is the total production cost per toaster?

4. Will the total production cost per toaster always be the same? Justify your answer.

5. Write a function rule representing the total production cost per toaster $P(x)$ for producing x toasters.

6. Using your graphing calculator, create a graph of your function rule from question 5. Use either the graph or algebraic methods to answer the following questions:

a. What is the production cost per toaster if 300 toasters are produced in one week? If 500 toasters are produced in one week?

b. What happens to the total production cost per toaster as the number of toasters produced increases? Explain your answer.

c. How many toasters must be produced to have a total production cost per toaster of $8?

Community Service, Sequences, and Functions (Performance Task)

Introduction

In this task, students will explore the relationship between arithmetic and geometric sequences and exponential functions. They will convert a recursive relationship into an explicit function.

Mathematical Goals

- Recognize that sequences are functions sometimes defined recursively
- Use technology to graph and analyze functions
- Convert a recursive relationship into an explicit function
- Construct linear and exponential function (including reading these from a table)
- Observe the difference between linear and exponential functions

Essential Questions

- How are sequences and functions related? How can I model one with the other?

Common Core Georgia Performance Standards

MCC9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *(Draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences.)*

MCC9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *(Focus on linear and exponential functions.)*

MCC9-12.F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

MCC9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

MCC9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MCC9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, ~~quadratically, or (more generally) as a polynomial function.~~

MCC9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Standards for Mathematical Practice

4. Model with mathematics.
Students will create sequences from context and model them with tables and equations.

7. Look for and make use of structure.
Students will use tables to formulate equations.

8. Look for and express regularity in repeated reasoning.
Students will recognize patterns within sequences.

Background Knowledge

- Students understand sequences as functions.
- Students can use and write explicit and recursive formulas for sequences.

Common Misconceptions

- Students may confuse explicit and recursive formulas and the parts that make them up.

Materials

- None

Grouping

- Partner / Individual

Differentiation

Extension:

- Have students write explicit and recursive formulas for the amount of money collected.
  #7: $a_n = 5n$; $a_n = a_{n-1} + 5$
  #8: $a_n = 5(2)^{n-1}$; $a_n = 2a_n$

Intervention:

- Provide students with two rows in the table
- Give formulas for the sequences.

Formative Assessment Questions

- How are sequences related to functions?
- What types of real-life situations can be modeled with functions?

Community Service, Sequences, and Functions – Teacher Notes

Comment:
Activities that require students to practice completing geometric and arithmetic sequences and generate an explicit and recursive formula from those sequences should occur prior to completing this task.

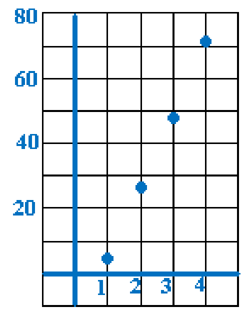
Larry, Moe, and Curly spend their free time doing community service projects. They would like to get more people involved. They began by observing the number of people who show up to the town cleanup activities each day. The data from their observations is recorded in the given table for the Great Four Day Cleanup.

x	*y*
1	5
2	27
3	49
4	71

1. Give a verbal description of what the domain and range presented in the table represents.

Solution:
The domain is the number of days. The range represents the number of people that showed up each day.

2. Sketch the data on the grid below.



3. Determine the type of function modeled in the graph above and describe key features of the graph.

Solution:
Answers may vary. The graph models a linear function. The sequence represents discrete data. Looking at the graph, the pattern appears to be increasing at a constant rate of change.

4. Based on the pattern in the data collected, what recursive process could Larry, Curly, and Moe write?

Solution:
$a_1 = 5, a_n = a_{n-1} + 22$

5. Write a linear equation to model the function.

Solution:
Students could answer in the form of the explicit formula, $a_n = 5 + 22(n - 1)$, or in slope-intercept form $f(x) = 22x - 17$. Students should understand the relationship between these two forms.

6. How would Larry, Curly, and Moe use the explicit formula to predict the number of people who would help if the cleanup campaign went on for 7 days?

Solution:
By evaluating f(x) = 22x – 17 (or $a_n = 5 + (n-1)22$) substituting 7 for the x value in the explicit formula, they could predict that 22(7) –17 or 137 people will show up on day 7 if the cleanup campaign continued.

Excited about the growing number of people participating in community service, Larry, Curly, and Moe decide to have a fundraiser to plant flowers and trees in the parks that were cleaned during the Great Four Day cleanup. It will cost them $5,000 to plant the trees and flowers. They decide to sell some of the delicious pies that Moe bakes with his sisters. For every 100 pies sold, it costs Moe and his sisters $20.00 for supplies and ingredients to bake the pies. Larry, Curly, and Moe decide to sell the pies for $5.00 each.

7. Complete the following table to find the total number of pies sold and the amount of money the trio collects.

a. On Day 1, each customer buys the same number of pies as his customer number. In other words the first customer buys 1 pie, the second customer buys 2 pies. Fill in the table showing the number of pies and the amount collected on Day 1. Then calculate the total number of pies sold and dollars collected.

b. Write a recursive and explicit formula for the pies sold on Day 1. Explain your thinking.

Customer Number	*Number of Pies Sold*	*Amount Collected*
1	*1*	*$5*
2	*2*	*$10*
3	*3*	*$15*
4	*4*	*$20*
5	*5*	*$25*
6	*6*	*$30*
7	*7*	*$35*
8	*8*	*$40*
9	*9*	*$45*
10	*10*	*$50*
total	*55*	*$275*

Solution:
Since the number of pies sold to each customer is the same as the customer number, we have the explicit formula that $a_n = n$, where a_n is the number of pies and n is the customer number. We can also notice that the number of pies increases by one each time so $a_n = a_{n-1} + 1$, where a_n is the number of pies, is the recursive formula for the number of pies sold.

Extension:
To obtain the explicit formula for the amount collected, we can multiply the number of pies sold by 5. This gives us $a_n = 5n$, where a_n is the cost of the pies sold and n is the customer number.

c. On Day 2, the first customer buys 1 pie, the second customer buys 2 pies, the third customer buys 4 pies, the fourth customer buys 8 pies, and so on**.** Complete table based on the pattern established. Then calculate the total

Customer Number	*Number of Pies Sold*	*Amount Collected*
1	*1*	*$5*
2	*2*	*$10*

number of pies sold and dollars collected.

3	*4*	*$20*
4	*8*	*$40*
5	*16*	*$80*
6	*32*	*$160*
7	*64*	*$320*
8	*128*	*$640*
9	*256*	*$1280*
10	*512*	*$2560*
total	*1023*	*$5115*

d. Write a recursive and explicit formula for the pies sold on Day 2. Explain your thinking.

Solution:
Since the number of pies sold to each customer doubles each time, we have the explicit formula that $a_n = 2^{n-1}$, where a_n is the number of pies and n is the customer number. We also have that the recursive formula is $a_n = 2a_{n-1}$, where a_n is the number of pies.
Extension: To obtain the explicit formula for the amount collected, we can multiply the number of pies sold by 5. This gives us $a_n = 5(2)^{n-1}$, where a_n is the price of the pies and n is the customer number. Looking at a recursive pattern, we notice that the price column still doubles each time. This gives a recursive formula of $a_n = 2a_{n-1}$ for the price of pies, where a_n is the cost of the pies.

8. Compare the rates of change on Day 1 and Day 2 for the number of pies sold.

Solution:
Answers may vary.
Possible answer:
When the change in x is 1, Day 1 (linear) the change in y is a constant (slope).
On day two, (exponential) the y-values are multiplied by a constant ratio to get the succeeding y-value.

9. Did Larry, Curly, and Moe earn enough in two days to fund their project? Consider costs incurred to bake the pies. Justify your reasoning.

Solution:
There are two ways students might answer this question. a) Students assume that $20 per 100 pies is really $0.20 per pie OR b)students assume that they must purchase pies in 100 pie increments.
Therefore…
On the first day, they sold 55 pies and made $275. a) At a cost of $20 per 100 pies, they spent $11 on ingredients. This yields a profit of $264. OR b) They spent $20 on 100 pies, of which they sold 55. They yield a profit $255.
On the second day, they sold 1023 pies and made $5115. a) At a cost of $20 per 100 pies, they spent $204.60 on ingredients. Their profit on day two was $4910.40. OR b) They spent $220 on 1100 pies, of which they sold 1023 pies. They yield a profit of $4895.

Combining their profit from day 1 and day 2 yields a total of a) $5174.40 OR b) $5150. Therefore, the trio reached their project goal of $5000.

Performance Task: Community Service, Sequences, and Functions

Name________________________________ **Date**__________________

Mathematical Goals

- Recognize that sequences are functions sometimes defined recursively
- Use technology to graph and analyze functions
- Convert a recursive relationship into an explicit function
- Construct linear and exponential function (including reading these from a table)
- Observe the difference between linear and exponential functions

Essential Questions

- How are sequences and functions related? How can I model one with the other?

Common Core Georgia Performance Standards

MCC9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *(Draw connection to F.BF.2, which requires students to write arithmetic and gemetric sequences.)*

MCC9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *(Focus on linear and exponential functions.)*

MCC9-12.F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

MCC9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

MCC9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MCC9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, ~~quadratically, or (more generally) as a polynomial function.~~

MCC9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

Standards for Mathematical Practice

4. Model with mathematics.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Performance Task: Community Service, Sequences, and Functions

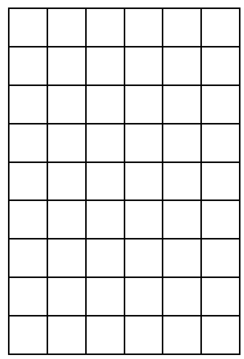
Name________________________________ **Date**_________________

Larry, Moe, and Curly spend their free time doing community service projects. They would like to get more people involved. They began by observing the number of people who show up to the town cleanup activities each day. The data from their observations is recorded in the given table for the Great Four Day Cleanup.

x	***y***
1	5
2	27
3	49
4	71

1. Give a verbal description of what the domain and range presented in the table represents.

2. Sketch the data on the grid below.



3. Determine the type of function modeled in the graph above and describe key features of the graph.

4. Based on the pattern in the data collected, what recursive process could Larry, Curly, and Moe write?

5. Write a linear equation to model the function.

6. How would Larry, Curly, and Moe use the explicit formula to predict the number of people who would help if the cleanup campaign went on for 7 days?

Excited about the growing number of people participating in community service, Larry, Curly, and Moe decide to have a fundraiser to plant flowers and trees in the parks that were cleaned during the Great Four Day cleanup. It will cost them $5,000 to plant the trees and flowers. They decide to sell some of the delicious pies that Moe bakes with his sisters. For every 100 pies sold, it costs Moe and his sisters $20.00 for supplies and ingredients to bake the pies. Larry, Curly, and Moe decide to sell the pies for $5.00 each.

7. Complete the following table to find the total number of pies sold and the amount of money the trio collects.

a. On Day 1, each customer buys the same number of pies as his customer number. In other words the first customer buys 1 pie, the second customer buys 2 pies. Fill in the table showing the number of pies and the amount collected on Day 1. Then calculate the total number of pies sold and dollars collected.

b. Write a recursive and explicit formula for the pies sold on Day 1. Explain your thinking.

Customer Number	*Number of Pies Sold*	*Amount Collected*
1	*1*	*$5*
2	*2*	*$10*
total		

c. On Day 2, the first customer buys 1 pie, the second customer buys 2 pies, the third customer buys 4 pies, the fourth customer buys 8 pies, and so on**.** Complete table based on the pattern established. Then calculate the total number of pies sold and dollars collected.

d. Write a recursive and explicit formula for the pies sold on Day 2. Explain your thinking.

Customer Number	*Number of Pies Sold*	*Amount Collected*
1	*1*	*$5*
2	*2*	*$10*
total		

8. Compare the rates of change on Day 1 and Day 2 for the number of pies sold.

9. Did Larry, Curly, and Moe earn enough in two days to fund their project? Consider costs incurred to bake the pies. Justify your reasoning.

Having Kittens (Formative Assessment Lesson)

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://map.mathshell.org/materials/download.php?fileid=1204

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=formative

The task, *Modeling: Having Kittens*, is a Formative Assessment Lesson (FAL) that can be found at the website: http://map.mathshell.org/materials/lessons.php?taskid=407&subpage=problem

The FAL document provides a clear lesson design, from the opening of the lesson to the closing of the lesson.

The PDF version of the task can be found at the link below:
http://map.mathshell.org/materials/download.php?fileid=1204

Mathematical Goals

- Interpret a situation and represent the constraints and variables mathematically.
- Select appropriate mathematical methods to use.
- Make sensible estimates and assumptions.
- Investigate an exponentially increasing sequence.

Essential Questions

- How can I use mathematical models to determine whether the poster's claim that one cat can have 2000 descendants in just 18 months is reasonable?

Common Core Georgia Performance Standards

MCC9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

MCC9-12.F.LE.1a Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

MCC9-12.F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

MCC9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

MCC9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MCC9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically

Background Knowledge

- The background knowledge required for this task is quite general. There are many entry points to this problem, all of which build from different types of background knowledge.

Common Misconceptions

- Students may forget that each new kitten can also have litters of its own after 4 months.
- Students must make assumptions in order to approach the problem. See discussion in "Solutions" of the FAL.

Materials

- see FAL website

Grouping

- Individual / small group

Building and Combining Functions (Learning Task)

Introduction

Students should be able to create a new function by using addition, subtraction, multiplication and division to combine two different linear and/or exponential functions. They will also need to be able to do simple compositions of functions. Discussions with students should include the impact on the domain and range when two functions are combined together. This can be done by looking at the graphs, the equations or even tables of values. Using a graphing calculator or other technology can be very useful when looking at the graphs of these combined functions. It will allow students to examine graphs that are more difficult to create by hand and might otherwise be inaccessible.

Mathematical Goals

- Calculate and interpret rate of change
- Combine functions
- Write explicit function rules

Essential Questions

- How can we use real-world situations to construct and compare linear and exponential models and solve problems?

Common Core Georgia Performance Standards

MCC9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *(Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers.)*

MCC9-12.F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1b Combine standard function types using arithmetic operations. *(Limit to linear and exponential functions.)*

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
Students will interpret patterns in data to determine relationships.

4. Model with mathematics.
Students will use knowledge of linear equations to model linear relationships in data.

Background Knowledge

- Students can apply properties of combining like terms and substitution.
- Students can interpret data.

Common Misconceptions

- This task requires students to use a more complex function notation. Students could get easily confused in the subscripts.
- Students might have difficulty seeing the data for two cities as distinct when written in one table
- $f(x)$ and $g(x)$ in the introduction do not have any function rule. Students may need an introduction to this type of notation.

Materials

- None

Grouping

- Partner / Individual

Differentiation

Extension:

- Students should graph the data for population and crime.
- Students could research the data for their own city and compare it to City A and City B.

Intervention:

- Relate #7-11 directly to the introduction to help students see the connection.

Formative Assessment Questions

- What are the different ways can we model functions?
- How can real-life situations be modeled by functions?

Building and Combining Functions – Teacher Notes

Given the functions $r(x) = 4x – 5$ and $s(x) = 3^x$

1. Find $r(x) + s(x)$

Solution:

$\mathbf{r(x) + s(x) = 4x - 5 + 3^{x}}$

2. Find $\dfrac{r(x)}{s(x)}$

Solution:

$\dfrac{r(x)}{s(x)} = \dfrac{4x - 5}{3^x}$

2. Given the functions f and g as defined in the table below, complete the table using the given function rules.

x	$f(x)$	$g(x)$	$n(x) = f(x) + g(x)$	$p(x) = 2f(x)g(x) - f(x)$	$q(x) = \dfrac{g(x)}{f(x)}$
1	3	2	5	9	2/3
2	4	1	5	4	1/4
3	1	4	5	7	4
4	2	3	5	10	3/2

The number of violent crimes committed in major cities is one statistic that is used to determine the safety rating of that city. In this task, we will examine data from two cities to examine the relationships of the crime rates to other factors relative to each city. In Table 1, the number of violent crimes committed in each city is given by year. In Table 2, the population of each city is given by year.

TABLE 1: Number of Violent Crimes

Year	2000	2001	2002	2003	2004	2005
# of Violent Crimes in City A	793	795	807	818	825	831
# of Violent Crimes in City B	448	500	525	566	593	652

4. By just looking at Table 1, which city would you predict is safer? Why?

Solution:
City B appears to be safer because of the number of crimes reported is lower that City A.

TABLE 2: Population

Year	2000	2001	2002	2003	2004	2005
Population of City A	61,000	62,100	63,220	64,350	65,510	66,690
Population of City B	28,000	28,588	29,188	29,801	30,427	31,066

5. By just looking at Table 2, which city would you predict is safer? Why?

Solution:
Answers may vary, but students may perceive City B to be safer because there are less people. Some start making connecting between the numbers of crimes in proportion to the population.

