Performance Assessment Task
Quadratic (2009) Grade 9
The task challenges a student to demonstrate an understanding of quadratic functions in various forms. A student must make sense of the meaning of relations and functions and select, convert flexibly among, and use various representations for them. A student must be able to approximate and interpret rates of change from graphic and numeric data. A student must be able to identify minimum point and solutions of a quadratic. A student must determine the solutions to a quadratic equation by using algebra.
Common Core State Standards Math - Content Standards
<u>**High School – Algebra – Reasoning with Equations and Inequalities**</u> **Solve equations and inequalities in one variable.** A-REI.4 Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x – p)2 = q that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. <u>**High School – Functions – Interpreting Functions**</u> **Interpret functions that arise in applications in terms of the context.** F-IF.4 For a function that models a relationship between two quantities, interpret key features of a graphs and tables in terms of quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function in increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.* **Analyze functions using different representations.** F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicates cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
Common Core State Standards Math – Standards of Mathematical Practice
MP.1 Make sense of problems and persevere in solving them. Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends.

Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MP.5 Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Assessment Results

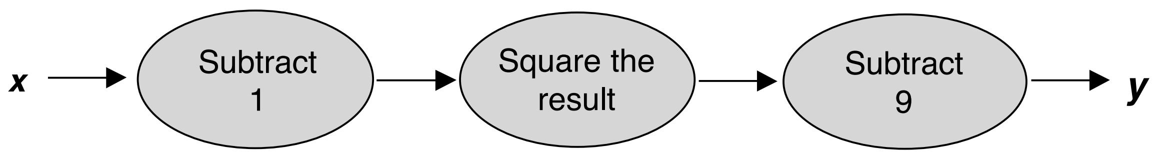
This task was developed by the Mathematics Assessment Resource Service and administered as part of a national, normed math assessment. For comparison purposes, teachers may be interested in the results of the national assessment, including the total points possible for the task, the number of core points, and the percent of students that scored at standard on the task. Related materials, including the scoring rubric, student work, and discussions of student understandings and misconceptions on the task, are included in the task packet.

Grade Level	Year	Total Points	Core Points	% At Standard
9	2009	9	5	33 %

Quadratic

This problem gives you the chance to:
• work with a quadratic function in various forms

This is a quadratic number machine.



1. a. Show that, if x is 5, y is 7. ______________________________

   b. What is y if x is 0? ______________________________

   c. Use algebra to show that, for this machine, $y = x^2 - 2x - 8$. ______________

   ____

   ____

The diagram on the next page shows the graph of the machine's quadratic function $y = x^2 - 2x - 8$ and the graphs of $y = 3$ and $y = x$.

2. a. Which point on the diagram shows the minimum value of y? ______________

   b. Which point(s) on the diagram show(s) the solution(s) to the equation $3 = x^2 - 2x - 8$?

   ____

   c. Which point(s) on the diagram show(s) the solution(s) to the equation $x = x^2 - 2x - 8$?

   ____

3\. a. Use the graph to solve the equation $x^2 - 2x - 8 = 0$. Mark the solutions on the graph.

x = __________ or x = ____________

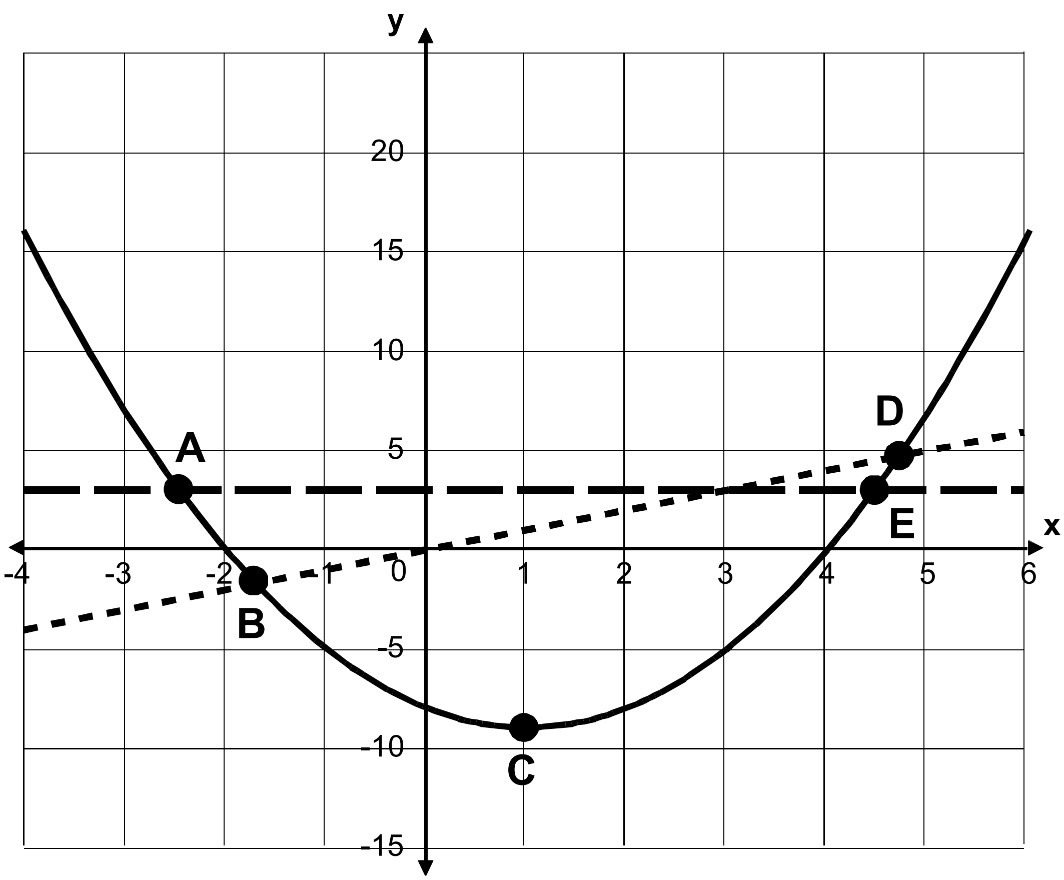
b. Use algebra to solve the same equation.

