

# **Transition Tasks** for Mathematics Grade 7





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> 1 2 3 4 5 6 7 8 9 10 ISBN 978-0-8251-7183-3 Copyright © 2012 J. Weston Walch, Publisher Portland, ME 04103 www.walch.com Printed in the United States of America



# **Table of Contents**

ntroduction	v
CCGPS Crosswalk	vi

#### **Ratios and Proportional Relationships Task**

# The Number System Tasks

MCC7.NS.2b: Green Fund-raiser	8
MCC7.NS.3: Making Cookies	17

#### **Expressions and Equations Tasks**

MCC7.EE.2: Saving Money	25
MCC7.EE.3: How Much Money Do You Need?	32
MCC7.EE.4a: Home Improvement Project	39
MCC7.EE.4b: Going Shopping	48

# **Geometry Tasks**

MCC7.G.1: Scale Drawing Challenge	56
MCC7.G.6: A Slimmer Jewel Case	65

#### **Statistics and Probability Tasks**

MCC7.SP.6: Blood Groups	71
MCC7.SP.8a: The Probability of Twinning	81

# Introduction

Use these engaging problem-solving tasks to help transition your mathematics program to the knowledge and skills required by the Common Core Georgia Performance Standards for Mathematics.

This collection of tasks addresses some of the new, rigorous content found in the Common Core Georgia Performance Standards (CCGPS) for seventh grade. The tasks support students in developing and using the Mathematical Practices that are a fundamental part of the CCGPS. You can implement these tasks periodically throughout the school year to infuse any math program with the content and skills of the CCGPS.

These tasks generally take 30 to 45 minutes and can be used to replace class work or guided practice during selected class periods. Depending on the background knowledge and structure of your class, however, the tasks could take less or more time. To aid with your planning, tasks are divided into two parts. This flexible structure allows you to differentiate according to your students' needs—some classes or advanced students may need only one class period for both parts, while others may need to defer Part 2 for another day or altogether. Use your own judgment regarding the amount of time your students will need to complete Parts 1 and 2. Another strategy for compressing the time necessary to complete a task is to divide the problems or calculation associated with a task among students or small groups of students. Then students can "pool" their information and proceed with solving the task.

Each Transition Task is set in a meaningful real-world context to engage student interest and reinforce the relevance of mathematics. Each is tightly aligned to a specific standard from the Grade 7 CCGPS. The tasks provide Teacher Notes with Implementation Suggestions that include ideas for Introducing, Monitoring/Facilitating, and Debriefing the tasks in order to engage students in meaningful discourse. Debriefing the tasks helps students develop and enhance their understanding of important mathematics, as well as their reasoning and communication skills. The Teacher Notes also offer specific strategies for Differentiation, Technology Connections, and Recommended Resources to access online.

Student pages present the problem-solving tasks in familiar and intriguing contexts, and require collaboration, problem solving, reasoning, and communication. You may choose to assign the tasks with little scaffolding (by removing the sequence of steps/questions after the task), or with the series of "coaching" questions that currently follow each task to lead students through the important steps of the problem.

We developed these Transition Tasks at the request of math educators and with advice and feedback from math supervisors and middle-school math teachers. Please let us know how they work in your classroom. We'd love suggestions for improving the tasks, or topics and contexts for creating additional tasks. Visit us at <u>www.walch.com</u>, follow us on Twitter (@WalchEd), or e-mail suggestions to <u>customerservice@walch.com</u>.

# **CCGPS Crosswalk**

The following crosswalk is provided for use in selecting appropriate transition tasks that correspond to the CCGPS mathematics unit being taught. The unit number and title as outlined in the Grade 7 CCGPS curriculum map are shown in the first column, followed by the corresponding transition task. The Common Core Georgia Performance Standard addressed in that task is given, along with the page number where the task may be found.