6. How might these two data sets be related?

Solution:
If in fact they are then we need to look at another relationship other than number of crimes per year and number of people per year. We need to look at the relationship between the number of crimes and the number of people or the per capita crime rate, that is the number of crimes per person in each city.

Let's define functions to represent the data we have. Let $C(t)$ be the function that represents the number of crimes t years after 2000.

That means that for city A, $C(0)$ = ___________, $C(1)$ = ___________, and $C(4)$ = ___________.

Solution:
793; 795; 825

Let $P(t)$ be the function that represents the population in t year, where t is measured in number of years since 2000.

That means that for city A, $P(0)$ = ___________, $P(1)$ = ___________, and $P(4)$ = ___________.

Solution:
61,000; 61,200; 65,510

7. We have just identified another notational issue. How can we adjust our notation to indicate the city to which we are referring?

Solution:
We can use subscripts to denote the city. So, $C_A(0)$ represents the number of crimes committed in city A during 2000 and $P_A(2)$ represents the population of city A in 2002.

8. Since the independent variable in our data is time, notice that each function written is dependent upon time. That means for us to find the per capita (per person) crime rate for each city, we need the ratio of these two functions. Let $R_A(t)$ be the per capita crime rate in city A and $R_B(t)$ be the per capita crime rate in city B. Using $C(t)$ and $P(t)$ for the appropriate cities, write the functional rule for $R_A(t)$ and $R_B(t)$.

Solution:
$R_A(t) = \dfrac{C_A(t)}{P_A(t)}$ ***is the appropriate function for the per capita crime rate in City A.***
$R_B(t) = \dfrac{C_B(t)}{P_B(t)}$ ***is the appropriate function for the per capita crime rate in City B.***

9. Now that you have the two functions R_A and R_B defined, complete the table below showing the per capita violent crime rate in both cities by year using the data from Table 1 and 2. Write each of the function values as a percent.

Solution:

Year	2000	2001	2002	2003	2004	2005
***t* (years since 2000)**	***0***	***1***	***2***	***3***	***4***	***5***
$R_A(t)$	***1.3%***	***1.28%***	***1.276%***	***1.271%***	***1.259%***	***1.246%***
$R_B(t)$	***1.6%***	***1.749%***	***1.799%***	***1.899%***	***1.949%***	***2.099%***

10. Now, using this data, which city is safer? Why?

Solution:
City A is actually safer because the per capita crime rate is actually decreasing while the per capita crime rate of City B is increasing.

11. Make any conclusions about the trends you see in the data. Specifically address rates of change.

Solution:
Answers will vary, however, students should use trends and rate of change in their discussion.

12. Write a function rule for $C_A(t)$, $C_B(t)$, $P_A(t)$, $P_B(t)$, and then using these function rules, write an explicit function rule for $R_A(t)$ and $R_B(t)$. Verify that each function gives the correct value that you calculated from the data in the table above.

Solution:

$C_A(t) = 8.314x + 790.714$

$C_B(t) = 38.286x + 451.619$

$P_A(t) = 1137.429x + 60968.095$

$P_B(t) = 613.142x + 27978.809$

$$R_A(t) = \frac{8.314x + 790.714}{1137.429x + 60968.095}$$

$$R_B(t) = \frac{38.286x + 451.619}{613.142x + 27978.809}$$

13. Using the functions, can you make predictions about crime rates in the future if the trends in the given data continue?

Solution:
Have students use the graphing calculator to predict crime rates using the functions and describe the results of their predictions. Answers will vary.

Learning Task: Building and Combining Functions

Name________________________________ **Date**_________________

Mathematical Goals
- Calculate and interpret rate of change
- Combine functions
- Write explicit function rules

Essential Questions
- How can we use real-world situations to construct and compare linear and exponential models and solve problems?

Common Core Georgia Performance Standards

MCC9-12.F.IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *(Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers.)*
MCC9-12.F.LE.1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MCC9-12.F.BF.1	Write a function that describes a relationship between two quantities. *(Limit to linear and exponential functions.)*
MCC9-12.F.BF.1b	Combine standard function types using arithmetic operations. *(Limit to linear and exponential functions.)*

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
4. Model with mathematics.

Learning Task: Building and Combining Functions

Name________________________________ **Date**____________________

adapted from Functions Modeling Change, A Preparation for Calculus, Connally, Hughes-Hallett, Gleason, et al. John Wiley & Sons, 1998.

Given the functions $r(x) = 4x - 5$ and $s(x) = 3^x$

1. Find $r(x) + s(x)$

2. Find $\frac{r(x)}{s(x)}$.

3. Given the functions f and g as defined in the table below, complete the table.

x	$f(x)$	$g(x)$	$n(x) = f(x) + g(x)$	$p(x) = 2f(x)g(x) - f(x)$	$q(x) = \frac{g(x)}{f(x)}$
1	3	2			
2	4	1			
3	1	4			
4	2	3			

The number of violent crimes committed in major cities is one statistic that is used to determine the safety rating of that city. In this task, we will examine data from two cities to examine the relationships of the crime rates to other factors relative to each city. In Table 1, the number of violent crimes committed in each city is given by year. In Table 2, the population of each city is given by year.

TABLE 1: Number of Violent Crimes

Year	2000	2001	2002	2003	2004	2005
# of Violent Crimes in City A	793	795	807	818	825	831
# of Violent Crimes in City B	448	500	525	566	593	652

4. By just looking at Table 1, which city would you predict is safer? Why?

TABLE 2: Population

Year	2000	2001	2002	2003	2004	2005
Population of City A	61,000	62,100	63,220	64,350	65,510	66,690
Population of City B	28,000	28,588	29,188	29,801	30,427	31,066

5. By just looking at Table 2, which city would you predict is safer? Why?

6. How might these two data sets be related?

Let's define functions to represent the data we have. Let $C(t)$ be the function that represents the number of crimes t years after 2000.

That means that for city A, $C(0)$ = ___________, $C(1)$ = ___________, and $C(4)$ = ___________.

Let $P(t)$ be the function that represents the population in t year, where t is measured in number of years since 2000.

That means that for city A, $P(0)$ = ___________, $P(1)$ = ___________, and $P(4)$ = ___________.

7. We have just identified another notational issue. How can we adjust our notation to indicate the city to which we are referring?

8. Since the independent variable in our data is time, notice that each function written is dependent upon time. That means for us to find the per capita (per person) crime rate for each city, we need the ratio of these two functions. Let $R_A(t)$ be the per capita crime rate in city A and $R_B(t)$ be the per capita crime rate in city B. Using $C(t)$ and $P(t)$ for the appropriate cities, write the functional rule for $R_A(t)$ and $R_B(t)$.

9. Now that you have the two functions R_A and R_B defined, complete the table below showing the per capita violent crime rate in both cities by year using the data from Table 1 and 2. Write each of the function values as a percent.

Year	2000	2001	2002	2003	2004	2005
***t* (years since 2000)**						
$R_A(t)$						
$R_B(t)$						

10. Now, using this data, which city is safer? Why?

11. Make any conclusions about the trends you see in the data. Specifically address rates of change.

12. Write a function rule for $C_A(t)$, $C_B(t)$, $P_A(t)$, $P_B(t)$, and then using these function rules, write an explicit function rule for $R_A(t)$ and $R_B(t)$. Verify that each function gives the correct value that you calculated from the data in the table above.

13. Using the functions, can you make predictions about crime rates in the future if the trends in the given data continue?

Interpreting Functions (Short Cycle Task)

Source: Formative Assessment Lesson Materials from Mathematics Assessment Project
http://www.map.mathshell.org/materials/download.php?fileid=840

Task Comments and Introduction

Tasks and lessons from the Mathematics Assessment Project are specifically designed to help teachers effectively formatively assess their students. The way the tasks and lessons are designed gives the teacher a clear understanding of what the students are able to do and not do. Within the lesson, teachers will find suggestions and question prompts that will help guide students towards understanding. For more information access the MAP website:
http://www.map.mathshell.org/materials/background.php?subpage=summative

The task, *Interpreting Functions*, is a Mathematics Assessment Project Assessment Task that can be found at the website: http://www.map.mathshell.org/materials/tasks.php?taskid=294&subpage=novice

The PDF version of the task can be found at the link below:
http://www.map.mathshell.org/materials/download.php?fileid=840

The scoring rubric can be found at the following link:
http://www.map.mathshell.org/materials/download.php?fileid=841

Mathematical Goals

- Interpret graphs of functions in context.

Essential Questions

- How can I relate graphs to the context they represent?

Common Core Georgia Performance Standards

MCC9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *(Draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences.)*

MCC9-12.F.IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~. *(Focus on linear and exponential functions.)*
MCC9-12.F.IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *(Focus on linear and exponential functions.)*
MCC9-12.F.IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *(Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers.)*
MCC9-12.F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*
MCC9-12.F.IF.7a	Graph linear ~~and quadratic~~ functions and show intercepts, maxima, and minima.
MCC9-12.F.IF.7e	Graph exponential ~~and logarithmic~~ functions, showing intercepts and end behavior, and ~~trigonometric functions, showing period, midline, and amplitude.~~
MCC9-12.F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

Standards for Mathematical Practice

2. Reason abstractly and quantitatively.
6. Attend to precision.

Background Knowledge

- Students have experience relating graphs to real-life situations.

Common Misconceptions

- Students may believe that the vertical line, rather than the horizontal line, shows that the car is not moving.

Materials

- see FAL website

Grouping

- Individual / partner

**Birthday Gifts and Turtle Problem*

INTRODUCTION TO THIS FORMATIVE ASSESSMENT LESSON

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Write linear and exponential functions from verbal sentences
- Understand the rates of change of linear functions are constant, while the rates of change of exponential functions are not constant.

COMMON CORE STATE STANDARDS

This lesson involves mathematical content in the standards from across the grades, with emphasis on:

MCC9-12.F.LE.1 - Distinguish between situations that can be modeled with linear functions and with exponential functions.
Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MCC9-12.F.LE.2 - Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MCC9-12.F.LE.3 - Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly.
SMP1. Make sense of problems and persevere in solving them.
SMP2. Reason abstractly and quantitatively.
SMP4. Model with mathematics.
SMP6. Attend to precision.

INTRODUCTION

This lesson is structured in the following way:

- Before the lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work, and create questions for students to answer in order to improve their solutions.
- Students are grouped into pairs by common misconceptions.
- After a whole-class interactive introduction, students work in pairs on the collaborative discussion task first to match the situation with the mathematical equation, and then to match the graph to the pair.

OPTIONAL: teacher may have students create charts of the equations before he/she hands out the graphs.

OPTIONAL: teacher could ask students to do one of the following on chart paper:

- Put cards in order from changing the slowest to changing the fastest
- Put cards in order from increasing the slowest to increasing the fastest.
- For the linear equations, ask students to compute the slope of each line
- For exponential equations, ask students to compute slope between points A and B, then between points B and C, then between points C and D, to note the way that the rate of change is changing. Aim discussions toward Calculus thought (difference quotient…how the slope is more accurate when the points chosen are closer together) for advanced learners.
- Students write their own equations on the blank equation cards for the situations (2) that had no match.
- After a plenary discussion, students return to their original assessment tasks, and try to improve their own responses.
- A different post-assessment task (which is similar to the pre-assessment task) will be administered to determine if growth has been made in understanding linear/exponential equations and their graphs.

MATERIALS REQUIRED

Each individual student will need:

- A copy of the Pre-Assessment
- A copy of the Post-Assessment

Each pair of students will need:

- Chart Paper
- Glue Sticks or Tape
- Card Sets (Situations, Equations and Graphs)

TEACHER PREP REQUIRED

Teacher will need to cut out Card Sets so the order can be scrambled.

TIME NEEDED

For Pre-Assessment: 15 minutes For Lesson: 45 minutes For Post: 15 minutes

Timings are approximate. Exact timings will depend on the needs of the class.

FRAMING FOR THE TEACHER:

CCGPS Mathematics compares exponential to linear functions during 9th grade in order for students to understand the difference between a constant rate of change and a non-constant rate of change. These two are chosen for comparison because of the arithmetic vs. geometric link to sequences.

Students learn that a constant difference creates an arithmetic sequence when the domain is comprised of integer values, whereas a constant factor creates a geometric sequence under the same circumstances. This task highlights the differences between the two types of functions and their rates of change. The collaborative activity requires that student understand how to create equations to model situations that may be described by one of these types of functions.

FRAMING FOR THE KIDS:

Say to the students:

This activity will take about one to two days for us to complete.

The reason we are doing this is to be sure that you understand the difference between linear functions with a constant rate of change and exponential functions with a non-constant rate of change before we move on to a new idea.

You will have a chance to work with a partner to correct any misconceptions that you may have. After the partner work, you will be able to show me what you have learned!

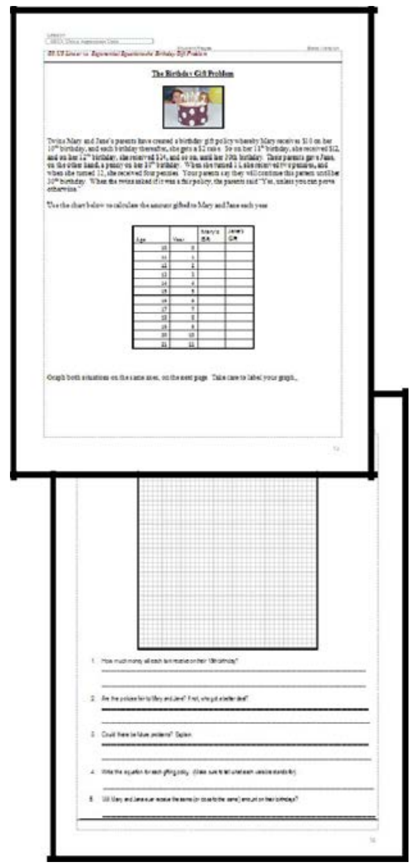
PRE-ASSESSMENT BEFORE THE LESSON

ASSESSMENT TASK:

Name of Assessment Task: Birthday Gifts and Turtle Problem

Time This Should Take: 15 minutes

Have the students do this task in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will them be able to target your help more effectively in the follow-up lesson.



Give each student a copy of the Pre-Assessment:

Briefly introduce the task and help the class to understand the problem and its context.

Spend 15 minutes working individually on this task. Read through the task and try to answer it as carefully as you can. Show all your work so that I can understand your reasoning. Don't worry if you can't complete everything. There will be a lesson that should help you understand these concepts better. Your goal is to be able to confidently answer questions similar to these by the end of the next lesson.

Note: The teacher may consider adding labels to the axes on the coordinate plane if students appear to be struggling with that.

Students should do their best to answer these questions, without teacher assistance. It is important that students are allowed to answer the questions on their own so that the results show what students truly do not understand.

Students should not worry too much if they cannot understand or do everything on the pre-assessment, because in the next lesson they will engage in a task which is designed to help them.. Explain to students that by the end of the next lesson, they should expect to be able to answer questions such as these confidently.

This is their goal.

COLLABORATION TIME/READING STUDENTS RESPONSES

You Will Not "Grade" These!

Collect students' responses to the task. It is helpful to read students' responses with colleagues who are also analyzing student work. Make notes (on your own paper, not on their pre-assessment) about what their work reveals about their current levels of understanding, and their approaches to the task. You will find that the misconceptions reveal themselves and often take similar paths from one student to another, and even from one teacher to another. Some misconceptions seem to arise very organically in students' thinking. Pair students in the same classes with other students who have similar misconceptions. This will help you to address the issues in fewer steps, since they'll be together. (Note: pairs are better than larger groups for FAL's because both must participate in order to discuss!)

You will begin to construct Socrates-style questions to try and elicit understanding from students. We suggest you write a list of your own questions; however some guiding questions and prompts are also listed below as a jumping-off point.

GUIDING QUESTIONS

Common Issues	Suggested Questions and Prompts
Student Was Unable to Begin Activity Example: *I don't know how to write an equation from a situation.*	• What do you start out with? (i.e. what money do you pay initially how much do you have initially, where are you at first, etc. . .) • What is changing (variable) in this situation? (i.e. the number of miles, the number of years, the number of chores, etc. . .)
Student was Unable to Begin Assessment Example: *I don't know how to put these answers in the chart.*	• If Mary is 10, how much money does she get? According to this situation, how much money will she get next year? • What is different about how Mary's gift amount increases and how Jane's gift amount increases?
Student thinks all graphs are linear *Example: only graphs two points of Jane's gift amount, and then connects those dots.*	• Have you looked to see if all of the points on your chart match this graph?
Student does not know where the variable goes in the equation *Example: Student confuses* $f(x)=2x+25$ *with* $f(x)=25*2^x$	• Which variable is independent? • Which variable is dependent? • Can you plug values in to help you match these?
Student does not realize that exponential equations will change very quickly over time. *Example: It is obvious that Mary got a better present amount, because she started out with more. No way can a penny change into more than what Mary had at first.*	• What if they lived to 110? How much would each of them receive on their birthday? • Have you checked the last payment they will receive to see?
Student does not use proper function notation. *Example: Student writes* $y=2x + 25$ *or* $y = 25*2^x$.	• Is there another, more proper, notation to use than "y" when writing an equation?
Student does not realize that 30 minutes or 50 cents is equivalent to the fraction ½. *Example: The numbers in this situation aren't even found in any of these equation cards.*	• What is another word for a 50-cent piece? • What is another way of saying 30 minutes?