____

____

____

____



9

Quadratic		Rubric
The core elements of performance required by this task are: • work with a quadratic function in various forms Based on these, credit for specific aspects of performance should be assigned as follows	points	section points
1. a. Gives a correct answer: $5 \rightarrow 4 \rightarrow 16 \rightarrow \mathbf{7}$	1	
b. Gives a correct answer: **- 8**	1	
c. Gives a correct answer: $y = (x - 1)^2 - 9$ $= x^2 - 2x + 1 - 9$ $= x^2 - 2x - 8$	2	4
2. a. Gives a correct answer: **C**	1	
b. Gives a correct answer: **A and E**	1	
c. Gives a correct answer: **B and D**	1	3
3. a. Gives a correct answer: **-2, 4**	1	
b. Gives a correct answer such as: $(x+2)(x-4) = 0$, so $x = -2$ or 4 or uses completing the square to find 1 or 2 correct answers or uses quadratic formula.	1	2
Total Points		**9**

Quadratic

Work the task and look at the rubric. What are the key mathematical ideas being assessed? ____

Look at student work on part 1b, using the using the number machine. How many of your students put:

-8	-10	0	7	10	Other

What caused some of the student errors? What did they struggle with?

Now look at work on part 1c, using algebra to show that the number machine is equivalent to the equation $y = x^2 - 2x - 8$. How many of your students:

- Wrote an expression for the number machine and then carried out the algebraic manipulations to show that the result was equal to the equation?
- How many tried to use substitution of values to show that both would give the same answer?
- How many students were unwilling to attempt this part of the task?

How often do students in your class get opportunities to use algebra to make a proof or justification?

Do students get opportunities to rectify different equations that come up when identifying patterns?

What do students in your class seem to understand about justification?

Look at work on part 2b. How many of your students put:

A & E	E	B & D	B	C	All	No answer	Other

What are some of the misconceptions behind these responses?

Now look at work on part 2c. How many of your students put:

B &D	D	C	B	A &E	All	No answer	Other

Now look at student work on part 3a.

- How many of your students could use the graph to find the correct values for x?_________
- How many students made errors with the signs for x?_________
- How many of the students were not able to give a response?_________

What other errors did you see?

Now look at student work for solving the quadratic for x. How many of your students:

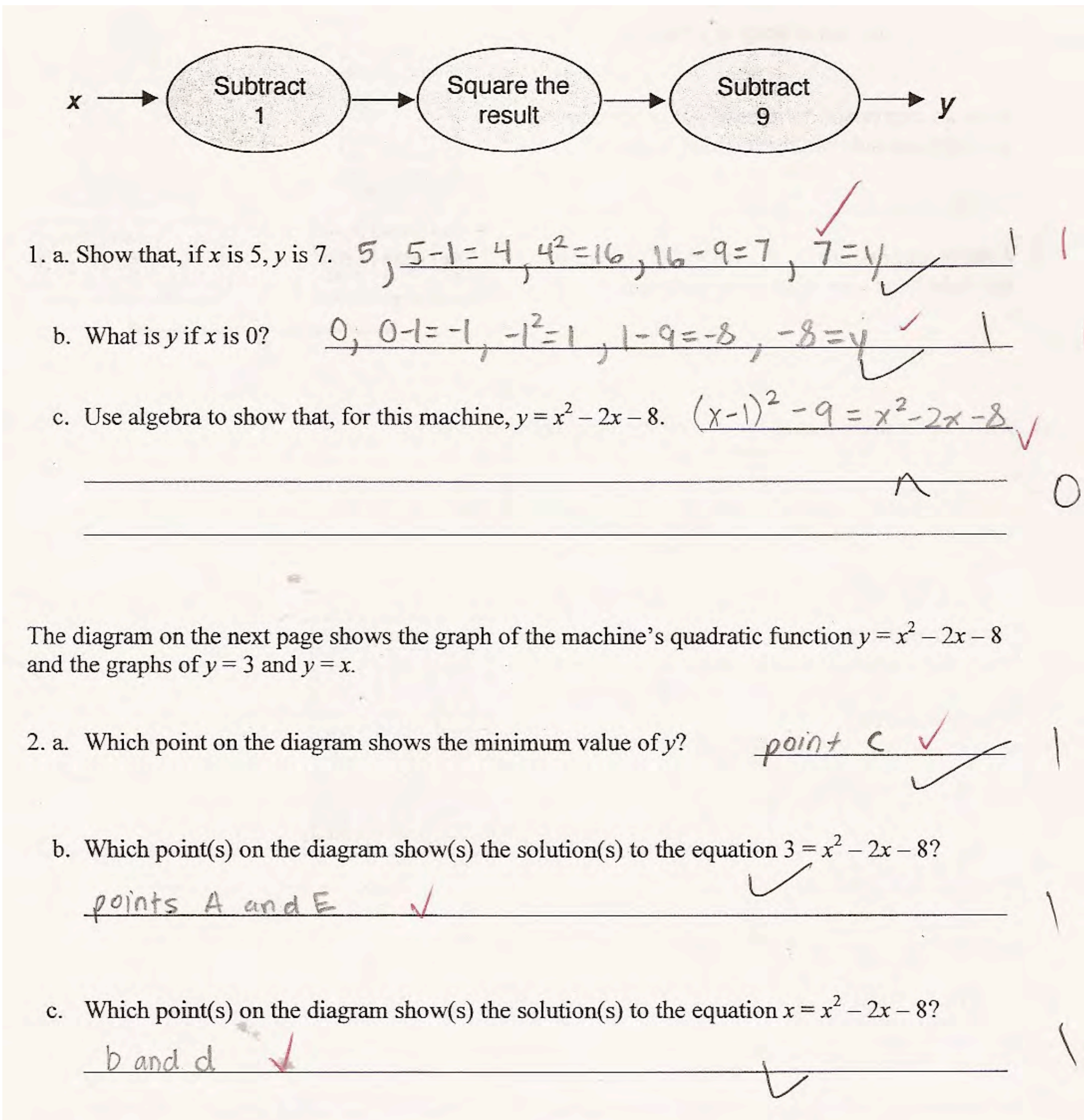
- Used factoring?____________
- Used substitution?__________
- Attempted the quadratic formula?__________
- Had no strategies for using the equation?___________

How much work have your students had with factoring? Do you see evidence that they understand or see the purpose for factoring to solve an equation? What are the big ideas or underlying conceptual knowledge that we want students to have about quadratics and their solutions?

Looking at Student Work on Quadratic

Student A is able to use the number machine to calculate values for y. The student understands how to encode the number machine into an algebraic expression. In trying to prove that the machine is equal to the equation y = x^2 –2 – 8, the student sets it equal to the expression representing the machine. The student doesn't understand that the steps for transforming the expression need to be shown to complete the argument. The student is able to use factoring and setting the factors equal to zero to find the solutions to the quadratic equation.