Unit number and title	Transition task	CCGPS addressed	Page number
Unit 1: Operations with Rational Numbers	Green Fund-raiser	MCC7.NS.2b	8
Unit 1: Operations with Rational Numbers	Making Cookies	MCC7.NS.3	17
Unit 2: Expressions & Equations	Saving Money	MCC7.EE.2	25
Unit 2: Expressions & Equations	How Much Money Do You Need?	MCC7.EE.3	32
Unit 2: Expressions & Equations	Home Improvement Project	MCC7.EE.4a	39
Unit 2: Expressions & Equations	Going Shopping	MCC7.EE.4b	48
Unit 3: Ratios and Proportional Relationships	Which Pizza Is the Best Value for My Money?	MCC7.RP.1	1
Unit 3: Ratios and Proportional Relationships	Scale Drawing Challenge	MCC7.G.1	56
Unit 5: Geometry	A Slimmer Jewel Case	MCC7.G.6	65
Unit 6: Probability	Blood Groups	MCC7.SP.6	71
Unit 6: Probability	The Probability of Twinning	MCC7.SP.8a	81

# MCC7.RP.1 Task • Ratios and Proportional Relationships Which Pizza Is the Best Value for My Money?

Instruction

# **Common Core Georgia Performance Standard**

MCC7.RP.1

# **Task Overview**

# Background

Pizza menus often contain house specials and a "create your own" section. It may be cheaper to make your own version of a house favorite. In this task, students will determine the best value for their money when ordering pizza for themselves (using cost per topping) and for a group (using cost per person).

Prior to this task, students should have experience working with setting up proportional relationships. Students should also have experience with ordering fractional and decimal values.

Students may struggle with setting up the proportions for each problem. It is important that the proportions are set up consistently and with regard to the text in each problem.

This task also provides practice with:

- ordering and calculating decimal and fraction values
- rounding decimal values
- working with dollar values

# **Implementation Suggestions**

Students may work individually or in groups for Part 1 of this task. During Part 2, it may be helpful for students to collaborate.

# MCC7.RP.1 Task • Ratios and Proportional Relationships Which Pizza Is the Best Value for My Money?

# Instruction

# Introduction

Ask students to describe their experiences with being charged different prices for the same item. Students might mention different prices in different stores, value meal pricing in fast food restaurants, or student discounts.

Ask students how they would decide what foods to order for a large group if they needed to save money and satisfy different tastes. Ask them to discuss how they know they are getting a value for their dollar. Explain that in some situations the better deal may be obvious, as when a coupon is used. Other times, discovering the better deal may require math.

# Monitoring/Facilitating the Task

Ask questions and prompt student thinking so that they:

- Set up their proportion for each pizza type. They should first think of the phrase given in the problem, "cost per topping," and then transition to the unit value in order to compare each specialty pizza.
- Think about why they need to use the unit value to compare the pizzas.
- Explore multiple scenarios in Part 2 by selecting different pizza combinations that fit the constraints.
- Think in terms of cost per person to compare their scenarios.

# **Debriefing the Task**

- Explore problems 1–3 by asking students to present their responses. Stress the importance of calculating the **unit value** to compare the pizzas. Ask students how the unit value can help them justify their response to problem 3.
- Students should have followed a procedure similar to that outlined in problems 1–3 to complete problems 4 and 5, remembering to divide the per-topping cost for the whole pizza by the number of people.
- Have groups present their scenarios for Part 2. Each group should present their pizza order and their cost per person. The class discussion should focus on the different ways each group met the needs of the individuals while spending the least amount of money per person. Each solution must include at least a small Uno pizza, at least one medium or large Giardino pizza, and at least one medium or large Carne, Uno, or Tutto pizza. Solutions do not have to include a Formaggio pizza.