LESSON DAY

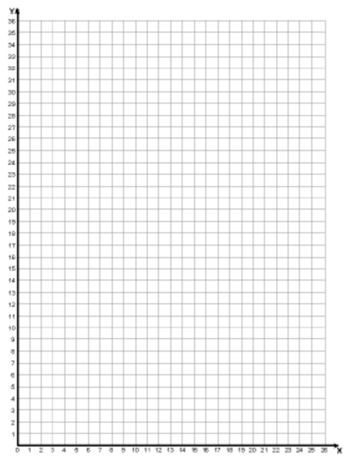
SUGGESTED LESSON OUTLINE:

Part 1: Whole-Class Introduction: Time to Allot: (15 minutes)

Display the "Warm Up" question provided.

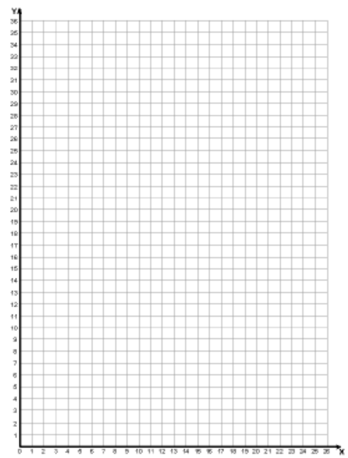
Situation 1: You have 20 liters of water on a very hot day. You realize that the volume of your water halves each hour. How many liters do you have remaining at the end of 1, 2, 3, and 10 hours?

Hours	*Volume of Water*
0	
1	
2	
3	
10	
20	
100	
1000	



Situation 2: You have 20 liters of water, but this time, it is leaking out at a rate of ½ liter per hour. How many liters do you have remaining at the end of 1, 2, 3, and 10 hours?

Hours	*Volume of Water*
0	
1	
2	
3	
10	
20	
100	
1000	



Situation 1: You have 20 liters of water on a very hot day. You realize that the volume of your water halves each hour. How many liters do you have remaining at the end of 1, 2, 3, and 10 hours?

To find out, track the math you did on the previous slide:

1) Initial amount: 20
2) Multiply by: $20 * ½ = 20(½)^1$ (you just multiplied once by ½)
3) Multiply again: $20 * ½ * ½ = 20(½)^2$ (you just multiplied twice by ½)
4) Multiply again: $20 * ½ * ½ * ½ = 20(½)^3$ (you just multiplied three times by ½)…
5) Multiply again: $20 * ½ * ½ * ½ * ½ = 20(½)^4$
6) Multiply again: $20 * ½ * ½ * ½ * ½ * ½ = 20(½)^5$
7) Multiply again: $20 * ½ * ½ * ½ * ½ * ½ * ½ = 20(½)^6$
8) Multiply again: $20 * ½ * ½ * ½ * ½ * ½ * ½ * ½ = 20(½)^7$

So what if you multiplied "x" times by ½ ?

$$f(x) = 20(½)^x$$

Situation 2: You have 20 liters of water, but this time, it is leaking out at a rate of ½ liter per hour. How many liters do you have remaining at the end of 1, 2, 3, and 10 hours?

To find out, track the math you did on the previous slide:

1) Initial amount: 20
2) Subtract ½ : $20 - ½ = 20 – ½$ (you just subtracted ½)
3) Subtract again: $20 -½ - ½ = 20 - ½ *2$(you just subtracted ½ twice)
4) Subtract again: $20 -½ - ½- ½= 20 - ½ *3$ (you just subtracted ½ three times)
5) Subtract again: $20 -½ - ½- ½- ½= 20 - ½ *4$
6) Subtract again: $20 -½ - ½- ½- ½- ½= 20 - ½ *5$
7) Subtract again: $20 -½ - ½- ½- ½- ½- ½= 20 - ½ *6$
8) Again: $20 -½ - ½- ½- ½- ½- ½- ½= 20 - ½ *7$

So what if you subtracted ½ "x" times?

$$f(x) = 20 – (½)*x$$
$$\text{or } f(x) = - ½ x + 20$$

Suggested Prompts:

Questions to Ask (Situation 1):

- *When will the water be all gone?*
- *How fast does the water evaporate?*
- *What do you multiply by to "half" a quantity of something?*

Questions to ask (Situation 2)

- *When will the water be all gone?*
- *How fast does the water evaporate?*
- *What do you multiply by to "half" a quantity of something?*

Below is an extra optional warm up.

Situation 1: You have $230 when you go to work, and you earn $5 an hour in tips.	*Situation 2: You have $230, and you are playing "Who Wants to Be a Hundredaire." If you get a question right, your money quintuples.*
f(x) =	**f(x) =**

Part 2: Collaborative Activity: Time to Allot: (30 minutes)

Do/Say the Following:

- Group students in pairs by common errors found in Formative Assessment "The Birthday Gift Problem"
- Post collaborative activity instructions (projector resources, included)
- After a whole-class interactive introduction, students work in pairs on the collaborative discussion. First they match the situation with the mathematical equation (functions).
- Then they match the graph with the appropriate situation/equation cards.
- Students write their own equations on the blank equation cards for the situations (2) that had no match.

The purpose of this structured group work is to ensure that students know how to write linear and exponential equations from real-world situations. This activity demonstrates students' understandings of how linear and exponential functions are graphed and modeled. The goal is to develop an understanding of the nature of "change" in linear and exponential functions.

During the Collaborative Activity, the Teacher has 3 tasks:

- Circulate to students' whose errors you noted from the pre-assessment and support their reasoning with your guiding questions.
- Circulate to other students also to support their reason in the same way.
- Make a note of student approaches for the summary (plenary discussion). Some students have interesting and novel solutions!

Note different student approaches to the task and any common mistakes. For example, students may

- Have a tough time getting started with modeling
- They may choose to graph all functions as lines.
- They may not realize that exponential functions change very rapidly over time and that the rates of change differ over time.
- Students may confuse the structure of linear and exponential functions.
- Students may try to match similar numbers together (the cards were created so that this cannot be done, since numbers are repeated, and some numbers are "hidden" in context, such as 30 minutes…students should pick up on this equaling half an hour.

Also *notice the ways students may:*

- Use improper notation. Correct this error with "is this proper function notation?"
- Over-simplify exponential graphs by only showing two points
- Be disturbed that their graph in the pre-assessment won't hold all of the domain points because the amounts are too high. Reassure them (later) that this happens all the time. Just go with it. We cannot always predict at the beginning of graphing what the graph will look like, and if we could, there would be no need to graph.

You can use this information to focus a plenary whole-class discussion.

Support Student Reasoning

Try not to make suggestions that steer students towards a particular "correct" answer or response. Instead, ask questions to help students to reason together.

If you find one student has produced a correct response, challenge another student in the group to provide an explanation.

Example:
Sherry, why do you think that this situation is exponential rather than linear?
John, what tells you that the y-intercept is this number?
Hal, how can you tell where the variable goes in this equation?

If you find students have difficulty articulating their decisions, use the sheet Suggested questions and prompts to support your own questioning of students.

Sample ways to jump-start students' work in the group collaboration:

- *Can you narrow down any that seem linear?*

- *Can you rule out any that definitely do not go together?*
- *What is the starting value or amount?*

If the whole class is struggling on the same issue, you could write a couple of questions on the board and hold an interim, whole-class discussion. You could ask students who performed well in the assessment to help struggling students.

Part 3: Plenary (Summary) Discussion: Time to Allot: (15 minutes)

Gather students together, share solutions. Discussion prompts should be made up of your original guiding questions and notes about student approaches. Some other discussion prompts are listed below:

NOTE: *"Scribing" helps to increase student buy-in and participation. When a student answers your question, write the student's name on the board and scribe his/her response quickly. You will find that students volunteer more often when they know you will scribe their responses – this practice will keep the discussions lively and active!*

Bring the class together and use the following to guide discussion:

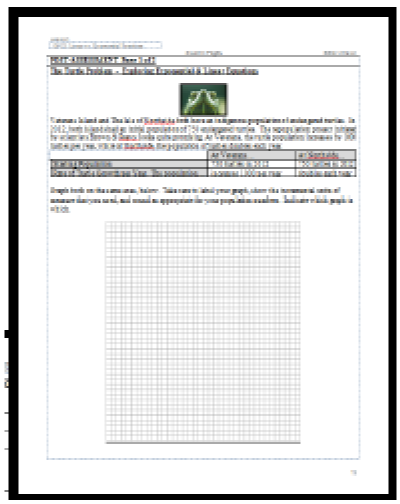
- *How can you tell if the situation is exponential or linear? What clues do you look for?*
- *How can you tell if a graph is exponential or linear?*
- *Where do you find the "initial value" in exponential equations?*
- *Where do you find the "initial value" in linear equations?*
- *Where do you find the "initial value" in exponential graphs? Linear graphs? All graphs?*

Part 4: Improving Solutions to the Assessment Task Time to Allot: (15 minutes)

The Shell MAP Centre advises handing students their original assessment tasks back to guide their responses to their new Post-Assessment (which is sometimes the exact same as the Pre-Assessment). In practice, some teachers find that students mindlessly transfer incorrect answers from their Pre- to their Post-Assessment, assuming that no “X” mark means that it must have been right. . Until students become accustomed to UNGRADED FORMATIVE assessments, they may naturally do this. Teachers often report success by handing students a list of the guiding questions to keep in mind while they improve their solutions.

Practice will make perfect, and teachers should do what makes them most comfortable with their students/finds and kills misconceptions!

Return to the students their original assessment task if you choose (some teachers choose not to return the original assessment task because students tend to just copy the old answers without realizing that they had incorrect answers previously; they misinterpret a lack of “X” marks as meaning they got the problems all correct – it may be worthwhile to mention to students who are new to FAL’s that the lack of an “X” does not mean they got it right if you do choose to give students their original tasks back).



Look at your original responses and think about what you have learned this lesson.

Using what you have learned, try to answer the questions on this new assessment. The situation and questions are different, but the same basic concepts are used..

If you have not added questions to individual pieces of work then write your list of questions on the board or hand out copies of the questions.

Students should select from this list only the questions they think are appropriate to their own work.

Give students a blank copy of the post assessment task “Turtle Repopulation.” They may use their original pre-assessment task to help, if the teacher wishes.

PRE-ASSESSMENT (Answer Key)

ASSESSMENT TASK:

Name of Assessment Task:

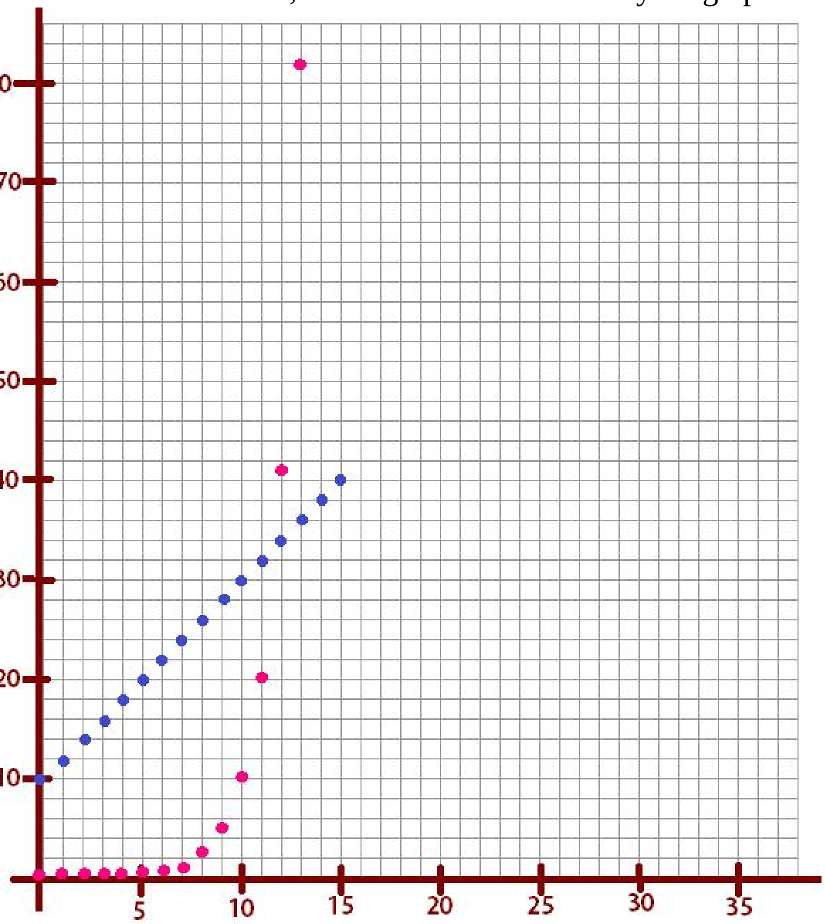
The Birthday Gift Problem

Twins Mary and Jane's parents have created a birthday gift policy whereby Mary receives $10 on her 10th birthday, and each birthday thereafter, she gets a $2 raise. So on her 11th birthday, she received $12, and on her 12th birthday, she received $14, and so on, until her 30th birthday. Their parents gave Jane, on the other hand, a penny on her 10th birthday. When she turned 11, she received two pennies, and when she turned 12, she received four pennies. Your parents say they will continue this pattern until her 30th birthday. When the twins asked if it was a fair policy, the parents said "Yes, unless you can prove otherwise."

Use the chart below to calculate the amount gifted to Mary and Jane each year.

Age	Year	Mary's Gift	Jane's Gift
10	0	*10*	*.01*
11	1	*12*	*.02*
12	2	*14*	*.04*
13	3	*16*	*.08*
14	4	*18*	*.16*
15	5	*20*	*.32*
16	6	*22*	*.64*
17	7	*24*	*1.28*
18	8	*26*	*2.56*
19	9	*28*	*5.12*
20	10	*30*	*10.24*
21	11	*32*	*20.48*
22	12	*34*	40.96
23	13	*36*	*81.92*
24	14	*38*	*163.84*
25	15	*40*	*327.68*
26	16	*42*	*655.36*
27	17	*44*	*1310.72*
28	18	*46*	*2621.44*
29	19	*48*	*5242.88*
30	20	*50*	*10,486.76*

Graph both situations on the same axes, below. Take care to label your graph.



1. How much money will each twin receive on their 15th birthday?
   The 15^{th} birthday is x = 5.
   Mary receives $20 and Jane receives $0.32.

2. Are the policies fair to Mary and Jane? If not, who got a better deal?
   Even though teachers know that fair is not always equal, this is not fair. Mary receives more money at first than Jane, but after about 12 years, Jane's gifts are far more than Mary's.

3. Could there be future problems? Explain.
   Besides fighting between Mary and Jane, their parents may need to save money. By the last year, they need more than $10,000 to give to Jane!

4. Write the equation for each gifting policy. (Make sure to tell what each variable stands for).
   M(t)=2t+10 {t = time in years, M(t) = amount of money received by Mary)
   $J(t) = 10*(2^t)$ {t = time in years, J(t) = amount of money received by Jane)

5. Will Mary and Jane ever receive the same (or close to the same) amount on their birthdays?
   On their 22^{th} birthday, they receive amounts that are only $6.96 apart. That's the closest they ever come.

Collaborative Activity (Answer Key)

SOLUTIONS TO MATCHING in the COLLABORATIVE ACTIVITY

SITUATIONS IN WORDS	A	B	C	D	E	F	G	H	I	J
MATH SENTENCES	f(x)	q(x)	h(x)	n(x)	k(x)	r(x)	m(x)	w(x)	$f(x) = 2x+5$	$f(x) = 5(2)^x$
GRAPHS	9	3	5	7	8	4	10	2	1	6

POST-ASSESSMENT (Answer Key)

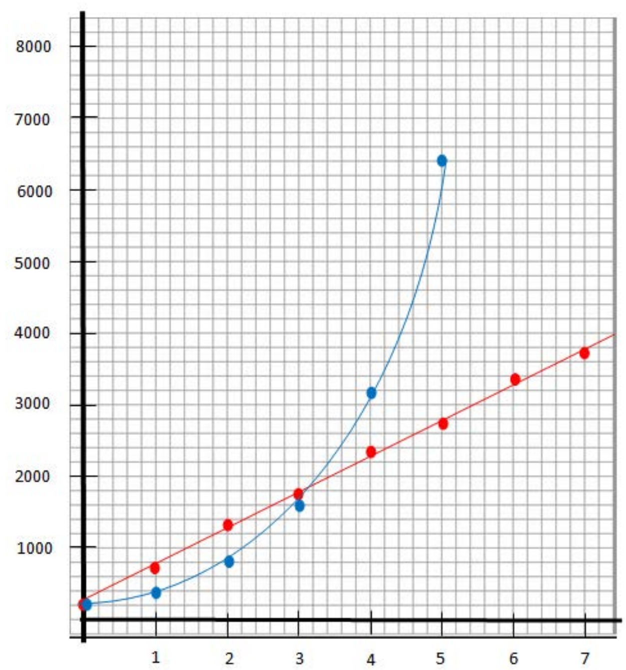
The Turtle Problem

Veterans Island and The Isle of Northside both have an indigenous population of endangered turtles. In 2012, both islands had an initial population of 200 endangered turtles. The repopulation project initiated by scientists Brown & Peavy looks quite promising. At Veterans, the turtle population increases by 500 turtles per year, while at Northside, the population of turtles doubles each year.