Student A


$x \rightarrow$ Subtract 1 $\rightarrow$ Square the result $\rightarrow$ Subtract 9 $\rightarrow y$

1. a. Show that, if x is 5, y is 7. 5, 5-1=4, 4^2=16, 16-9=7, 7=y

b. What is y if x is 0? 0, 0-1=-1, -1^2=1, 1-9=-8, -8=y

c. Use algebra to show that, for this machine, $y = x^2 - 2x - 8$. $(x-1)^2 - 9 = x^2 - 2x - 8$

0

The diagram on the next page shows the graph of the machine's quadratic function $y = x^2 - 2x - 8$ and the graphs of $y = 3$ and $y = x$.

2. a. Which point on the diagram shows the minimum value of y? point C

b. Which point(s) on the diagram show(s) the solution(s) to the equation $3 = x^2 - 2x - 8$?

points A and E

c. Which point(s) on the diagram show(s) the solution(s) to the equation $x = x^2 - 2x - 8$?

b and d

Student A, continued

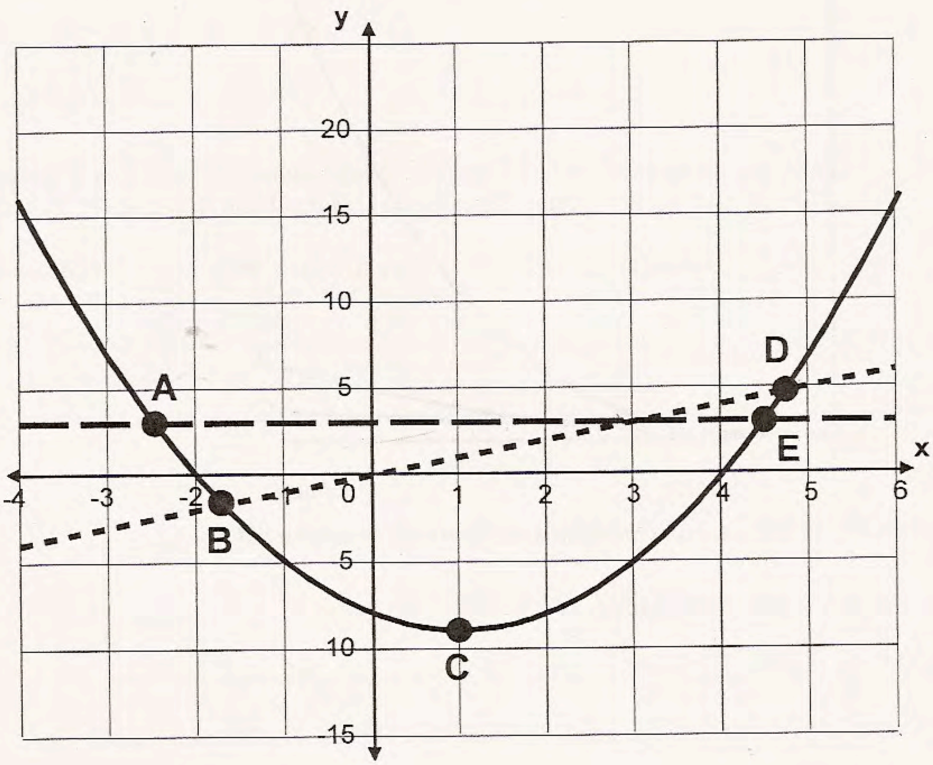
3. a. Use the graph to solve the equation $x^2 - 2x - 8 = 0$. Mark the solutions on the graph.

x = 4 or x = -2

b. Use algebra to solve the same equation.

$x^2-2x-8=0$ $\quad$ $-4 \times 2 = -8$, $-4 + 2 = -2$ $\quad$ $(x-4)(x+2) = x^2-2x-8=0$

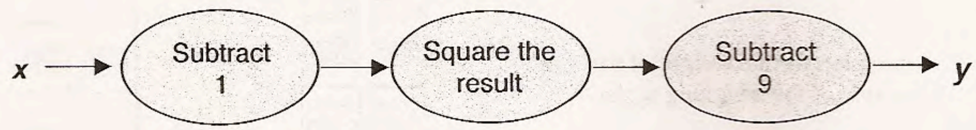
$(x-4)(x+2)=0$ $\quad$ $x = 4$ or -2



Algebra

Student B tries to use substitution to prove that the number machine is equal to the equation $y = x^2 - 2 - 8$. To find the solution to the equation $x = x^2 - 2 - 8$, the student uses factoring. However when needing the same information in part 3b, the student uses the quadratic equation. *Do you think the student knows that both are yielding the same solution? What question would you like to pose to the student to probe for understanding?*

Student B



1. a. Show that, if x is 5, y is 7. $x = 5 - 1 = 4 \cdot 4 = 16 - 9 = 7$ $y = 7$

b. What is y if x is 0? $y = -8$ $x = 0 - 1 = -1 \cdot -1 = 1 - 9 = -8$

c. Use algebra to show that, for this machine, $y = x^2 - 2x - 8$. If $x = 0$, then $y = 0^2 - 2(0) - 8$, $y = -8$, $(0, -8)$; if $x = 5$, then $y = 5^2 - 2(5) - 8$, $y = 7$, $(5, 7)$.

The diagram on the next page shows the graph of the machine's quadratic function $y = x^2 - 2x - 8$ and the graphs of $y = 3$ and $y = x$.

2. a. Which point on the diagram shows the minimum value of y? point C

b. Which point(s) on the diagram show(s) the solution(s) to the equation $3 = x^2 - 2x - 8$?

