The Impossible Task

Time required
60 minutes

Activity Overview

In this problem, students are given a manufacturing situation and asked to write an inequality to represent it. Once they have written the inequality, students examine its solution setting by testing values of the variable using lists and viewing its graph. In Problem 2, a second constraint and a second variable are added to the situation. Students graph the second inequality on top of the first and compare the solution set shown by the graph with that found by testing values. In Problem 3, a third constraint is introduced, creating a system of inequalities that has no solution, which is explored in a similar fashion.

Topic: Linear Systems

- *Graph a linear inequality in two variables and describe the three regions into which it divides the plane.*
- *Graph a pair of linear inequalities in two variables and describe the region of their intersection.*
- *Determine whether a given point belongs to the solution set of a pair of linear inequalities in two variables.*

Teacher Preparation and Notes

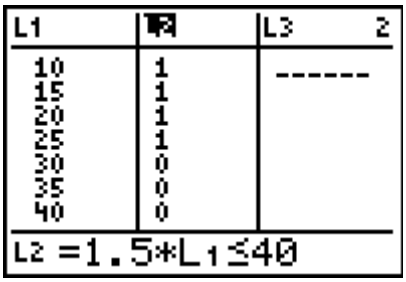
- *This activity is appropriate for students in Algebra 1. It is assumed that students are familiar with linear inequalities and their graphs, as well as systems of linear equations.*
- *Before beginning the activity, have students clear all lists by selecting ClrAllLists from the Catalog.*
- *This activity can be easily extended to include linear programming by introducing a profit function P(x, y).*

Associated Materials

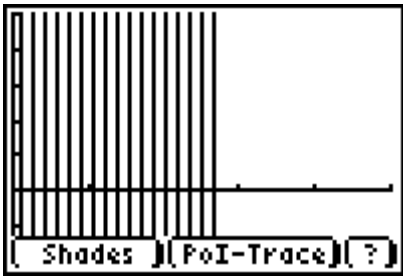
- *Student sheet.doc*

Problem 1 – The first constraint

In this problem, students are given a manufacturing situation and asked to write an inequality to represent it. Caution students to be aware that some of the information in the problem is given in hours and others in minutes—they will need to convert one to the other to write their inequalities.

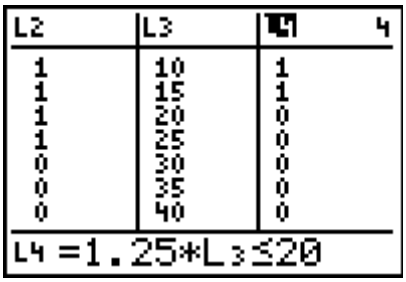


Once they have written the inequality, students examine its solution setting by testing values of the variable using lists and viewing its graph.

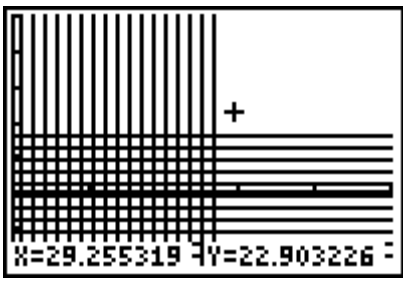


Problem 2 – Another constraint

A second constraint and a second variable are added to the situation. Again, students should be aware of units as they write their inequalities. The solution set to this inequality is again explored in the list editor. The idea of a system of inequalities, with a solution set equal to the intersection of the two inequalities in the system, is introduced and further explored.

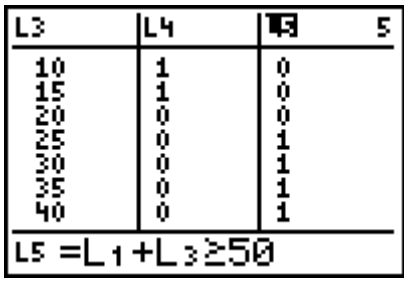


Students then graph the second inequality on top of the first and compare the solution set shown by the graph with that found by testing values. Students must solve the inequality for *y* before they can graph it.

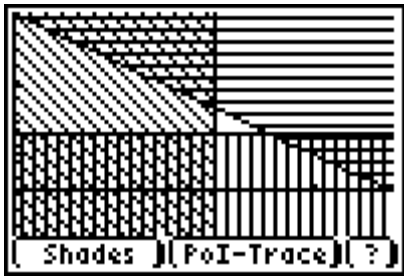


Problem 3 – A final constraint

A third constraint is introduced and explored in a similar fashion. First the solution set of the inequality by itself is discussed, then students are prompted to search for solutions to the system created by all three inequalities taken together.



They should find that there are no such (*x*, *y*) pairs and conclude that the task is impossible. This conclusion is verified when they graph the system on page 1.19 and see that there is no area where all three solution sets (shaded areas) overlap.



Solutions

1. 10: yes; 20: yes; 30: no
2. 10: yes; 20: no; 30: no
3. It means that in one week, the owner can make 10 birdhouses and the expert can make 15.
4. Answers will vary. Sample answer: (10, 10), (10, 15), (15, 10), (15, 15), (20, 10), (20, 15), (25, 10), (25, 15).
5. Answers will vary. Sample answer: (10, 10), (10, 15), (15, 10), (15, 15), (20, 10), (20, 15), (25, 10), (25, 15).
6. The solutions to the system (the answer to question 4) are points within the intersection (answer to question 5).
7. Answers will vary. Sample answer: (25, 25), (25, 30), (25, 40), (30, 25), (30, 30), (30, 40), (40, 25), (40, 30), (40, 40)
8. There are no such rows.
9. This system has no solution.
10. No.
11. This system has no solution.
12. At most, they can make 32 birdhouses (the owner can make a maximum of 26 birdhouses a week and the expert can make a maximum of 16 birdhouses a week).