	At Veterans…	At Northside…
Starting Population	200 turtles in 2012	200 turtles in 2012
Rate of Turtle Growth per Year. The population….	increases 500 per year	doubles each year

Graph both on the same axes, below. Take care to label your graph, show the incremental units of measure that you used, and round as appropriate for your population numbers. Indicate which graph is which.

Calendar Year	Year	Veterans Island	Isle of Northside
2012	0	200	200
2013	1	700	400
2014	2	1200	800
2015	3	1700	1600
2016	4	2200	3200
2017	5	2700	6400
2018	6	3200	12,800
2019	7	3700	25,600
2020	8	4200	51,200
2021	9	4700	102,400
2022	10	5200	204,800



POST- ASSESSMENT page 2 of 2

1. What will the population of turtles on each island be after 5 years?
   Veterans turtle population will be 2700 *Isle of Northside will have 6400*
2. Which island saw the greatest change in turtle population?
   Isle of Northside showed the greatest change from 200 to 204,800 over a ten year period.
3. Is it possible that the most effective repopulation program may experience future turtle problems? Explain. *Eventually, the resources of the Isle of Northside could not support the turtle population growth any longer. Animals living in close proximity to one another tend to spread disease. Animals that are plentiful also take resources from other animals, and therefore the population growth of turtles may cause extinction of another species. The model does not mention death of*

turtles and so it may not be 100% accurate over long periods of time. However, turtles do live a very long time….

4. Write the equation for each island. Include a legend for each of your variables!

Veterans Island $p(t) = 500(t) + 200$ t = time in years, p(t) = turtle population

Isle of Northside $p(t) = 200^t$ t = time in years, p(t) = turtle population

The Birthday Gift Problem

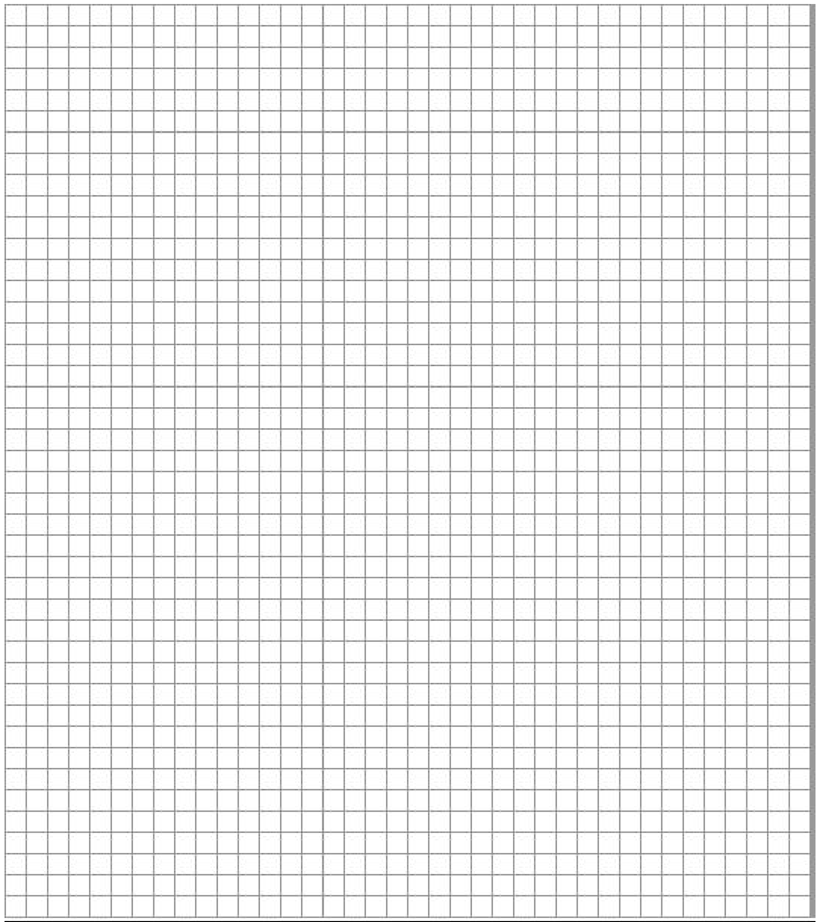
Twins Mary and Jane's parents have created a birthday gift policy whereby Mary receives $10 on her 10th birthday, and each birthday thereafter, she gets a $2 raise. So on her 11th birthday, she received $12, and on her 12th birthday, she received $14, and so on, until her 30th birthday. Their parents gave Jane, on the other hand, a penny on her 10th birthday. When she turned 11, she received two pennies, and when she turned 12, she received four pennies. Your parents say they will continue this pattern until her 30th birthday. When the twins asked if it was a fair policy, the parents said "Yes, unless you can prove otherwise."

Use the chart below to calculate the amount gifted to Mary and Jane each year.

Age	Year	Mary's Gift	Jane's Gift
10	0		
11	1		
12	2		
13	3		
14	4		
15	5		
16	6		
17	7		
18	8		
19	9		
20	10		
21	11		

Graph both situations on the same axes, on the next page. Take care to label your graph.

The Birthday Gift Problem



1. How much money will each twin receive on their 15th birthday?

    ____

2. Are the policies fair to Mary and Jane? If not, who got a better deal?

    ____

    ____

3. Could there be future problems? Explain.

    ____

    ____

4. Write the equation for each gifting policy. (Make sure to tell what each variable stands for).

    ____

5. Will Mary and Jane ever receive the same (or close to the same) amount on their birthdays?

    ____

COLLABORATIVE ACTIVITY

Name of Assessment Task: Card Match   Card Set 1 : A-J

Situation A	Situation B
To hire a taxi, you pay $4 to get in the car, and $3 per mile.	You have four dollars in your piggy bank, which triples every year.
Situation C	**Situation D**
Your parents pay you four dollars a week for allowance, plus 50₡ for each chore that you do.	You are four miles from your date's house. You walk half the distance there. Then you walk half of the remaining distance. Then you walk half of the remaining-remaining distance...you do this forever. Technically, you never get there.
Situation E	**Situation F**
Your wonderful math teacher gives you three M&M's at the beginning of class, and four additional M&M's for each correct answer on your homework.	A population of cockroaches in a school's kitchen (not yours, of course) starts with just three cockroaches. It quadruples each week.

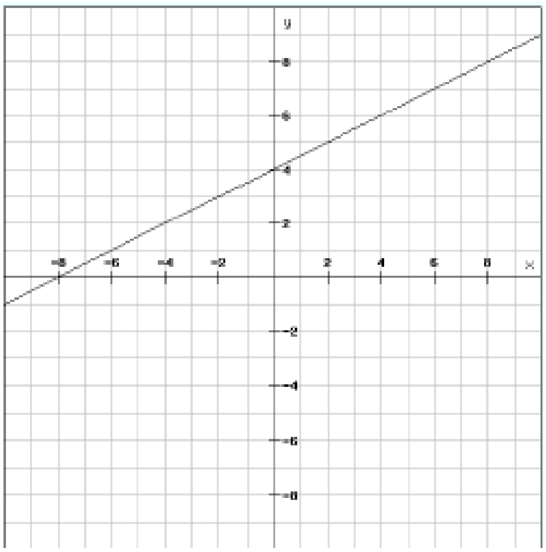
Situation G	Situation H
It takes you 30 minutes to set up the "Sunglasses, etc." kiosk in the middle of the mall, and then you work there 4 hours per day.	You have half a gallon of water in the kiddie pool, and as you fill it, the volume quadruples each hour.
Situation I	**Situation J**
You have $5 that you put into an account, and you deposit $2 every Friday.	You have $5 that you put into an account that doubles each year

Card Set 2

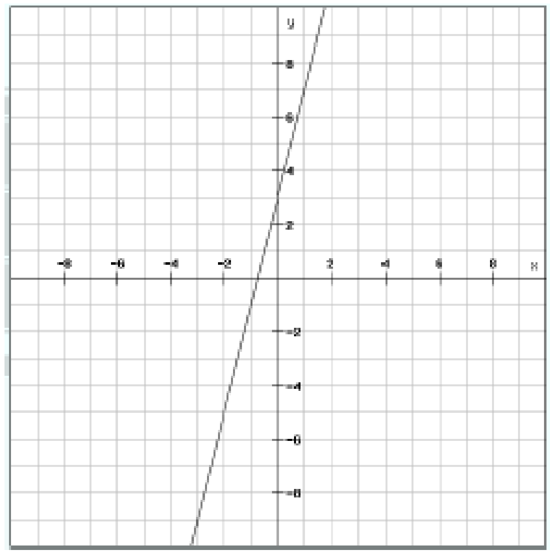
$f(x)=3x+4$	$q(x) = 4*3^x$
$h(x)=(^1/_2)x+4$	$n(x) = 4(^1/_2)^x$
$k(x)=4x+3$	$r(x) = 3*4^x$
$m(x)=4x+^1/_2$	$w(x) =(^1/_2)4^x$

Card Set 3

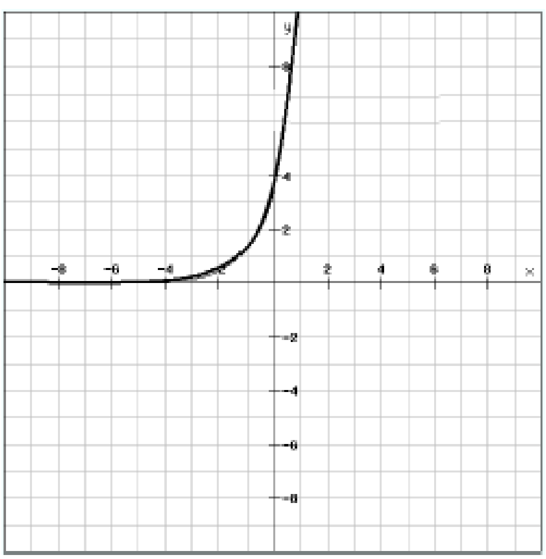
Graph 5



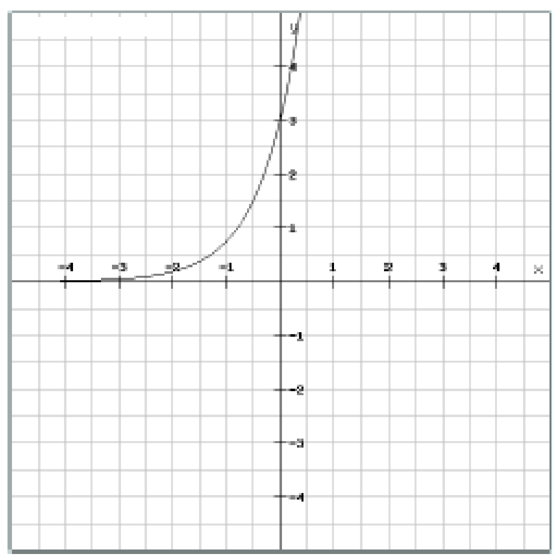
Graph 8

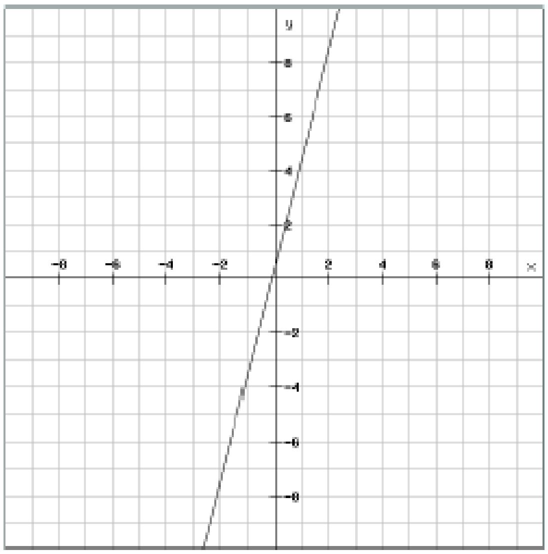
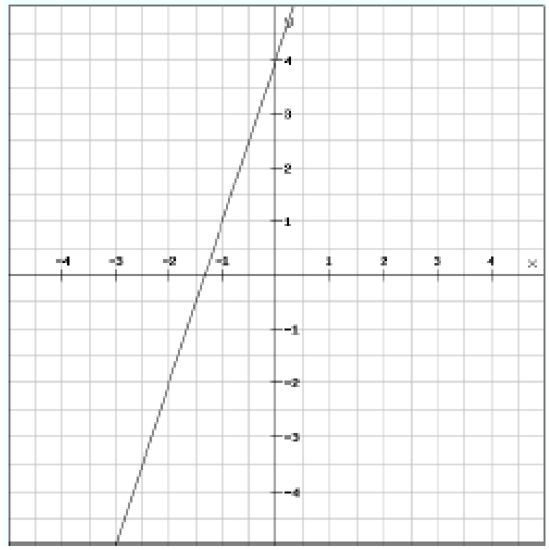
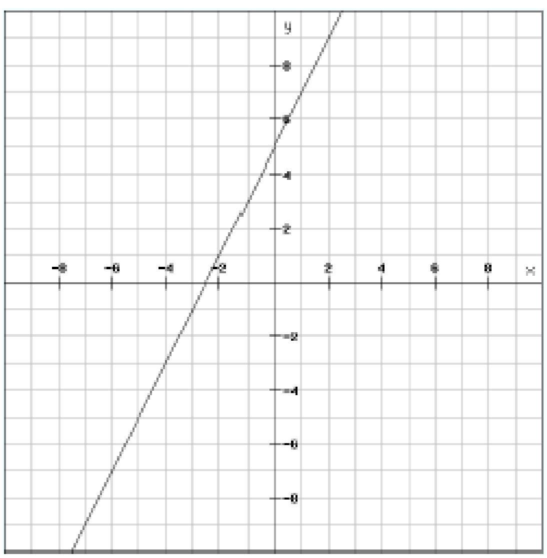
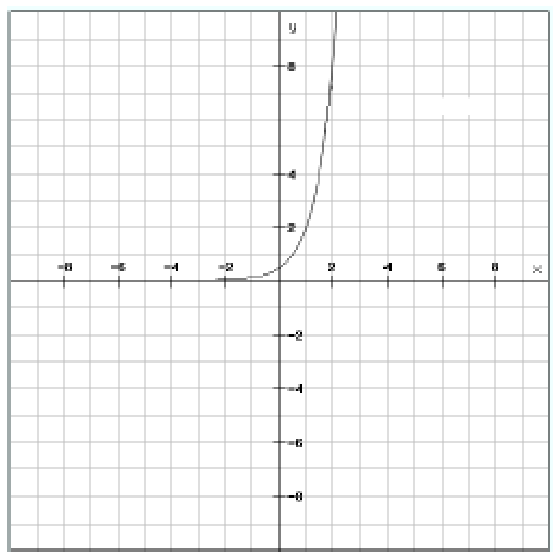


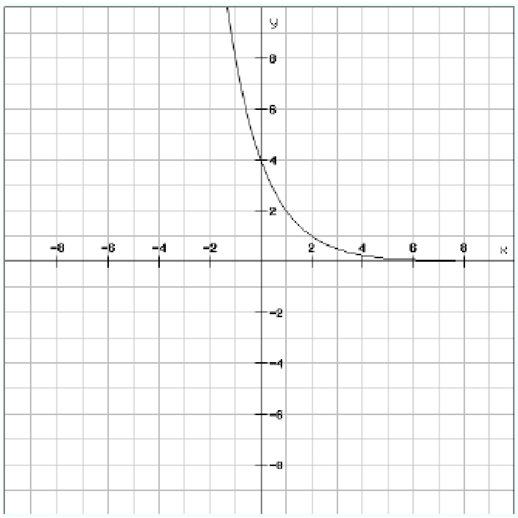
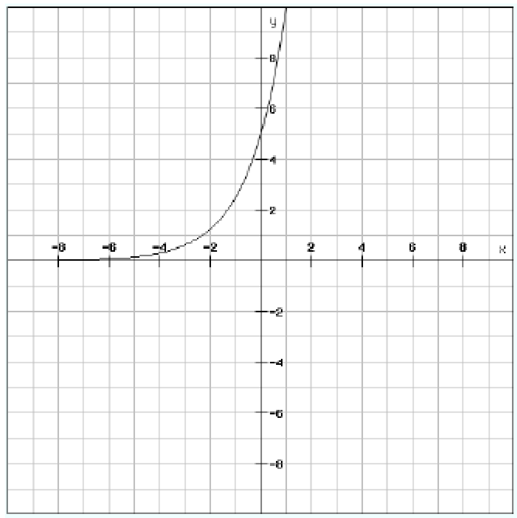
Graph 3