points A & E

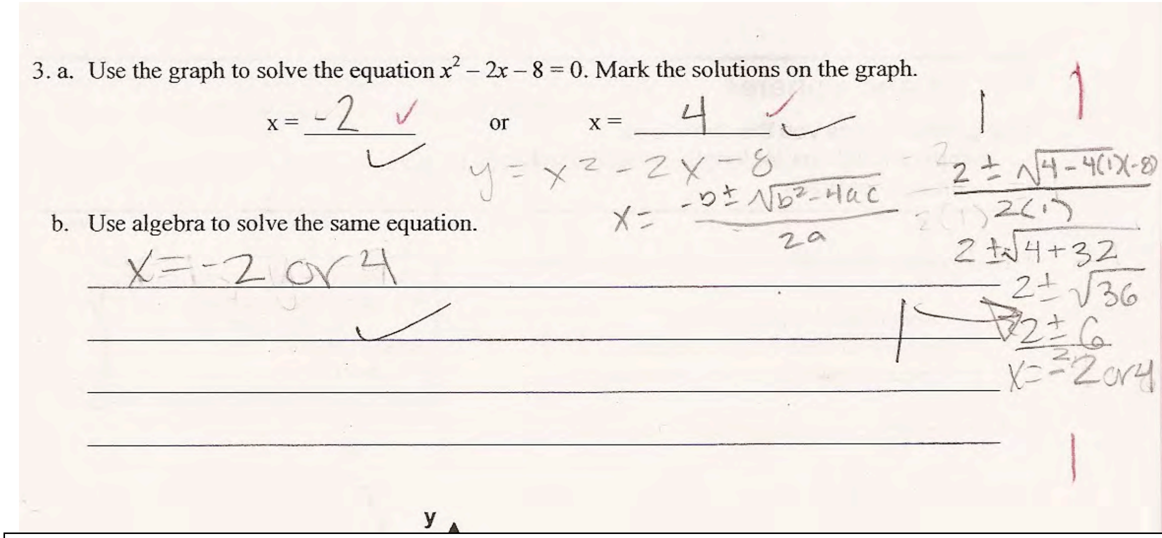
c. Which point(s) on the diagram show(s) the solution(s) to the equation $x = x^2 - 2x - 8$?

B & D

$x = -4$ or 2 $(x - 4)(x + 2)$

-8, -4, 2, -2

Student B, continued


3. a. Use the graph to solve the equation $x^2 - 2x - 8 = 0$. Mark the solutions on the graph.

x = -2 or x = 4

$y = x^2 - 2x - 8$

b. Use algebra to solve the same equation.

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\frac{2 \pm \sqrt{4 - 4(1)(-8)}}{2(1)}$

$2 \pm \sqrt{4 + 32}$

$2 \pm \sqrt{36}$

2 ± 6

$x = -2$ or 4

x = -2 or 4

y

Student C is able to calculate y using the number machine. The student does not attempt to show why the number machine is equal to the equation $y = x^2 - 2 - 8$. The student understands the minimum value on the parabola. But the student doesn't connect the equations to solutions on the graph in part 2. In part 3, the student does seem to understand that the solution of a quadratic is the value of x when y = 0. However the student has not mastered how to find these solutions using algebra. Instead the student uses guess and check to find the solution.

Student C

1. a. Show that, if x is 5, y is 7. $5-1=4, 4^2=16, 16-9=7$

5-1=-1, 1²=1, 1-9=-8

b. What is y if x is 0? -8

c. Use algebra to show that, for this machine, $y = x^2 - 2x - 8$.

The diagram on the next page shows the graph of the machine's quadratic function $y = x^2 - 2x - 8$ and the graphs of $y = 3$ and $y = x$.

2. a. Which point on the diagram shows the minimum value of y? C

b. Which point(s) on the diagram show(s) the solution(s) to the equation $3 = x^2 - 2x - 8$?

c. Which point(s) on the diagram show(s) the solution(s) to the equation $x = x^2 - 2x - 8$?

C

x	y
3	-5
2	-8
1	-9
0	-8
-1	-5

$3 = x^2 - 2x - 8$
$+8 \quad +8$
$12 = x^2 - 2x$
$-x^2 \quad -x^2$

3. a. Use the graph to solve the equation $x^2 - 2x - 8 = 0$. Mark the solutions on the graph.

x = -2 or x = 4

x	y
4	0
3	-5
2	-8
1	-9
0	-8
-1	-5
-2	0

b. Use algebra to solve the same equation.

I made a table I put in different numbers for x and then filled out what the y would be.

Student D tries to use examples to prove the equality between the equation $y = x^2 - 2 - 8$ and the number machine. While the student can identify solutions on a graph in part 2, the student can't name the solutions in 3a. The student attempts to use the quadratic equation to solve in part 3b, but struggles with the computations.

Student D

1. a. Show that, if x is 5, y is 7. $5-1=4^2=16-9=7$ ✓ 1 1

b. What is y if x is 0? $0-1=-1 \rightarrow -1^2=1 \rightarrow 1-9=-8$ ✓ 1 1

So $y=-8$

c. Use algebra to show that, for this machine, $y = x^2 - 2x - 8$. $y=5^2-2(5)-8$

$y=25-10-8$ $y=15-8=7$ so $y=7$ × 0

$y=7$

0

The diagram on the next page shows the graph of the machine's quadratic function $y = x^2 - 2x - 8$ and the graphs of $y = 3$ and $y = x$.

2. a. Which point on the diagram shows the minimum value of y? C ✓ 1 1

b. Which point(s) on the diagram show(s) the solution(s) to the equation $3 = x^2 - 2x - 8$?

A and E ✓ 1 1

c. Which point(s) on the diagram show(s) the solution(s) to the equation $x = x^2 - 2x - 8$? 1

B and D ✓ 1

Student D, continued

3. a. Use the graph to solve the equation $x^2 - 2x - 8 = 0$. Mark the solutions on the graph.

x = ____ or x = ____

$x = 2 \pm \sqrt{-28}$ $x = 2 \pm \sqrt{4-32}$ $x = \frac{2 \pm \sqrt{2^2 - 4(1)(8)}}{2(1) = 2}$

b. Use algebra to solve the same equation.

0 0 0 0

Student E struggles with detail and signs. In 1b the student either doesn't square the –1 or adds – 1 and –9 incorrectly. In 1c the student factors the equation y = x^2 –2 – 8, which yields an equality, but doesn't connect the equation to the number machine. The student loses track of what needs to be proved. In part 3 the student understands that the equation needs to be factored, but doesn't use the factors to set up equations equal to zero. Thus, the student's solutions have incorrect signs.

Student E

1. a. Show that, if x is 5, y is 7. 5-1=4 ; 16-7=9 i have subtracted the information together. 1 1

b. What is y if x is 0? 0-1=-1-9=-8 $\boxed{x=-8}$ I have subtracted the quadratic #'s. 1 1

c. Use algebra to show that, for this machine, $y = x^2 - 2x - 8$. If x does = to 5 and y = 7 than it might work if you substitute the #'s in to get the answer. 0 0

$x(x-4)(x+2) = -2$ So $-2x$ is $x^2 -2x - 8$

The diagram on the next page shows the graph of the machine's quadratic function $y = x^2 - 2x - 8$ and the graphs of $y = 3$ and $y = x$.

2. a. Which point on the diagram shows the minimum value of y? C 1 1

b. Which point(s) on the diagram show(s) the solution(s) to the equation $3 = x^2 - 2x - 8$? I think points A and E. 1 1

c. Which point(s) on the diagram show(s) the solution(s) to the equation $x = x^2 - 2x - 8$? Points B and D. 1 1

Student E, continued

3. a. Use the graph to solve the equation $x^2 - 2x - 8 = 0$. Mark the solutions on the graph.

x = +2 X or x = -4 X

$(x+2)(x-4)$

0

b. Use algebra to solve the same equation.