# **Answer Key**

- 1. Carne = \$2.00; Giardino = \$1.06; Formaggio = \$2.75; Uno = \$7.00; Tutto = \$1.29
- 2. Giardino, Tutto, Carne, Formaggio, Uno
- 3. A small Giardino pizza is the best value for one person because it has the lowest cost per topping.
- 4. At \$1.53 per topping for the whole pizza, a medium Giardino pizza offers the best topping value. Split between two people, the cost per topping is further reduced to \$0.76.
- 5. A large Giardino pizza offers the best topping value at \$1.89 per topping. Split among five people, this lowers the cost per topping to \$0.38. (Some students may insist the large Tutto pizza offers the same value—with rounding, the topping value amounts to \$1.90 per topping, but splitting it five ways results in a cost per topping of \$0.38. Accept this as a reasonable answer, given appropriate student mathematical evidence.)
- 6. Answers will vary but should provide support that the order will address everyone's requests for the lowest price possible. Sample answer: I should order 1 large and 1 medium Uno pizza (for the seven people who like meat and the one person who only likes pepperoni), 2 medium Giardino pizzas (for the six vegetarians), and 1 medium Formaggio pizza (for the one person who is allergic to mushrooms and for me, since I love cheese pizza). The total cost is \$59.50 or about \$3.72 per person.

# Differentiation

Some students may benefit from the use of calculators for this task.

Some students may want to explore alternate measures of value in a pizza. Dimensions for each size pizza and portion sizes for each topping could be developed to allow for the calculation of cost per square inch or cost per pound of topping.

# **Technology Connection**

A spreadsheet program (for example, Excel or Google Spreadsheet) could be used to organize the calculations necessary for Part 1. Students could also review the information on pizza surface area and value found at the Pete's Pizza Web site, listed in the Recommended Resources.

# **Choices for Students**

Allow students to design their own specialty pizza, name it, price it, and include it in their calculations. Offer students the chance to determine their own "best value" in pizza, such as how to get the most vegetables for their dollar.

# MCC7.RP.1 Task • Ratios and Proportional Relationships Which Pizza Is the Best Value for My Money?

# **Meaningful Context**

Value per dollar is an important measure when shopping for all items, not just pizza. Have students explore bulk pricing vs. individual pricing using online grocery and specialty stores. Students can consider if bulk shopping is always the cheapest option.

# **Recommended Resources**

- Domino's Value for Pizza <u>www.walch.com/rr/CCTTG7PizzaCompare</u> This blog post outlines the methods and results from an in-depth comparison of pizza selection available from a chain restaurant.
- Fun with Dimensional Analysis
   <u>www.walch.com/rr/CCTTG7DimenAnalysis</u>
   This site describes how to use dimensional analysis (the unit-factor method) to
   determine time ratios, such as the number of hours in a year. It includes step-by-step
   examples for finding the proportions necessary to end up with a unit rate of time. Not
   written for the middle-school level.
- Pete's Pizza: The Geometry of Pizza <u>www.walch.com/rr/CCTTG7PizzaValue</u> This Web site gives a three-part analysis of how to find the best value in pizza, complete with surface area calculations, decimal equivalents, diagrams, and example word problems.
- Savings Experiment: Slicing Up the Best Pizza Values
   <u>www.walch.com/rr/CCTTG7ValueSearch</u>
   This blog entry details the author's search for the best value in pizza. It includes
   discussion and results from several methods of determining the best pizza.
- Unit Price

www.walch.com/rr/CCTTG7CostPerUnit

This Web site reviews the concept of cost per unit through examples and an interactive Web-based game; it also provides links to a unit price game and other resources.

Instruction

# MCC7.RP.1 Task • Ratios and Proportional Relationships Which Pizza Is the Best Value for My Money?

# Part 1

Sal's Pizzeria offers several specialty pizzas. Each pizza includes tomato sauce, mozzarella cheese, a thin crust, and additional toppings as listed. A small pizza serves 1 person, a medium serves 3 people, and a large serves 5 people. Sal's regular pizzas are priced by the number of toppings, but the specialty pizzas each have their own price, listed below.