Graph 4



Graph 10	Graph 9
	
Graph 1	**Graph 2**
	

Graph 7	Graph 6
	

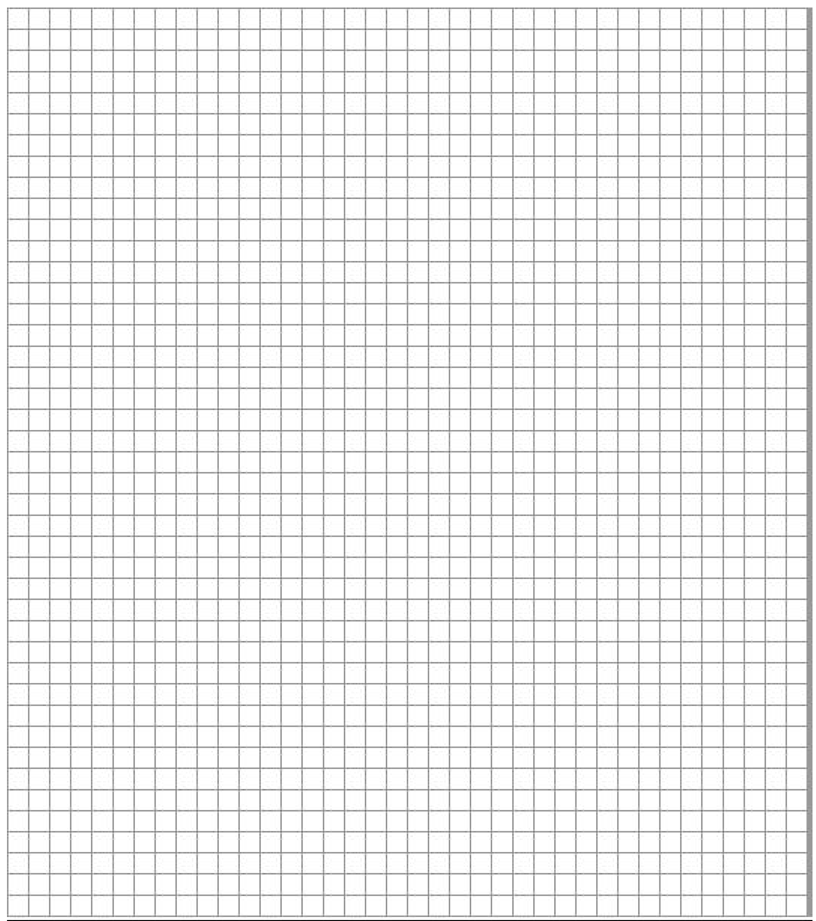
The Turtle Problem

Veterans Island and The Isle of Northside both have an indigenous population of endangered turtles. In 2012, both islands had an initial population of 200 endangered turtles. The repopulation project initiated by scientists Brown & Peavy looks quite promising. At Veterans, the turtle population increases by 500 turtles per year, while at Northside, the population of turtles doubles each year.

	At Veterans…	At Northside…
Starting Population	200 turtles in 2012	200 turtles in 2012
Rate of Turtle Growth per Year. The population….	increases 500 per year	doubles each year

Graph both on the same axes, below. Take care to label your graph, show the incremental units of measure that you used, and round as appropriate for your population numbers. Indicate which graph is which.

Calendar Year	Year	Veterans Island	Isle of Northside
2012	0	200	200
2013	1		
2014	2		
2015	3		



page 2 of 2

1. What will the population of turtles on each island be after 5 years?

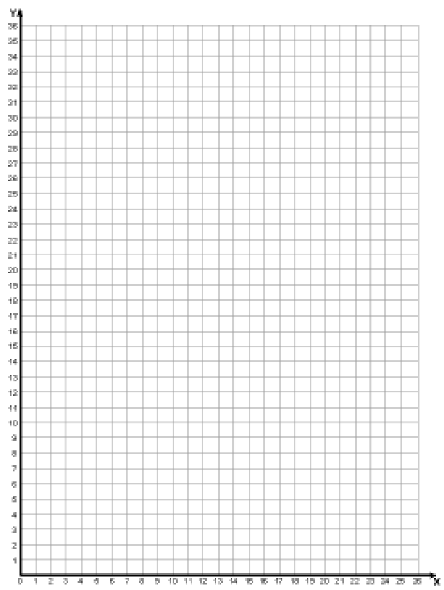
2. Which island saw the greatest change in turtle population?

3. Is it possible that the most effective repopulation program may experience future turtle problems? Explain.

4. Write the equation for each island. Include a legend for each of your variables!

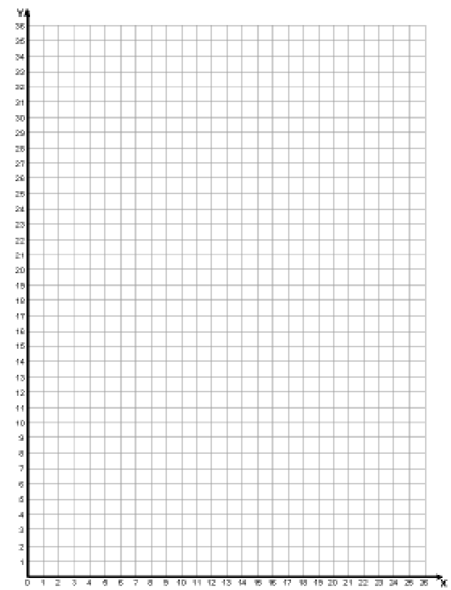
Situation 1: You have 20 liters of water on a very hot day. You realize that the volume of your water halves each hour. How many liters do you have remaining at the end of 1, 2, 3, and 10 hours?

Hours	*Volume of Water*
0	
1	
2	
3	
10	
20	
100	
1000	



Situation 2: You have 20 liters of water, but this time, it is leaking out at a rate of ½ liter per hour. How many liters do you have remaining at the end of 1, 2, 3, and 10 hours?

Hours	*Volume of Water*
0	
1	
2	
3	
10	
20	
100	
1000	



Situation 1: You have 20 liters of water on a very hot day. You realize that the volume of your water halves each hour. How many liters do you have remaining at the end of 1, 2, 3, and 10 hours?

To find out, track the math you did on the previous slide:

1) Initial amount: 20
2) Multiply by: $20 * ½ = 20(½)^1$ (you just multiplied once by ½)
3) Multiply again: $20 * ½ * ½ = 20(½)^2$ (you just multiplied twice by ½)
4) Multiply again: $20 * ½ * ½ * ½ = 20(½)^3$ (you just multiplied three times by ½)…
5) Multiply again: $20 * ½ * ½ * ½ * ½ = 20(½)^4$
6) Multiply again: $20 * ½ * ½ * ½ * ½ * ½ = 20(½)^5$
7) Multiply again: $20 * ½ * ½ * ½ * ½ * ½ * ½ = 20(½)^6$
8) Multiply again: $20 * ½ * ½ * ½ * ½ * ½ * ½ * ½ = 20(½)^7$

So what if you multiplied "x" times by ½ ?

$$f(x) = 20(½)^x$$

Situation 2: You have 20 liters of water, but this time, it is leaking out at a rate of ½ liter per hour. How many liters do you have remaining at the end of 1, 2, 3, and 10 hours?

To find out, track the math you did on the previous slide:

1) Initial amount: 20
2) Subtract ½ : 20 - ½ = 20 – ½ (you just subtracted ½)
3) Subtract again: 20 -½ - ½ = 20 - ½ *2(you just subtracted ½ twice)
4) Subtract again: 20 -½ - ½- ½= 20 - ½ *3 (you just subtracted ½ three times)
5) Subtract again: 20 -½ - ½- ½- ½= 20 - ½ *4
6) Subtract again: 20 -½ - ½- ½- ½- ½= 20 - ½ *5
7) Subtract again: 20 -½ - ½- ½- ½- ½- ½= 20 - ½ *6
8) Again: 20 -½ - ½- ½- ½- ½- ½- ½= 20 - ½ *7

So what if you subtracted ½ "x" times?

$$f(x) = 20 – (½)*x$$
$$\text{or } f(x) = - ½ x + 20$$

Situation 1: You have $230 when you go to work, and you earn $5 an hour in tips.	*Situation 2: You have $230, and you are playing "Who Wants to Be a Hundredaire." If you get a question right, your money quintuples.*
f(x) =	**f(x) =**

Instructions for Collaborative Activity:

1. You have been grouped in pairs based on your answers to the Formative Assessment "The Birthday Gift Problem"
2. After our whole-class interactive introduction, you should work in pairs on the collaborative task. Your first goal is to match the situation cards you are given with the mathematical equation (functions).
3. When you are finished, raise your hand and I will give you a third set of graph cards to match the graph with the appropriate situation/equation cards.
4. Two of the cards were blank; write your own equations on the blank equation cards for the situations (2) that had no match.
5. After a plenary discussion (I will tell you the correct matches at that time), you will have the opportunity to return to your original assessment tasks, and try to improve their own responses.
6. Then you will take a post-assessment to show growth in the area of linear vs. exponential functions.

*Exploring Paths

INTRODUCTION TO THIS FORMATIVE ASSESSMENT LESSON

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Utilize what they already know about linear functions and exponential functions in the context of different graphs.
- Reasoning qualitatively, compares linear and exponential models verbally, numerically, algebraically, and graphically.

COMMON CORE STATE STANDARDS

This lesson involves mathematical content in the standards from across the grades, with emphasis on:

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. ★ (Limit to linear and exponential functions.)
MCC9-12.F.LE.1 - Distinguish between situations that can be modeled with linear functions and with exponential functions.

- o Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
- o Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- o Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

This lesson involves a range of mathematical practices, with emphasis on:

SMP4. Model with mathematics.
SMP7. Look for and make use of structure.
SMP8. Look for and express regularity in repeated reasoning.

INTRODUCTION

This lesson is structured in the following way:

Before the Lesson, students work individually on an assessment task that is designed to reveal their current understandings and difficulties. You then review their work and create questions for students to answer in order to improve their solutions.

At the Start of the Lesson, a whole class discussion should include real-world situations resulting in both a linear model and an exponential model. See teacher guide pages.

During the Lesson, students work in pairs on a collaborative discussion task in which:

- Students first match verbal situation (Card Set A) with the modeling function (Card Set B).
- Students will then match a possible graph (Card Set C) that could go with the verbal situation and modeling function.

After the Whole-Group Class Discussion, students return to their original assessment tasks, and try to improve their own responses.

MATERIALS REQUIRED

Each individual student will need a copy of the Pre-Assessment and a copy of the Post-Assessment

Each small group of students will need:

a set of card set A, a set of card set B, a set of card set C, Poster Paper, Glue Sticks or Tape

TEACHER PREP REQUIRED

Teacher, be advised that prior to the lesson, the following preparations/copies will need to be made:

- Copies of the pre and post-assessment for each student
- Group sets of card sets A, B, C (copy and cut apart)
- Poster paper , glue or tape
- Projector resource slide of linear and exponential graph

For Pre-Assessment: 15 minutes For Lesson: 45 minutes For Post: 15 minutes

FRAMING FOR THE TEACHER:

CCGPS Mathematics compares exponential to linear functions during 9th grade in order for students to understand the difference between a constant rate of change and a non-constant rate of change. These two are chosen for comparison because of the arithmetic vs. geometric link to sequences.

Students learn that a constant difference creates an arithmetic sequence when the domain is comprised of integer values, whereas a constant factor creates a geometric sequence under the same circumstances. This task highlights the differences between the two types of functions and their rates of change. The collaborative activity requires that student understand how to create equations to model situations that may be described by one of these types of functions.

FRAMING FOR THE KIDS:

Say to the students:

This activity will take about _2_ *days for us to complete.*

The reason we are doing this is to be sure that you understand the difference between linear functions with a constant rate of change and exponential functions with a non-constant rate of change before we move on to a new idea.

You will have a chance to work with a partner to correct any misconceptions that you may have. After the partner work, you will be able to show me what you have learned!

PRE-ASSESSMENT BEFORE THE LESSON

ASSESSMENT TASK:

Name of Assessment Task: *Understanding Paths*

Time This Should Take: 15 minutes

Have the students do this task in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and to find out the kinds of difficulties students have with it. You will them be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the Pre-Assessment:

Briefly introduce the task and help the class to understand the problem and its context.

Spend 15 minutes working individually on this task. Read through the task and try to answer it as carefully as you can. Show all your work so that I can understand your reasoning. Don't worry if you can't complete everything. There will be a lesson that should help you understand these concepts better. Your goal is to be able to confidently answer questions similar to these by the end of the next lesson.

Students should do their best to answer these questions, without teacher assistance. It is important that students are allowed to answer the questions on their own so that the results show what students truly do not understand.

Students should not worry too much if they cannot understand or do everything on the pre-assessment, because in the next lesson they will engage in a task which is designed to help them with *Knowing how to read a graph from left to right.* Explain to students that by the end of the next lesson, they should expect to be able to answer questions such as these confidently.

This is their goal.

Linear & Exponential Functions

1. Fill in the tables below to represent the indicated function.

Linear – positive slope

x	y
5	
10	
15	
20	

Linear – Negative slope

x	y
5	
10	
15	
20	

Exponential Growth

x	y
5	
10	
15	
20	

Exponential Decay

x	y
5	
10	
15	
20	

2. David says that the values of $f(x) = 6^x$ will increase faster than the values of the linear function $f(x) = 6x$. Do you agree or disagree? Justify your answer.

3. Given the tables represented below which shows a linear relation and which shows an exponential relation? Explain your reasoning.

Weeks	Price
1	$ 105.00
2	$ 95.00
3	$ 85.00
4	$ 75.00
5	$ 65.00

Weeks	Price
1	$ 105.00
2	$ 95.00
3	$ 88.00
4	$ 84.00
5	$ 83.00

COLLABORATION TIME/READING STUDENTS RESPONSES

You Will Not "Grade" These!

Collect students' responses to the task. It is helpful to read students' responses with colleagues who are also analyzing student work. Make notes (on your own paper, not on their pre-assessment) about what their work reveals about their current levels of understanding, and their approaches to the task. You will find that the misconceptions reveal themselves and often take similar paths from one student to another, and even from one teacher to another. Some misconceptions seem to arise very organically in students' thinking. Pair students in the same classes with other students who have similar misconceptions. This will help you to address the issues in fewer steps, since they'll be together. (Note: pairs are better than larger groups for FAL's because both must participate in order to discuss!)

You will begin to construct Socrates-style questions to try and elicit understanding from students. We suggest you write a list of your own questions, however some guiding questions and prompts are also listed below as a jumping-off point.

GUIDING QUESTIONS

COMMON ISSUES	*SUGGESTED QUESTIONS AND PROMPTS*
Knowing how to read a graph from left to right.	• *How do you read a graph?*
Understanding slope	• *What is slope?* • *How can you find the slope?* • *How can you find slope using two points?* • *What does it say about a graph when the slope decreases? Increases? Is constant?*
Comparing a number raised to a power versus a number times a number. Ex. 6^3 and 6(3)	• *What does it mean to raise a number to a power?* • *Compare a number to a power with a number times a number.*

LESSON DAY

SUGGESTED LESSON OUTLINE:

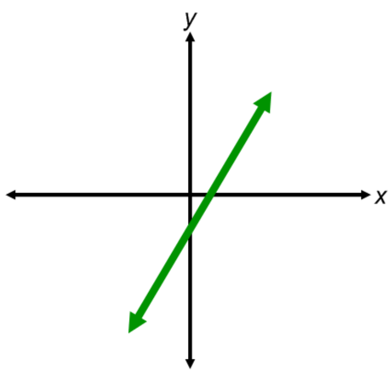
Part 1: Whole-Class Introduction: Time to Allot: (10 minutes)

Display the "Warm Up" question provided.

Whole-class interactive introduction (10 minutes)

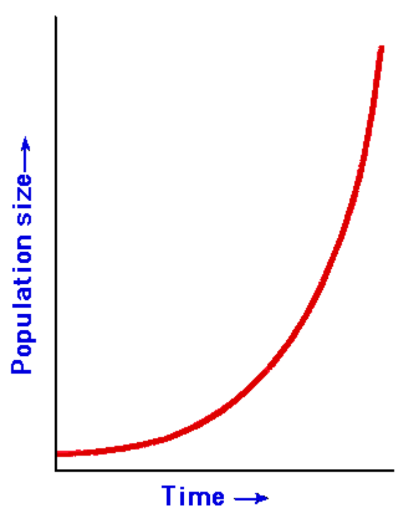
Discuss linear and exponential functions. There is a slide in Teacher Resources that the teacher may display during this discussion.