All I did was find the factors so I started with $x^2 - 2x - 8$ and I know so far dat it is $(x - \ )(x + \ )$. So then I thought for a moment on what the multiples of 8 were and then I got my final answer which is: $(x+2)(x-4) = x^2 - 2x - 8$ ✓

0

Y 2

Student F is able to do the calculations in 1a and 1b. The student doesn't seem to understand what is meant by proving that the calculations in the number machine are the same as the equation. The student writes about what is confusing in the equation. While the student answers part of 2 correctly, the uncertainty or tentativeness of the solution is clear. In part 3, the student attempts to manipulate the equation in a manner that would solve for x. However when solving a quadratic the equation needs to be in the form $0 = ax^2 + bx + cy^2$. The scorer should not have given the process point.

Student F

This is a quadratic number machine.

x → Subtract 1 → Square the result → Subtract 9 → y

5   5-1=4 $4^2=4\cdot4=16$ 16-9= 7   7

1. a. Show that, if x is 5, y is 7.

b. What is y if x is 0?   0-1=-1 → -1·-1=1+9= $\boxed{-8}$

c. Use algebra to show that, for this machine, $y = x^2 - 2x - 8$. $y = x^2 - 1 - 9$

9-1=8

The x throws ~~me off, the 2 makes sense~~ because in total in the original =tion we sub. 10. w/o the x in total in the 2nd =tion we sub. 10. but the x_2 just confuses me.

The diagram on the next page shows the graph of the machine's quadratic function $y = x^2 - 2x - 8$ and the graphs of $y = 3$ and $y = x$.

2. a. Which point on the diagram shows the minimum value of y?   9

b. Which point(s) on the diagram show(s) the solution(s) to the equation $3 = x^2 - 2x - 8$?

??? A B E ???

c. Which point(s) on the diagram show(s) the solution(s) to the equation $x = x^2 - 2x - 8$?

??? B B D ???

how?

3. a. Use the graph to solve the equation $x^2 - 2x - 8 = 0$. Mark the solutions on the graph.

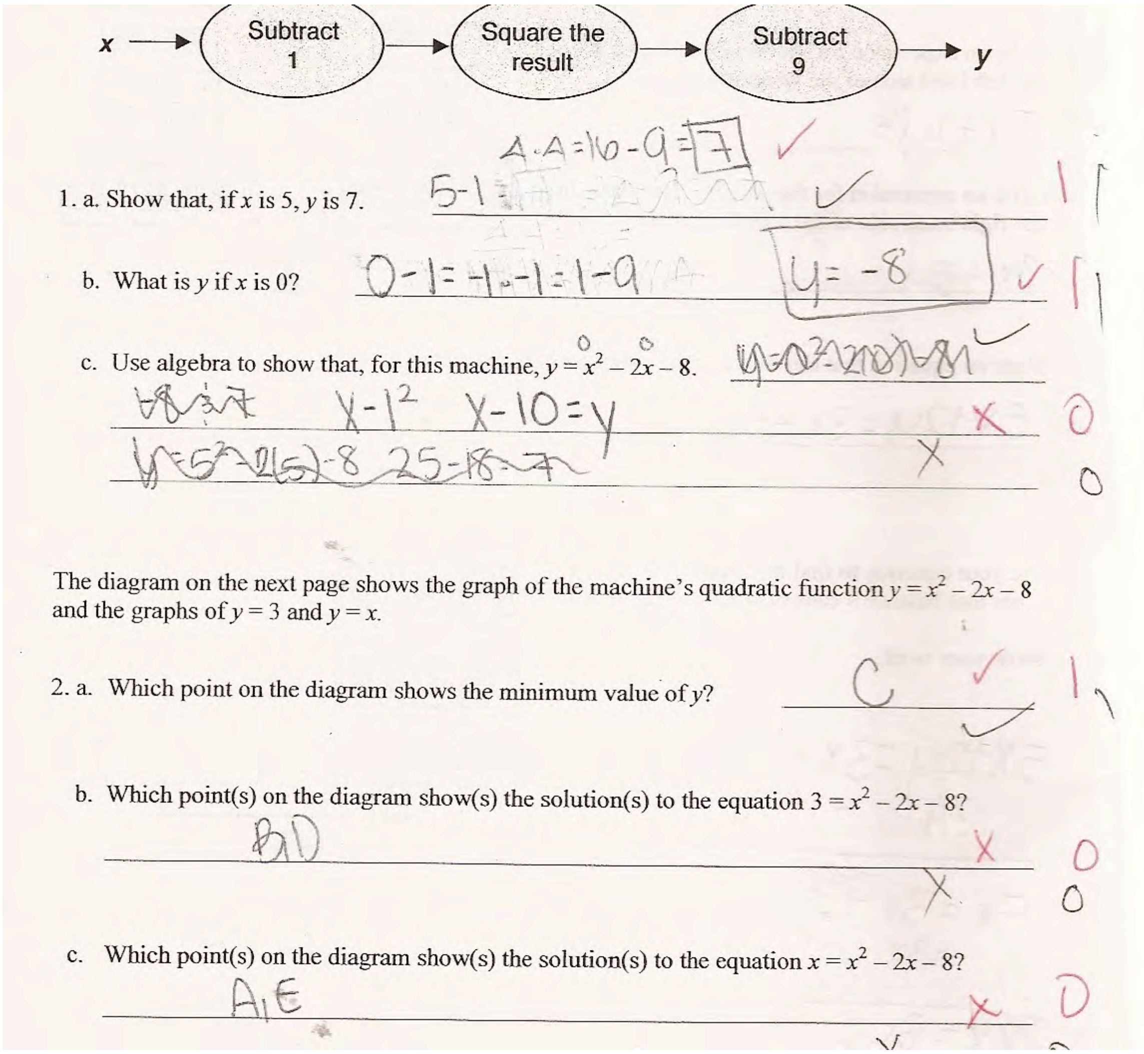
$x^2-2x-8=0$
$+8 \quad +8$
$x^2-2x = 8$
$-2 \quad -2$
$x^2+x=-4$

x = ________ or x = ________

b. Use algebra to solve the same equation.

Student G struggles with representing ideas symbolically. The student can do the step-by-step calculations in the number machine. When attempting to express those actions in algebra the student only squares the (-1) rather than the whole quantity (x-1). The student doesn't know how to connect solutions to the graphical representation in part 2. Student G also understands that the solution for a quadratic can be found by factoring, but doesn't know how to complete the process.