Linear   Real-world Situation: Hourly Wage



Possible Equation: ___________________

Exponential   Real-World Situation: Population increasing over time.



Possible Equation: __________________________

Part 2: Collaborative Activity: Time to Allot: (20 minutes)

Put students into their pairs according to your analysis of student errors.

- Group students in pairs by common errors found in Formative Assessment.
- Students are given card sets A and B, already cut apart.
- Students should match one card from A with one card from B.
- After 10-15 minutes, give students card set C to match with previous sets. There are only 4 cards in card set C and so each card will match multiple cards from Sets A and B.
- Using construction paper glue the matching cards from sets A & B underneath the appropriate graph from set C.

Do/Say the Following:

Make a note of student approaches to the task.

Listen and watch students carefully. Note different student approaches to the task and any common mistakes. For example, students may

- Group all the positive slopes together and all the negative slopes together.
- Match cards that have the same numbers.
- Compare graphs with constant slopes and those with rapidly changing slopes.

Support student reasoning

Try not to make suggestions that steer students towards a particular "correct" answer or response. Instead, ask questions to help students to reason together.

If you find one student has produced a correct response, challenge another student in the group to provide an explanation.

Example:

- *Why do you think this is an exponential growth?*
- *Why do you think this is an exponential decay?*
- *What words in this problem made you think it is a linear function?*
- *What words in this problem made you think it is an exponential function?*

If you find students have difficulty articulating their decisions, use the sheet Suggested Questions and Prompts to support your own questioning of students.

Sample ways to jump-start students' work in the group collaboration:

• *How do you read a graph?*
• *What is slope?* • *How can you find the slope?*

- *What does it mean to raise a number to a power?*
- *Compare a number to a power with a number times a number.*

If the whole class is struggling on the same issue, you could write a couple of questions on the board and hold an interim, whole-class discussion. You could ask students who performed well in the assessment to help struggling students.

Allow students time to collaborate as much as possible.

During the Collaborative Activity, the Teacher has 3 tasks:

- Circulate to students' whose errors you noted from the pre-assessment and support their reasoning with your guiding questions.
- Circulate to other students also to support their reason in the same way.
- Make a note of student approaches for the summary (plenary discussion). Some students have interesting and novel solutions!

Part 3: Plenary (Summary) Discussion: Time to Allot: (15 minutes)

Gather students together, share solutions. Discussion prompts should be made up of your original guiding questions and notes about student approaches. Some other discussion prompts are listed below:

NOTE: *"Scribing" helps to increase student buy-in and participation. When a student answers your question, write the student's name on the board and scribe his/her response quickly. You will find that students volunteer more often when they know you will scribe their responses – this practice will keep the discussions lively and active!*

- *Students will share their card matching activity..*
- *Why did you choose the given graph for the given situation?*
- *How can the equation of a function help us to determine the shape of the graph?*
- *In a given equation, what effect does the "m" have on the graph? The "n"?*

Part 4: Improving Solutions to the Assessment Task Time to Allot: (20 minutes)

The Shell MAP Centre advises handing students their original assessment tasks back to guide their responses to their new Post-Assessment (which is sometimes the exact same as the Pre-Assessment). In practice, some teachers find that students mindlessly transfer incorrect answers from their Pre- to their Post-Assessment, assuming that no "X" mark means that it must have been right. . Until students become accustomed to UNGRADED FORMATIVE assessments, they may naturally do this. Teachers often report success by handing students a list of the guiding questions to keep in mind while they improve their solutions.

Practice will make perfect, and teachers should do what makes them most comfortable with their students/finds misconceptions!

Look at your original responses and think about what you have learned this lesson.

Using what you have learned, try to improve your work.

If you have not added questions to individual pieces of work then write your list of questions on the board.

Students should select from this list only the questions they think are appropriate to their own work.

If you find you are running out of time, then you could set this task in the next lesson or for homework.

ASSESSMENT TASK: Key

ANSWERS WILL VARY

Linear – positive slope

x	y
5	5
10	15
15	25
20	35

Linear – Negative slope

x	y
5	-11
10	-16
15	-21
20	-26

Exponential Growth

x	y
5	32
10	1,024
15	32,768
20	1,048,576

Exponential Decay

x	y
5	1.9
10	.6
15	.2
20	.07

2. David says that the values of $f(x) = 6^x$ will increase faster than the values of the linear function $f(x) = 6x$. Do you agree or disagree? Justify your answer.

Agree, $f(x) = 6^x$ grows exponentially and $f(x) = 6x$ is a linear function growing at a constant rate.

3. Given the tables represented below which shows a linear relation and which shows an exponential relation?

Weeks	Price
1	$ 105
2	$ 95
3	$ 85
4	$ 75
5	$ 65

Weeks	Price
1	$ 105
2	$ 95
3	$ 86
4	$ 78
5	$ 71

Explain your reasoning.

The first table is linear because it shows a constant rate of -10 and the second is exponential and shows a non-constant rate of change with a constant ratio of 0.9.

4. Write a real-life situation that could be represented by the function $f(t) = 3000(1.07)^t$. Explain.

Answers will vary : Deposit $3000 in a bank that pays 7% yearly rate.

Collaborative Activity (Answer Key)

A1	B3	C4
A2	B5	C4
A3	B4	C1
A4	B6	C1
A5	B1	C2
A6	B2	C3
A7	B8	C3
A8	B7	C2
A9	B10	C1
A10	B9	C4

Linear & Exponential Functions

1. Fill in the tables below to represent the indicated function.

Linear – positive slope

x	y
5	
10	
15	
20	

Linear – Negative slope

x	y
5	
10	
15	
20	

Exponential Growth

x	y
5	
10	
15	
20	

Exponential Decay

x	y
5	
10	
15	
20	

2. David says that the values of $f(x) = 6^x$ will increase faster than the values of the linear function $f(x) = 6x$. Do you agree or disagree? Justify your answer.

3. Given the tables represented below which shows a linear relation and which shows an exponential relation?

Weeks	Price
1	$ 105
2	$ 95
3	$ 85
4	$ 75
5	$ 65

Weeks	Price
1	$ 105
2	$ 95
3	$ 86
4	$ 78
5	$ 71

Explain your reasoning.

4. Write a real-life situation that could be represented by the function $f(t) = 3000(1.07)^t$. Explain.

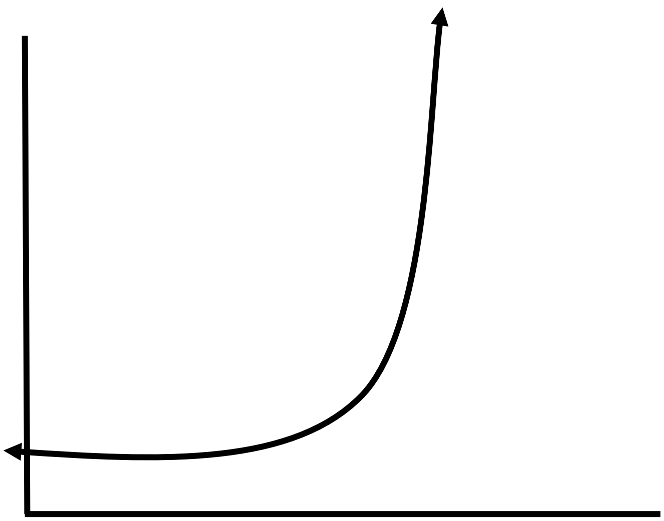
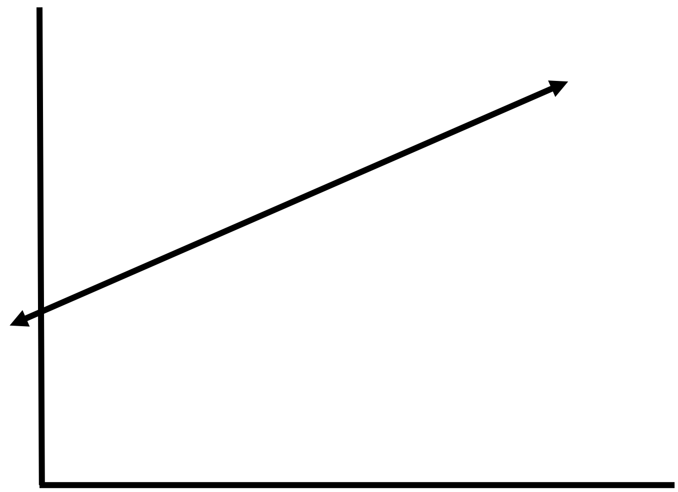
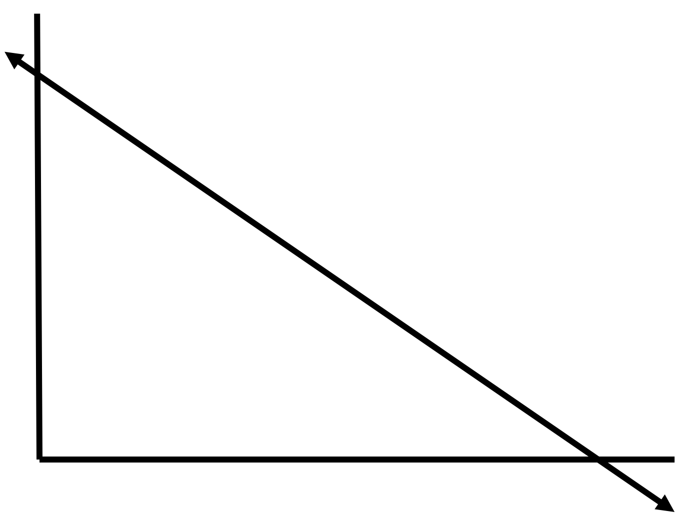
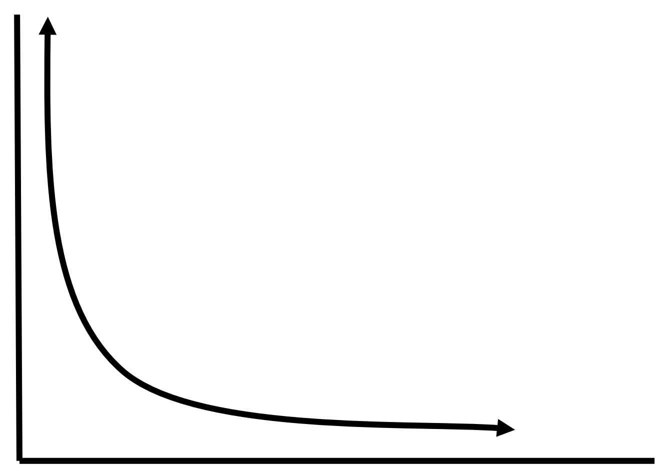
COLLABORATIVE ACTIVITY

Name of Assessment Task:

Card Set A: (Verbal)		
A1. The population of Waycross in 2009 is 14,200. The population has dropped by 4% since 2000.	A2. The purchase price of a new car is $18,000. The value of the car depreciates yearly by 12%.	A3. A mouse population of 14,200 is increasing in size at a rate of 4% per year.
A4. The value of a certain stock is $18,000 and is growing annually at a rate of 12%.	A5. A mechanic charges $40 per hour plus a flat rate of $25.	A6. The temperature was 25° and it dropped 40° every 30 minutes.
A7. Mary had *n* dollars in the bank and spends *m* dollars per week on CD's.	A8. Mom pays Mary *n* dollars each semester to keep her room clean and *m* dollars per A on her report card.	A9. Mary's hometown is experiencing a yearly population growth of *n*%. The original population is *m*.
A10. The price of oil is *m* per barrel. Because of low demand, the price has decreased n% per week.		

Card Set B (Modeling Function)	
B1. $y = 40x + 25$	B2. $y = -40x + 25$
B3. $y = 14{,}200(.96)^t$	B4. $y = 14{,}200(1.04)^t$
B5. $y = 18{,}000(.88)^t$	B6. $y = 18{,}000(1.12)^t$
B7. $y = mx + n$	B8. $y = -mx + n$
B9. $y = m\left(1 - \frac{n}{100}\right)^t$	B10. $y = m\left(1 + \frac{n}{100}\right)^t$

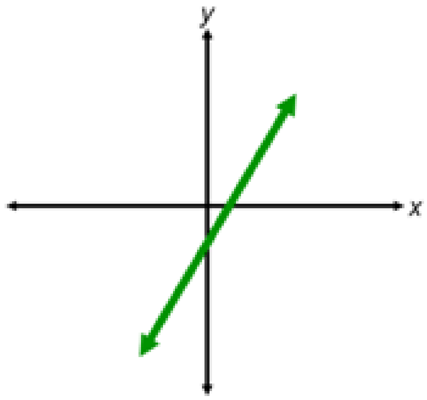
Card Set C (Possible Graphs)

C1	C2
	
C3	C4
	

Collaborative Activity Instructions:

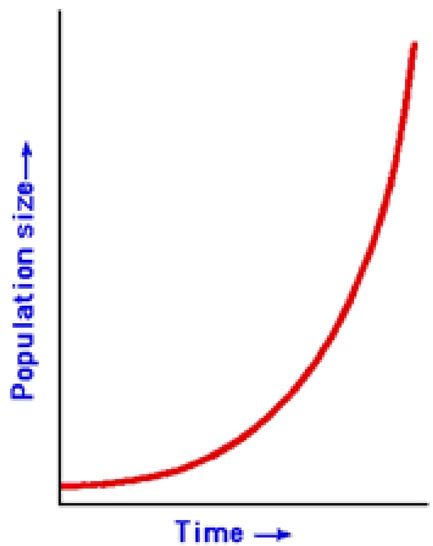
1) You have been grouped in pairs.
2) You are given card sets A (verbal situations) and B (modeling functions), already cut apart.
3) Read each situation carefully. You and your partner should match each situation from card set A with a modeling function from card set B. Discuss to ensure that you both agree.
4) Once you are sure that you have completed the matching correctly and you are instructed to do so, take out card set C. Match card set C to card sets A & B previously matched. Note, more than one set of matched A&B cards will be used with each graph in set C.
5) On the construction paper provided, glue or tape the matching sets down (grouped under card set C). Be sure to explain in writing your reasoning.

<u>Linear</u> Real-world Situation: Hourly Wage



Possible Equation: ____________________

<u>Exponential</u> Real-World Situation: Population increasing over time.



Possible Equation: ____________________________

High Functioning! (Practice Task)

Introduction

Students are introduced to transformations of functions. In this task, students will focus on vertical translations of graphs of linear and make connections to the *y*-intercept. Transformations are approached from the perspective of using a constant, *k*, to make changes to the function. They will be able to answer questions such as: What happens when you add *k* to the input? What happens when *k* is a negative number?

Mathematical Goals

- Use graphs of vertical translations to determine function rules.
- Relate vertical translations of a linear function to its *y*-intercept.
- Identify even and odd functions

Essential Questions

- How are functions affected by adding or subtracting a constant to the function?
- How does the vertical translation of a linear function model translations for other functions?

Common Core Georgia Performance Standards

MCC9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. *(Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.)*

Standards for Mathematical Practice

3. Construct viable arguments and critique the reasoning of others.
Students will make predictions based on observations of shifts and defend their reasoning.

6. Attend to precision.
Students will need to graph equations precisely and use specific points to show the shifts.

7. Look for and express regularity in repeated reasoning.
Students will look for patterns and use them to determine general rules.

Background Knowledge

- Students can graph linear equations.
- Students can read, write, and interpret function notation.
- Students have a basic idea of rotations and reflections and/or rotational and line symmetry.

Common Misconceptions

- Students may think that odd and even functions are denoted by odd/even numbers.
- Students may make very general/tangential observations and need to be directed.
- Students may not understand function notation well enough to write rules.

Materials

- Graphing Calculator (optional)

Grouping

- Partner / Individual

Differentiation

Extension:

- Students can use a graphing calculator to model more examples of linear and exponential functions and see the transformations that occur.

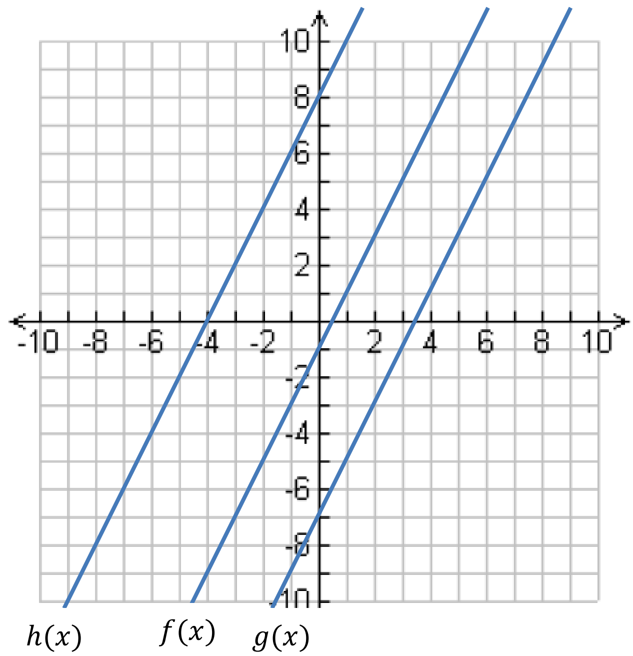
Intervention:

- Use strategic grouping to pair struggling students with resident experts.

Formative Assessment Questions

- How are translations of linear functions related to the y-intercept?
- How can we determine the relative location of a function given a translation in function notation?

High Functioning – Teacher Notes



1. Graph and label the following functions.

$f(x) = 2x - 1$
$g(x) = 2x - 7$
$h(x) = 2x + 8$

2. What observations can you make about the three functions? Be sure to include observations about the characteristics and the location of the functions.

Solution:
Answers may vary. Students should not that the slopes of the functions are the same and that the lines are parallel. They should see that the y-intercept has changed in each function causing the function to move up and down.

3. Analyze specifically what happens to the y-intercepts $f(0)$, $g(0)$, and $h(0)$ in the three functions.
How does the *y*-intercept change from…

a. $f \rightarrow g$?
Solution:
from -1 to -7; down 6

b. $g \rightarrow h$?
Solution:
from -7 to +8; up 15

c. $f \rightarrow h$?
Solution:
from -1 to +8; up 9

d. $h \rightarrow f$?
Solution:
from +8 to -1; down 9

4. Find…

a. $f(1)$

Solution:

2(1) – 1 = 1

b. $g(1)$

Solution:

2(1) – 7 = -5

c. $h(1)$

Solution:

2(1) + 8 = 10

5. What changes in the output as you go from…

a. $f(1) \rightarrow g(1)$?

Solution:

from 1 to -5; down 6

b. $g(1) \rightarrow h(1)$?

Solution:

from -5 to 10; up 15

c. $f(1) \rightarrow h(1)$?

Solution:

from 1 to 10; up 9

d. $h(1) \rightarrow f(1)$?

Solution:

from 10 to 1; down 9

6. Comparing your answers to 3 and 5, what predictions can you make about other inputs?

Solution:

The shifts in #3 (input x = 0) and #5 (input x = 1) are the same. We can predict that the shifts will be the same regardless of input.

7. Write an algebraic rule for the following shifts.

a. $f(x) \to g(x)$

Solution:

$g(x) = f(x) - 6$

b. $g(x) \to h(x)$

Solution:

$h(x) = g(x) + 15$

c. $f(x) \to h(x)$

Solution:

$h(x) = f(x) + 9$

d. $h(x) \to f(x)$

Solution:

$f(x) = h(x) - 9$

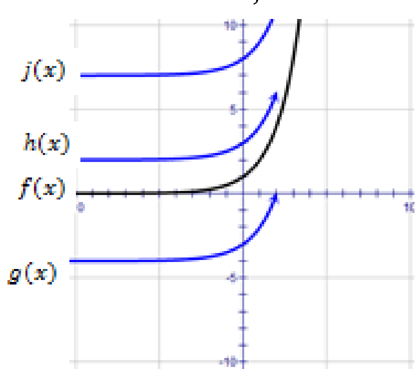
8. Write a general rule for a vertical translation.

Solution:

Answers may vary. A vertical translation is a shift up or down on the coordinate plane caused by the addition (or subtraction) of a constant to the function.

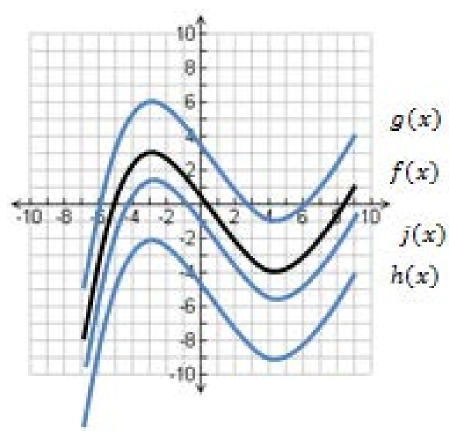
Using the functions below, draw and label the given translations.

9.



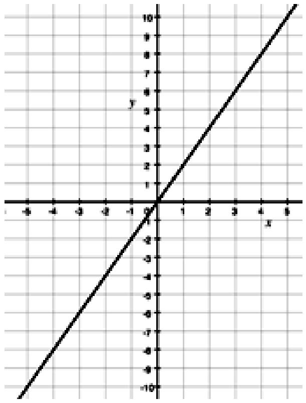
a. $g(x) = f(x) - 4$
b. $h(x) = f(x) + 2$
c. $j(x) = f(x) + 7$

10.



a. $g(x) = f(x) + 3$
b. $h(x) = f(x) - 5$
c. $j(x) = f(x) - 2$

11. The graph of the **odd function** $f(x) = 2x$ is shown below. Fill in the table.



$f(-1) =$ ***-2***	$f(1) =$ ***2***
$f(-4) =$ ***-8***	$f(4) =$ ***8***
$f(-8) =$ ***-16***	$f(8) =$ ***16***
$f(-25) =$ ***-50***	$f(25) =$ ***50***

12. What characteristics do you notice about odd functions based on the points in the table?

Solution:

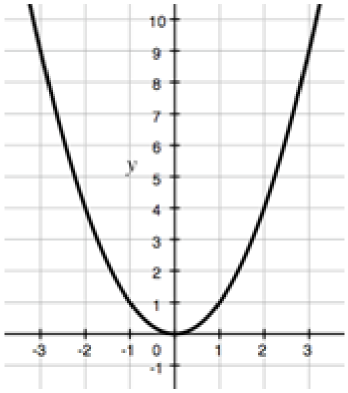
Opposite inputs give opposite outputs.

13. Now rotate your paper 180˚ so that the graph is upside down. What further observation can you make about characteristics of odd functions?

Solution:

Odd functions have rotational symmetry about the origin. Since f(x) is a linear function passing through the origin, its image looks the same after the graph is rotated so the graph is upside down.

14. The graph of the **even function** $g(x) = x^2$ is shown below. Fill in the table.



$g(-1) =$ ***1***	$g(1) =$ ***1***
$g(-2) =$ ***4***	$g(2) =$ ***4***
$g(-3) =$ ***9***	$g(3) =$ ***9***
$g(-4) =$ ***16***	$g(4) =$ ***16***

15. What characteristics do you notice about even functions based on the table?

Solution:

Opposite inputs give the same output.

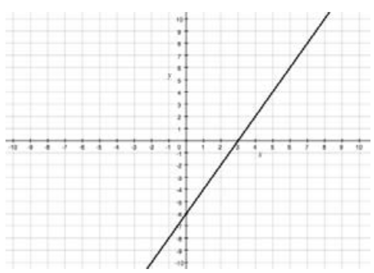
16. Fold the graph of $g(x)$ along the y-axis. What further observations can you make about the characteristics of even functions?

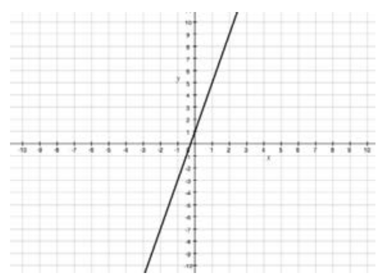
Solution:

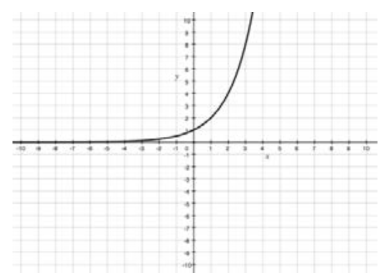
Even functions have symmetry over the y-axis. If an even function is reflected over the y-axis it will land back on itself.

17. Graph the three functions below and explain why they are neither even nor odd.

$f(x) = 2x - 6$ $\qquad$ $g(x) = 4x + 1$ $\qquad$ $h(x) = 2^x$







18. Now demonstrate algebraically that the three functions are neither even nor odd by using the inputs 2 and -2.

Solution:

$f(2) = 2(2) - 6 = -2$
$f(-2) = 2(-2) - 6 = -10$
These are neither the same nor opposite.

$g(2) = 4(2) + 1 = 9$
$g(-2) = 4(-2) + 1 = -7$
These are neither the same nor opposite.

$h(2) = 2^2 = 4$
$h(-2) = 2^{-2} = ¼$
These are neither the same nor opposite.

Practice Task: High Functioning

Name________________________________ **Date_________________**

Mathematical Goals

- Use graphs of vertical translations to determine function rules.
- Relate vertical translations of a linear function to its *y*-intercept.
- Identify even and odd functions

Essential Questions

- How are functions affected by adding or subtracting a constant to the function?
- How does the vertical translation of a linear function model translations for other functions?

Common Core Georgia Performance Standards

MCC9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. *(Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.)*

Standards for Mathematical Practice

3. Construct viable arguments and critique the reasoning of others.
6. Attend to precision.
7. Look for and express regularity in repeated reasoning.

Practice Task: High Functioning

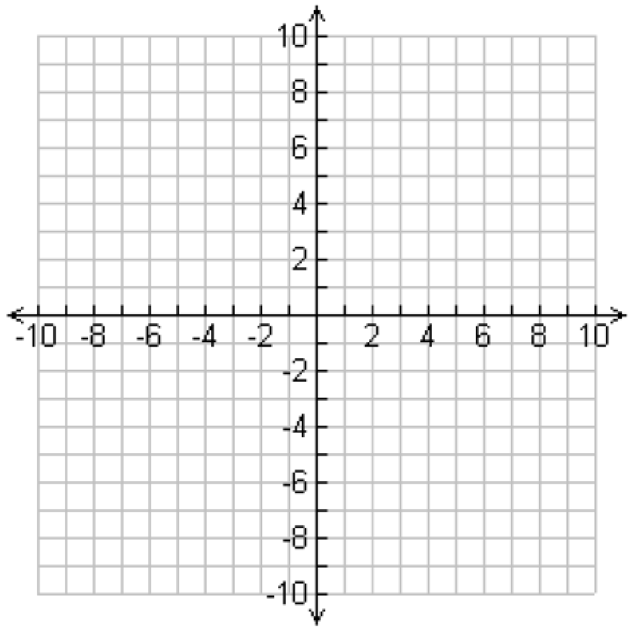
Name________________________________ **Date**__________________

19. Graph and label the following functions.

$f(x) = 2x - 1$

$g(x) = 2x - 7$

$h(x) = 2x + 8$



20. What observations can you make about the three functions? Be sure to include observations about the characteristics and the location of the functions.

21. Analyze specifically what happens to the y-intercepts $f(0)$, $g(0)$, and $h(0)$ in the three functions.
How does the y-intercept change from…

a. $f \rightarrow g$?

b. $g \rightarrow h$?

c. $f \rightarrow h$?

d. $h \rightarrow f$?

22. Find…

a. $f(1)$

b. $g(1)$

c. $h(1)$

23. What changes in the output as you go from…

a. $f(1) \rightarrow g(1)$?

b. $g(1) \rightarrow h(1)$?

c. $f(1) \rightarrow h(1)$?

d. $h(1) \rightarrow f(1)$?

24. Comparing your answers to 3 and 5, what predictions can you make about other inputs?

25. Write an algebraic rule for the following shifts.

a. $f(x) \rightarrow g(x)$

b. $g(x) \rightarrow h(x)$

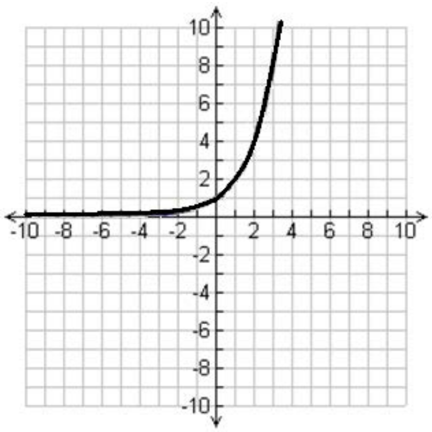
c. $f(x) \rightarrow h(x)$

d. $h(x) \rightarrow f(x)$

26. Write a general rule for a vertical translation.

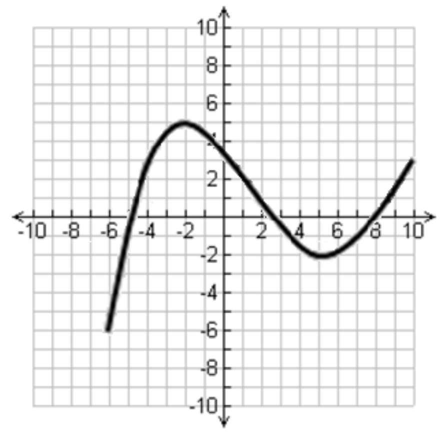
Using the functions below, draw and label the given translations.

27.



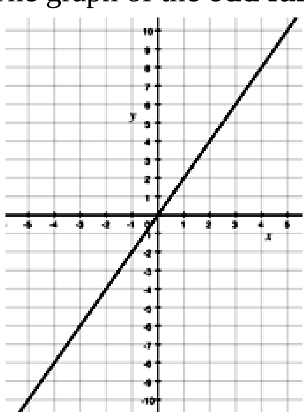
a. $g(x) = f(x) - 4$
b. $h(x) = f(x) + 2$
c. $j(x) = f(x) + 7$

28.



d. $g(x) = f(x) + 3$
e. $h(x) = f(x) - 5$
f. $j(x) = f(x) - 2$

29. The graph of the **odd function** $f(x) = 2x$ is shown below. Fill in the table.

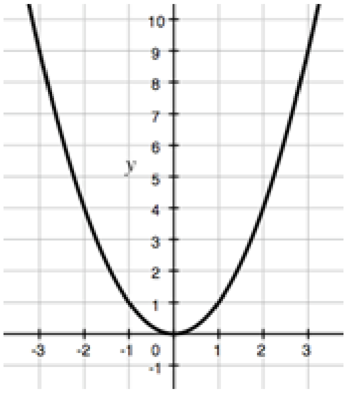


$f(-1) =$	$f(1) =$
$f(-4) =$	$f(4) =$
$f(-8) =$	$f(8) =$
$f(-25) =$	$f(25) =$

30. What characteristics do you notice about odd functions based on the points in the table?

31. Now rotate your paper 180˚ so that the graph is upside down. What further observation can you make about characteristics of odd functions?

32. The graph of the **even function** $g(x) = x^2$ is shown below. Fill in the table.



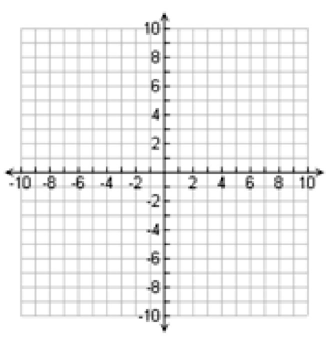
$g(-1) =$	$g(1) =$
$g(-2) =$	$g(2) =$
$g(-3) =$	$g(3) =$
$g(-4) =$	$g(4) =$

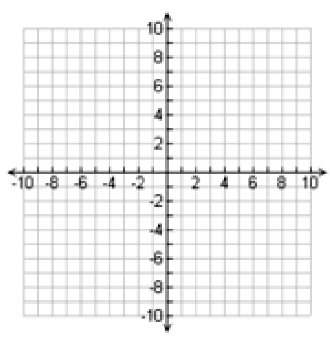
33. What characteristics do you notice about even functions based on the table?

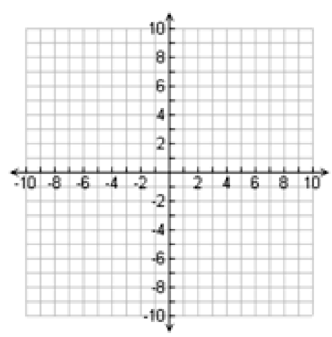
34. Fold the graph of $g(x)$ along the y-axis. What further observations can you make about the characteristics of even functions?

35. Graph the three functions below and explain why they are neither even nor odd.

$f(x) = 2x - 6$ $\qquad$ $g(x) = 4x + 1$ $\qquad$ $h(x) = 2^x$







36. Now demonstrate algebraically that the three functions are neither even nor odd by using the inputs 2 and -2.

Summing It Up: Putting the "Fun" in Functions (Culminating Task)

Introduction

In this culminating task, students will create a functions booklet or webpage using googlesites (or a hosting site of similar nature). They will use all of their resources and previous tasks to help them think through ways to create scenarios, examples, and model the standards in this unit. Finally students will write a one page reflection on what they've learned throughout the unit and create a work cited page using MLA format of all resources used to design their book or webpage.