Student G



$x \rightarrow$ Subtract 1 $\rightarrow$ Square the result $\rightarrow$ Subtract 9 $\rightarrow y$

1. a. Show that, if x is 5, y is 7. 5-1 = 4·4 = 16 - 9 = $\boxed{7}$

b. What is y if x is 0? 0-1 = -1 · -1 = 1 - 9 $\boxed{y = -8}$

c. Use algebra to show that, for this machine, $y = x^2 - 2x - 8$. $y = 0^2 - 2(0) - 8$

X-1² X-10 = y

$y = 5^2 - 2(5) - 8$ 25-18 = 7

The diagram on the next page shows the graph of the machine's quadratic function $y = x^2 - 2x - 8$ and the graphs of $y = 3$ and $y = x$.

2. a. Which point on the diagram shows the minimum value of y? C

b. Which point(s) on the diagram show(s) the solution(s) to the equation $3 = x^2 - 2x - 8$?

B, D

c. Which point(s) on the diagram show(s) the solution(s) to the equation $x = x^2 - 2x - 8$?

A, E

Algebra

Student G, continued

3. a. Use the graph to solve the equation $x^2 - 2x - 8 = 0$. Mark the solutions on the graph.

x = ~~2~~ or x = ~~4~~ 0

	-8	
-4	2	
X	-2	
X	X^2	-4X
2	2X	-8

0

b. Use algebra to solve the same equation.

$X^2-4X+2X-8 = (X-4)(X+2) = \cancel{8}$ $X = 4 \; \& \; 2$ 0

0

Student H is also confused by the graphical representations. First the student doesn't seem to connect the description of the graph with the idea that 3 different equations are being displayed. The student doesn't realize that minimum is a term used with quadratics and parabolas. The student does lots of symbolic manipulation but can't make sense of the information.

Student H

x → Subtract 1 → Square the result → Subtract 9 → y

a. $5-1$ $4^2 = 16-9 = 7$ b. $0-1$ $-1^2 = -1-9 = -10$

1. a. Show that, if x is 5, y is 7. 7 ✓ 1 1

b. What is y if x is 0? $\boxed{y = -8}$ ✓ 1 1

c. Use algebra to show that, for this machine, $y = x^2 - 2x - 8$. If you plug in the numbers for example A, then you get the same answer 2 0

$7 = 5^2 - 2(5) - 8$ | $7 = 25 - 10 - 8$ | $7 = 15 - 8$ $7 = 7$ ✓

The diagram on the next page shows the graph of the machine's quadratic function $y = x^2 - 2x - 8$ and the graphs of $y = 3$ and $y = x$. this means $x = 3$ also

2. a. Which point on the diagram shows the minimum value of y? 3 0

are the same

b. Which point(s) on the diagram show(s) the solution(s) to the equation $3 = x^2 - 2x - 8$?

no Solution $3 \neq -5$ 0

(work on bottom of graph)

c. Which point(s) on the diagram show(s) the solution(s) to the equation $x = x^2 - 2x - \cancel{8}$?

Same as example B. 0

Algebra

Student H, continued

$x^2 - 2x - 8 = 0$
$3^2 - 6 - 8 = 0$
$9 - 6 - 8 = 0$
$3 - 8 = 0$
$-5 = 0$
0

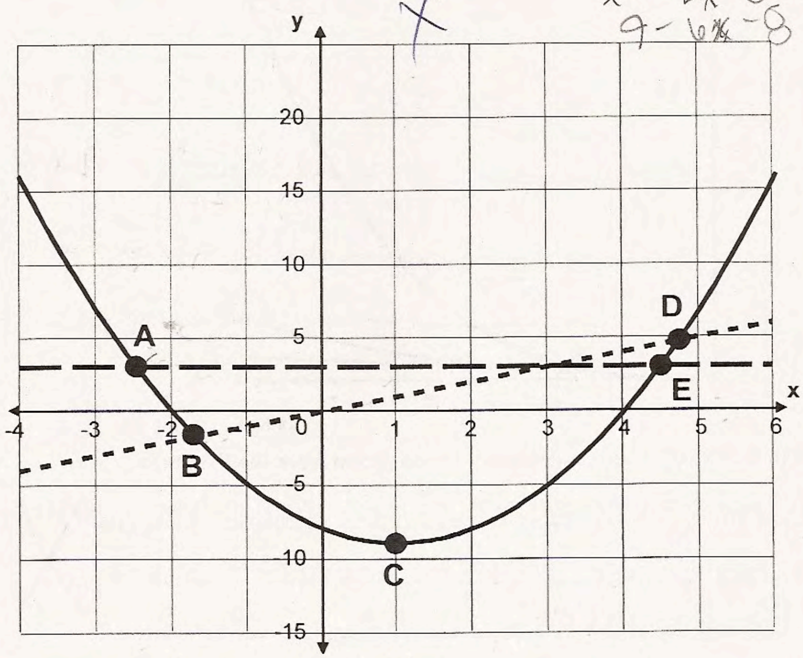
3\. a. Use the graph to solve the equation $x^2 - 2x - 8 = 0$. Mark the solutions on the graph.

x = -5 or x = 0 2

b. Use algebra to solve the same equation.

$x = -5$ $x = 0$
find an equation that fits these two points on a graph
$x^2 - 2x - 8 = 0$

$x^2 - 2x - 8 = 0$
$9 - 6x - 8$ $3 - 8$
0



$3 = 3^2 - 2x - 8$

$3 = x^2 - 2x - 8$
$3 = 3^2 - 6 - 8$
$3 = 9 - 6 - 8$
$3 = 3 - 8$
$3 = -5$

4

2

9

Student I struggles with calculations. The student should not be given credit for the solution in 1a because the 4 is never squared. 4-9 ≠ 7. The student also errs in solving 1b. The student does not realize that the solution given in part 1c is not equal. While the student is able to guess one of the 2 points correctly in 2b and 2c, it is unclear if this is just luck. *What do you think this student needs? What would be your next steps?*

Student I

1. a. Show that, if x is 5, y is 7. $5-1=4-9=7$ 1 1

   b. What is y if x is 0? 2 0 0

   c. Use algebra to show that, for this machine, $y = x^2 - 2x - 8$. $2 = 0^2 - 2(0) - 8$ 0

The diagram on the next page shows the graph of the machine's quadratic function $y = x^2 - 2x - 8$ and the graphs of $y = 3$ and $y = x$.

2. a. Which point on the diagram shows the minimum value of y? G 1 1

   b. Which point(s) on the diagram show(s) the solution(s) to the equation $3 = x^2 - 2x - 8$? A 0 0

   c. Which point(s) on the diagram show(s) the solution(s) to the equation $x = x^2 - 2x - 8$? B 0

3. a. Use the graph to solve the equation $x^2 - 2x - 8 = 0$. Mark the solutions on the graph.

   x = 2 or x = 4 0

   b. Use algebra to solve the same equation.

   $x^2 - 2x - 8 = 0$

   $+8 \quad +8$

   $x - 2x = 8$ 0

Algebra Task 2 Quadratic

Student Task	Work with a quadratic function in various forms.
Core Idea 3 Algebraic Properties and Representations	**Represent and analyze mathematical situations and structures using algebraic symbols.** • Approximate and interpret rates of change, from graphic and numeric data.
Core Idea 1 Functions and Relations	• Understand relations and functions and select, convert flexibly among, and use various representations for them.

Mathematics of the task:

- Calculations with exponents and negative numbers
- Codifying calculations into a symbolic string
- Rectifying two forms of an equation
- Interpreting graphical representations of linear and quadratic equations and identifying minimum point and solutions
- Using algebra to find the solution to a quadratic equation (using factoring or the quadratic equation)

Based on teacher observation, this is what algebra students know and are able to do:

- Calculate using the number machine
- Identify the minimum point on a parabola using a graph

Areas of difficulty for algebra students:

- Using algebra to show that two equations are equal
- Finding solutions to two equations on a graph
- Using algebra to find the solutions to a quadratic equation

MARS Test Task 2 Frequency Distribution and Bar Graph, Course 1

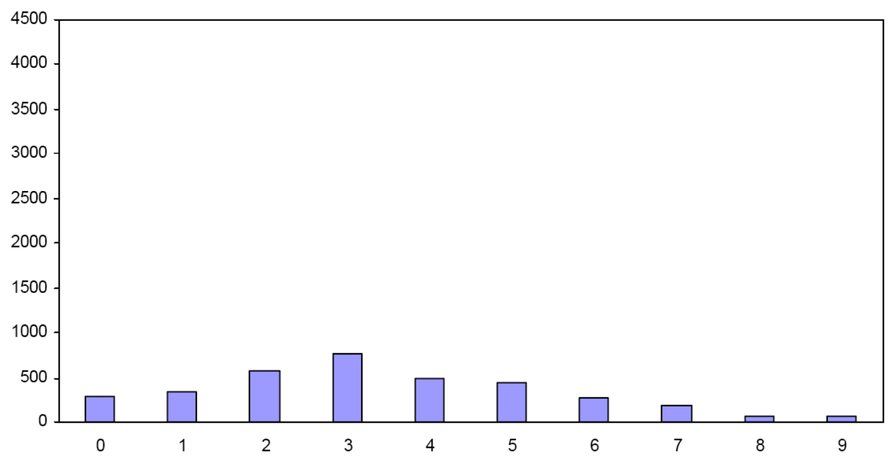
Task 2 - Quadratic

Mean: 3.47 StdDev: 2.13

Table 46: Frequency Distribution of MARS Test Task 2, Course 1

Task 2 Scores	Student Count	% at or below	% at or above
0	281	8.0%	100.0%
1	347	17.8%	92.0%
2	582	34.4%	82.2%
3	765	56.1%	65.6%
4	495	70.2%	43.9%
5	441	82.7%	29.8%
6	274	90.5%	17.3%
7	191	95.9%	9.5%
8	70	97.9%	4.1%
9	75	100.0%	2.1%

Figure 55: Bar Graph of MARS Test Task 2 Raw Scores, Course 1



The maximum score available on this task is 9 points.
The minimum score needed for a level 3 response, meeting standards, is 5 points.

Most students, 92%, could either identify the minimum or use the number machine to prove that an input of 5 equals an output of 7. 82% could do both calculations with the number machine. More than half the students, 66%, could use the number machine and find the minimum point on a parabola. Some students, 30%, could calculate with the number machine, find minimum, locate solutions to two equations on a graph, and use the graph to identify the solutions of the quadratic. About 2% of the students could meet all the demands of the task including showing that two equations are equal and using algebra to solve a quadratic. 8% of the students scored no points on this task. All the students in the sample with this score attempted the task.

Quadratic

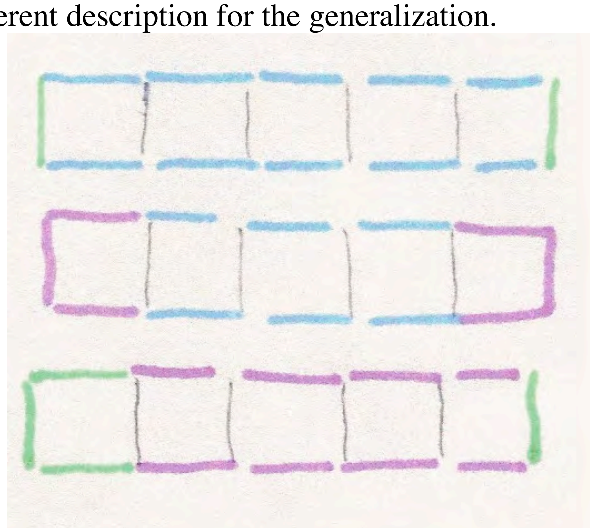
Points	Understandings	Misunderstandings
0	All the students in the sample with this score attempted the task.	Students did not know how to square numbers or calculate with negative numbers. They could not interpret the directions on the number machine.
1	Students could either solve the first number machine or locate the minimum on the graph.	Students struggled with the second number machine. 8% got an answer of -10. 8% got an answer of 0.
2	Students could solve both number machines or solve the first number machine and find the minimum.	Most students who struggled with the minimum put a value of –8 rather than identifying point C.
3	Students could use the number machine and find the minimum.	Students struggled with finding solutions to two equations on the graph. For the equation $3=x^2-2x-8$ common errors were D & B, E, E &D, and no attempt. For the equation $x = x^2-2x-8$ common errors were no attempt, D, C, B, and A & E.
5	Students could use the number machine, find the minimum, and locate solutions to two equations on a graph.	Students struggled with identifying the solution to a quadratic on a graph and using algebra to find the solution to a quadratic. 16% did not attempt a graphical solution. 26% did not attempt an algebraic solution. 19% used substitution from the graphical solution as their algebraic strategy.
7	Students could use the number machine, find the minimum, and locate solutions to two equations on a graph.	Students struggled with using algebra to prove that the number machine was equal to the equation $= x^2-2x-8$. 40% tried to use substitution for their proof. 19% did not attempt this part of the task.
9	Students could use the number machine, find the minimum, and locate solutions to two equations on a graph. Students could solve quadratics graphically and algebraically. They could also use algebra to show that two expressions were equal.	