Mathematical Goals

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations
- Building new functions from existing functions
- Construct and compare linear and exponential models and solve problems

Essential Questions

- How can I use and apply what I have learned about functions?

Common Core Georgia Performance Standards

MCC9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and *x* is an element of its domain, then *f(x)* denotes the output of f corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *(Draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences.)*

MCC9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~. *(Focus on linear and exponential functions.)*

MCC9-12.F.IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *(Focus on linear and exponential functions.)*
MCC9-12.F.IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *(Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers.)*
MCC9-12.F.IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*
MCC9-12.F.IF.7a	Graph linear ~~and quadratic~~ functions and show intercepts, maxima, and minima.
MCC9-12.F.IF.7e	Graph exponential ~~and logarithmic~~ functions, showing intercepts and end behavior, and ~~trigonometric functions, showing period, midline, and amplitude.~~
MCC9-12.F.IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*
MCC9-12.F.BF.1	Write a function that describes a relationship between two quantities. *(Limit to linear and exponential functions.)*
MCC9-12.F.BF.1a	Determine an explicit expression, a recursive process, or steps for calculation from a context. *(Limit to linear and exponential functions.)*
MCC9-12.F.BF.1b	Combine standard function types using arithmetic operations. *(Limit to linear and exponential functions.)*
MCC9-12.F.BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
MCC9-12.F.BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\ f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. *(Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.)*
MCC9-12.F.LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions.

MCC9-12.F.LE.1a	Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
MCC9-12.F.LE.1b	Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
MCC9-12.F.LE.1c	Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
MCC9-12.F.LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
MCC9-12.F.LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, ~~quadratically, or (more generally) as a polynomial function.~~
MCC9-12.F.LE.5	Interpret the parameters in a linear or exponential function in terms of a context. *(Limit exponential functions to those of the form $f(x) = b^x + k$.)*

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Students should use all eight SMPs when exploring this task.

Background Knowledge

- Students will apply everything they have learned in this unit.

Common Misconceptions

- Address misconceptions brought to light during the rest of the unit.

Materials

- Unit portfolio
- Culminating task guide
- Computer
- Graphing calculator
- Paper and pencil for planning
- Rubric

Grouping

- Individual / Partners

Summing It Up: Putting the "Fun" in Functions – Teacher Notes

In this unit you have learned the concept of a function and how to use function notation, interpret functions that arise in applications in terms of the context, analyze functions using different representations, building new functions from existing functions, and construct and compare linear and exponential models and solve problems.

Using the guide provided, you will construct a function booklet or create a webpage for students who will learn about linear and exponential functions next year. Before designing your booklet or webpage, use the guide to plan your pages or links. Make sure your use the graphing calculator to test all of your models prior to adding them to the booklet or webpage. Use the checklist to ensure that all parts of the task have been addressed.

***Comment*:**
Walk through the guide/checklist with the students. Model using examples and discuss the use of technology (such as spreadsheets to create graphs and graphing calculators to check multiple representations of a function) in completing this culminating task. Remind students to label all parts, tables, the scale and axis for all graphs. Also, remind them to use complete thoughts. This task may take about three days to complete.

Examples for various parts of the assignment are given below.

Summing it Up: Putting the "Fun" in Functions Booklet / Webpage
Planning Guide and Checklist

- ☐ **Booklet Cover/Home link on webpage: (1 point)**
  - ☐ Give your booklet/page a title
  - ☐ Use a mathematical symbol or symbols that are unique to learning about functions on your cover or home link
  - ☐ Include your name, date, and class period
- ☐ **Table of Contents Page or Link: (1 point)**
  - ☐ Page number for unit Definitions or link to Definitions
  - ☐ Page number or link for Function Notation
  - ☐ Page number or link for Interpreting Linear and Exponential Functions Arising in Applications
  - ☐ Page number or link for Analyzing Linear and Exponential Functions
  - ☐ Page number or link for Building Functions
  - ☐ Page number or link for Constructing and Comparing Linear and Exponential Models
  - ☐ Page number or link for Unit Reflection Summary
  - ☐ Page number or link for Works Cited
- ☐ **Definitions Page or Link: (8 points)**
  - ☐ Choose at least 10 important vocabulary words from the unit to define
  - ☐ Provide a model or example of each vocabulary word. (You may use symbols, graphs, tables, or pictures.)

- ☐ **Function Notation Page or Link: (20 points)**
  - ☐ Provide at least one example of a domain and range that illustrates a function and explain why it is a function.
  - ☐ Provide at least one example of a domain and range that is not a function and explain why.
  - ☐ Create one real world scenario in which function notation may be used to model a linear function. Show how the function might be evaluated for inputs in the domain based on the context of the scenario.
    Example: Marcus currently owns 200 songs in his iTunes collection. If he added 15 new songs each month, how many songs will he own in a year? The initial value for his function is 200 and the rate of change is 15 per month. With this information, we can write $f(x) = 15x + 200$. To show how to evaluate this function for the number of songs that he would have in one year, we would input 12.
  - ☐ Create one real world scenario in which function notation may be used to model an exponential function. Show how the function might be evaluated for inputs in the domain based on the context of the scenario.
    Example: The population of the popular town of Smithville in 2003 was estimated to be 35,000 people with an annual rate of increase (growth) of about 2.4%. We can write $f(x) = ab^x$, with x as the number of years we would input.
  - ☐ Use the scenarios to create a recursive formula
- ☐ **Interpreting Linear and Exponential Functions Arising in Applications: (20 point)**
  - ☐ Create a story that would generate a linear or exponential function and describe the meaning of key features (intercepts, intervals where the function is increasing, decreasing, positive, or negative; end behaviors) of the graph as they relate to the story.
  - ☐ Show the graph of your function and relate the domain to the quantitative relationship it describes. Describe the rate of change for a linear function or the rate a change over an interval for an exponential function.
    Example for the quantitative part of the story: You are hoping to make a profit on the school play and have determined the function describing the profit to be $f(t)= 8t - 2654$ where t is the number of tickets sold. What is a reasonable domain for this function? Explain.
- ☐ **Analyzing Linear and Exponential Functions: (10 points)**
  - ☐ Create one linear function expressed symbolically. Graph the function using technology (print for booklet or paste on web)
  - ☐ Create one exponential function expressed symbolically. Graph the function using technology (print for booklet or paste on web)
  - ☐ Create two different linear functions. Show one algebraically and the other using a verbal description. Compare the two functions.
    Example: Which has a greater slope?
    $f(x) = x + 5$ or a function representing the number of bottle caps in a shoebox where 5 are added each time
- ☐ **Building Functions: (10 points)**

- ☐ Explain how to find an explicit expression, a recursive process, or steps for calculation to complete a sequence/pattern. Write the sequence both recursively and with an explicit formula.
  Example: Find the number of objects (squares, toothpicks, etc.) needed to make the next three patterns in a series. Show the recursive and explicit formula for the pattern created.
- ☐ Use all four operations to illustrate combinations of linear and/or exponential functions. (4 problems and solutions).
- ☐ Explain vertical translations. Create three vertical translations for a linear or exponential function. Graph all three on a single axes and compare and contrast the graphs.

☐ **Constructing and Comparing Linear and Exponential Models (20 points)**

- ☐ Design a word problem that involves a linear and exponential model. Use a table or sequence to illustrate the relationships described in the models.
  Example: What's the better deal, earning $1000 a day for the rest of your life or earning $.01 the first day, and doubling it every day for the rest of your life? How do you know? Do you think an 80-year-old would make the same choice? Should she?
- ☐ Explain the constant rate and constant percent rate per unit interval relative to another for the word problem that you designed.
- ☐ Construct the graphs for each model in the word problem that you designed.
- ☐ Compare the linear and exponential models from your word problem. Interpret the parameters.

☐ **Reflection / Summary: (8 points)**

- ☐ Describe your learning journey throughout the unit. Reflect on topics that you found easy to learn and those that were most difficult.
- ☐ Are there any standards that you need more help grasping? Explain. If not, which standards do you have the best grasp? Explain.
- ☐ What advice would you give to other students that will learn about linear and exponential functions in the future?
- ☐ Which task(s) did you find the most beneficial to mastering key standards?
- ☐ Any other insight you would like to share about Unit 3.

☐ **Works Cited: (2 points)**

- ☐ Use MLA format to cite any books, websites, and any other references used to create your booklet or webpage.
  Comment:
  Point students toward online citation tools to assist them.

Culminating Task: Summing It Up: Putting the "Fun" in Functions

Name________________________________ **Date**__________________

Mathematical Goals

- Understand the concept of a function and use function notation
- Interpret functions that arise in applications in terms of the context
- Analyze functions using different representations
- Building new functions from existing functions
- Construct and compare linear and exponential models and solve problems

Essential Questions

- How can I use and apply what I have learned about functions?

Common Core Georgia Performance Standards

MCC9-12.F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and *x* is an element of its domain, then *f(x)* denotes the output of f corresponding to the input *x*. The graph of *f* is the graph of the equation *y = f(x)*. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. *(Draw examples from linear and exponential functions.)*

MCC9-12.F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *(Draw connection to F.BF.2, which requires students to write arithmetic and geometric sequences.)*

MCC9-12.F.IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; ~~and periodicity~~. *(Focus on linear and exponential functions.)*

MCC9-12.F.IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *(Focus on linear and exponential functions.)*

MCC9-12.F.IF.6 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. *(Focus on linear* *functions and intervals for exponential functions whose domain is a subset of the integers.)*

MCC9-12.F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

MCC9-12.F.IF.7a Graph linear ~~and quadratic~~ functions and show intercepts, maxima, and minima.

MCC9-12.F.IF.7e Graph exponential ~~and logarithmic~~ functions, showing intercepts and end behavior, and ~~trigonometric functions, showing period, midline, and amplitude.~~

MCC9-12.F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *(Focus on linear and exponential functions. Include comparisons of two functions presented algebraically.)*

MCC9-12.F.BF.1 Write a function that describes a relationship between two quantities. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.1b Combine standard function types using arithmetic operations. *(Limit to linear and exponential functions.)*

MCC9-12.F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.

MCC9-12.F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. *(Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept.)*

MCC9-12.F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions.

MCC9-12.F.LE.1a Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.

MCC9-12.F.LE.1b Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

MCC9-12.F.LE.1c Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

MCC9-12.F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

MCC9-12.F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, ~~quadratically, or (more generally) as a polynomial function.~~

MCC9-12.F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. *(Limit exponential functions to those of the form $f(x) = b^x + k$.)*

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Culminating Task: Summing It Up: Putting the "Fun" in Functions

Name________________________________ **Date_________________**

In this unit you have learned the concept of a function and how to use function notation, interpret functions that arise in applications in terms of the context, analyze functions using different representations, building new functions from existing functions, and construct and compare linear and exponential models and solve problems.

Using the guide provided, you will construct a function booklet or create a webpage for students who will learn about linear and exponential functions next year. Before designing your booklet or webpage, use the guide to plan your pages or links. Make sure you use the graphing calculator to test all of your models prior to adding them to the booklet or webpage. Use the checklist to ensure that all parts of the task have been addressed.

Booklet/ Webpage Planning Guide and Checklist

- ☐ **Booklet Cover/Home link on webpage: (1 point)**
  - ☐ Give your booklet/page a title
  - ☐ Use a mathematical symbol or symbols that are unique to learning about functions on your cover or home link
  - ☐ Include your name, date, and class period
- ☐ **Table of Contents Page or Link: (1 point)**
  - ☐ Page number for unit Definitions or link to Definitions
  - ☐ Page number or link for Function Notation
  - ☐ Page number or link for Interpreting Linear and Exponential Functions Arising in Applications
  - ☐ Page number or link for Analyzing Linear and Exponential Functions
  - ☐ Page number or link for Building Functions
  - ☐ Page number or link for Constructing and Comparing Linear and Exponential Models
  - ☐ Page number or link for Unit Reflection Summary
  - ☐ Page number or link for Works Cited
- ☐ **Definitions Page or Link: (8 points)**
  - ☐ Choose at least 10 important vocabulary words from the unit to define
  - ☐ Provide a model or example of each vocabulary word. (You may use symbols, graphs, tables, or pictures.)
- ☐ **Function Notation Page or Link: (20 points)**
  - ☐ Provide at least one example of a domain and range that illustrates a function and explain why it is a function.
  - ☐ Provide at least one example of a domain and range that is not a function and explain why.
  - ☐ Create one real world scenario in which function notation may be used to model a linear function. Show how the function might be evaluated for inputs in the domain based on the context of the scenario.
  - ☐ Create one real world scenario in which function notation may be used to model an exponential function. Show how the function might be evaluated for inputs in the domain based on the context of the scenario.

- ☐ Use the scenarios to create a recursive formula

☐ **Interpreting Linear and Exponential Functions Arising in Applications: (20 point)**
- ☐ Create a story that would generate a linear or exponential function and describe the meaning of key features (intercepts, intervals where the function is increasing, decreasing, positive, or negative; end behaviors) of the graph as they relate to the story.
- ☐ Show the graph of your function and relate the domain to the quantitative relationship it describes. Describe the rate of change for a linear function or the rate a change over an interval for an exponential function.

☐ **Analyzing Linear and Exponential Functions: (10 points)**
- ☐ Create one linear function expressed symbolically. Graph the function using technology (print for booklet or paste on web)
- ☐ Create one exponential function expressed symbolically. Graph the function using technology (print for booklet or paste on web)
- ☐ Create two different linear functions. Show one algebraically and the other using a verbal description. Compare the two functions.

☐ **Building Functions: (10 points)**
- ☐ Explain how to find an explicit expression, a recursive process, or steps for calculation to complete a sequence/pattern. Write the sequence both recursively and with an explicit formula.
- ☐ Use all four operations to illustrate combinations of linear and/or exponential functions. (4 problems and solutions).
- ☐ Explain vertical translations. Create three vertical translations for a linear or exponential function. Graph all three on a single axes and compare and contrast the graphs.

☐ **Constructing and Comparing Linear and Exponential Models (20 points)**
- ☐ Design a word problem that involves a linear and exponential model. Use a table or sequence to illustrate the relationships described in the models.
- ☐ Explain the constant rate and constant percent rate per unit interval relative to another for the word problem that you designed.
- ☐ Construct the graphs for each model in the word problem that you designed.
- ☐ Compare the linear and exponential models from your word problem. Interpret the parameters.

☐ **Reflection / Summary: (8 points)**
- ☐ Describe your learning journey throughout the unit. Reflect on topics that you found easy to learn and those that were most difficult.
- ☐ Are there any standards that you need more help grasping? Explain. If not, which standards do you have the best grasp? Explain.
- ☐ What advice would you give to other students that will learn about linear and exponential functions in the future?
- ☐ Which task(s) did you find the most beneficial to mastering key standards?
- ☐ Any other insight you would like to share about Unit 3.

☐ **Works Cited: (2 points)**
- ☐ Use MLA format to cite any books, websites, and any other references used to create your booklet or webpage.

The tasks featured in this table provide additional resources and supplemental tasks to be incorporated into unit 3 instruction as deemed appropriate by the instructor.

UNIT 3: Linear Functions	Standards Addressed in the Task
More Graphing Stories http://graphingstories.com Site Developed by Dan Meyer (1-3 Stories)	F-BF, F-IF
Double Sunglasses http://threeacts.mrmeyer.com/doublesunglasses Task Developed by Dan Meyer	F-LE.1
Relation Stations http://musingmathematically.blogspot.ca/2013/02/relation-stations.html Task Developed by Nat Banting	F-IF.1, F-BF.1, 1a, F-LE.1,1a,1b
Pixel Patterns http://threeacts.mrmeyer.com/pixelpattern/ Task Developed by Dan Meyer	F-LE.2
Math Taboo http://threeacts.mrmeyer.com/pixelpattern/ Task Idea Developed by Fawn Nguyen	F-LE 1,1a,1b
Moving on Up http://emergentmath.com/2011/05/11/u-haul-linear-systems-problem-updated-and-improved/ Task Developed by Geoff Krall on Emergent Math	F-BF.1a,b,c; F-LE.2, F-IF.6,7a
Taco Cart http://threeacts.mrmeyer.com/tacocart Task Developed by Dan Meyer	F-IF.4
"In and Out" hamburger math problem http://robertkaplinsky.com/work/in-n-out-100-x-100/ Task Developed by Robert Kaplinsky	F-LE.2

Taxi Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/243	A.REI.10, F.LE.5
Food Supply Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/645	A.REI.11, F.LE.2, F.LE.3
Points on a Graph Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/630	F.IF.1
The Customers Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/624	F.IF.1
Using Function Notation Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/598	F.IF.1
Do Two Points Always Determine a Straight Line? Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/377	F.IF.1, LE.2
Warming and Cooling Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/639	F.IF.4
The Random Walk Task from Illustrative Math https://www.illustrativemathematics.org/illustrations/626	F.IF.2