Implications for Instruction

Students need more experience using algebra as a tool. While many students may have been able to square an algebraic expression, they didn't think to pull out this tool to help show why two expressions are equal. Students didn't seem to understand how to put together algebra to make a convincing proof. They relied on the logic of proof by discrete examples. One of the big ideas of algebra is its usefulness in making and proving generalizations. This is a huge part of algebra.
Students need to be able to think about what solutions mean to quadratics or to two equations. They need experiences relating solutions graphically and algebraically. Students seemed unfamiliar with using the graph to find solutions.
Students also had difficult with the idea of solving quadratics. Some students knew that part of the procedure was to factor the expression, but then did not set the factors equal to zero and solve. Many students tried to solve the equation similar to the strategy of solving a linear equation putting the variable on one side and number values on the other. Other students have this idea that using substitution is a proof. Students need more experience with quadratics.

Ideas for Action Research

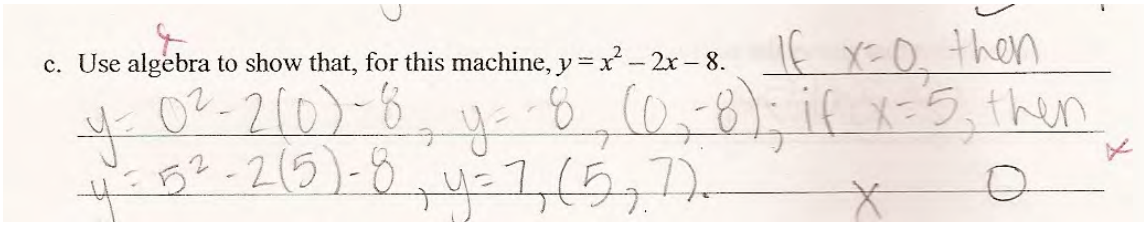
To help students learn to make generalizations and rectify equations give them an interesting pattern problem, like finding the perimeter of any number of squares in a row. As students work the task, they will come up with a variety of solutions or generalizations for the pattern. Here is a very natural way to have students work with the idea of proving the two equations are equal. Depending on how students see the pattern, they might write different description for the generalization.



The first pattern might be described as every square has two sides (top and bottom) plus two additional pieces at the end. This might be expressed algebraically as $p = 2x + 2$.
The second pattern might be described as the end squares have 3 sides and the middle squares have two sides. This might be expressed algebraically as $p = 2(x-2) + 6$.
The third pattern might be described as the first square has 3 sides and all other squares have 2 sides with a final end side. This might be expressed as $p = 2(x-1) + 4$.

Now students have organic expressions to work with and try to show the equality between the equations.

Now students have some background to help them re-engage with the mathematics of the task. Students did not understand what it meant to use algebra to prove that the number machine is the same as $y=x^2 - 2x - 8$. I might start by showing them the work of Student B:


c. Use algebra to show that, for this machine, $y = x^2 - 2x - 8$. If $x=0$, then $y=0^2-2(0)-8$, $y=-8$, $(0,-8)$; if $x=5$, then $y=5^2-2(5)-8$, $y=7$, $(5,7)$.

"I saw this work on a student's paper. What do you think the student was doing?" I would give students first some individual think time and then let them share their ideas with a neighbor. The idea of re-engagement is to maximize the amount of talking students get to do, so that they can talk their way into understanding the concept. After everyone has had a chance to form and verbalize an opinion, I would then open up the question to a group discussion. If it doesn't come out naturally, I would also add a prompt to get students to question whether or not this is a proof. We want students at this age to understand that it is impossible to test for every possible case and that proving it for one or even many cases is not a proof. A good reading on this topic is found in <u>Thinking Mathematically</u> by Carpenter, Franke, and Levi, chapter 7 Justification and Proof along with videos of young students making justifications. It has some good questions to push students' thinking about proof. "Is that always true? How do you know that is true for all numbers? Okay, so we have seen that it works for a lot of numbers, but how do we know that there is not some number – maybe a very, very big number – that it will not work for?"
How would you get students to move from this idea to actually setting up an equality? Work with colleagues to think about the types of questions that would help students know what to set equal and how to rectify the two sides. Are there any snippets of student work that you could use from your own student work or the toolkit that would help you?

Students also had difficulty with the idea of using algebra to find the solution to the equation $y=x^2 - 2x - 8$. Many students used substitution, which only works if you already know the answer or can read the answer off the graph. How can we push students to think beyond this.

I might start with a prompt: "*Dawn says that she can use factoring to find the solution to the equation. What do you think she did?*" This prompt gives a strategy but allows all students now the opportunity to try and apply what they know. A follow up question might be: "*How does this help her find a solution?"*

Next I might want students to confront a common misconception. So I might use the work of Student E:

3. a. Use the graph to solve the equation $x^2 - 2x - 8 = 0$. Mark the solutions on the graph.

x = +2 x or x = −4 x

(x+2) (x − 4)

0

b. Use algebra to solve the same equation.

All I did was find the factors so I started with $x^2 - 2x - 8$ and I know so far dat it is (x −)(x +). So then I thought for a moment on what the multiples of 8 were and then I got my final answer which is: $(x+2)(x-4) = x^2 - 2x - 8$ ✓

0

Why do you think Earl's paper is marked incorrect? What mistake has Earl made?

Finally, I might push students by saying that Anita said that it was just easier to use the quadratic formula. "*What do you think she did? How did this help her?"* After students have had a chance to think through this strategy, I think it is important for them to compare strategies. I might ask them to say which strategy they think is easier and why? Then I might give them a different equation that is not factorable so that they can see that sometimes the quadratic is a better choice. I want them to start to see the purpose for knowing more than one strategy and for thinking about making decisions up front about picking a strategy that is convenient and easy for the particular situation.

Look through your student work and work in the toolkits. Are there any other misconceptions that you want students to discuss explicitly? Are there some more important mathematical ideas that you want to address? How could you use student work to pose questions to address those issues?

What kind of question do you want to pose at the end of lesson to get students to reflect on what they have learned? Is there a follow up problem that you want to use in a few days to see if students can transfer what they have discussed to a new situation? Would you consider giving students back their original work with a red pen and allow them to edit their papers? How would this help further their understanding?