<b>The Carne</b> Pepperoni, ham, ground bee	f, bacon, sausage, chicl	ken
Small: \$12.00	Medium: \$16.25	Large: \$19.50
The Giardino		
Green peppers, tomatoes, or	nions, olives, mushroor	ns, pineapple, jalapenos,
broccoli, garlic		
Small: \$9.50	Medium: \$13.75	Large: \$17.00
The Formaggio		
Ricotta cheese, Parmesan ch	eese, Asiago cheese	
Small: \$8.25	Medium: \$11.00	Large: \$13.25
The Uno		
Pepperoni		
Small: \$7.00	Medium: \$9.50	Large: \$11.50
The Tutto		
Pepperoni, ham, ground bee onions, olives, mushrooms,	U	ken, green peppers, tomatoes,
Small: \$15.50	0	Large: \$22.75

1. Calculate the cost per topping for each small specialty pizza. Round to the nearest whole cent.

2. Arrange the small specialty pizzas in order from the least cost per topping to the most cost per topping.

# continued

#### NAME:

# MCC7.RP.1 Task • Ratios and Proportional Relationships Which Pizza Is the Best Value for My Money?

3. Determine which small specialty pizza offers the best value for toppings if you are eating alone and you aren't picky about toppings. Justify your response. Remember to round to the nearest whole cent.

4. Determine which medium specialty pizza offers the most toppings for your money if 2 people will split the cost of the pizza. Justify your response. Remember to round to the nearest whole cent.

5. Determine which large specialty pizza offers the most toppings for your money if 5 people will split the cost of the pizza. Justify your response. Remember to round to the nearest whole cent.



# MCC7.RP.1 Task • Ratios and Proportional Relationships Which Pizza Is the Best Value for My Money?

# Part 2

You are throwing a pizza party. You want to make your guests happy for the lowest cost possible. You know the following information.

- There are 16 guests total, including you.
- You love cheese pizza.
- Six guests are health-conscious vegetarians—their pizza must have vegetables on it but no meat.
- Seven guests insist on having meat on their pizza.
- One guest will eat only pepperoni on his pizza.
- One guest is allergic to mushrooms.
- 6. Decide how many of each pizza you should order. Make sure to consider the requests of your guests. Justify that your response meets the needs of all your guests at the lowest cost possible.

# Instruction

#### **Common Core Georgia Performance Standard**

MCC7.NS.2b\*

# **Task Overview**

# Background

Division of rational numbers is a difficult skill for many students to master. Students struggle with division of decimal and fractional numbers both from a conceptual standpoint and a procedural standpoint. Students must have this important skill to reinforce their number sense. In order to complete this task, students should have prior experience with dividing decimal numbers. They should also have a conceptual understanding of what happens when you divide by a number between 0 and 1 compared to what happens when you divide by a number greater than 1 (when the divisor is between 0 and 1, the quotient is larger than the dividend; when the divisor is greater than 1, the quotient is smaller than the dividend). Students should also have some prior experience with creating a goal chart.

This task is designed to give students practice dividing decimal numbers in the real-world context of organizing a collection for used printer cartridges. Students will determine how many printer cartridges must be collected to reach a goal. They will organize their volunteers and coordinate the collection. Then they will have to consider cost of gas in determining how many printer cartridges they must collect to reach their goal.

The task also provides practice with:

- multiplication of decimal numbers
- working with money
- determining reasonable answers for a given context
- creating a goal chart

# **Implementation Suggestions**

Students may work individually, in pairs, or in small groups to complete one or both parts of the task. Specifically, students may work in pairs or small groups to create the goal chart. Students may use one of the virtual manipulative tools as they complete the task.

\*partially addressed

# Introduction

Introduce the task by asking students if they have ever been involved in a project to raise money toward some goal. What role did they have? Did anyone assist in the goal setting? Did anyone help figure out how much they would have to collect to meet the goal?

If necessary, begin the task by reviewing the steps used when dividing by a decimal number. It may be helpful to do an example on the board for the whole class.

As needed, discuss some of the verbal clues that indicate division should be used to solve a problem (for example, the words "per" and "each"). It might be helpful to brainstorm these words and record them on the board prior to the task.

If desired, facilitate a class discussion around the different types of answers that can be obtained with a division problem (fraction, decimal, or integer) and how to determine what kind of number would be reasonable in certain contexts. Would you round using the traditional rules of rounding or would you always round up or round down?

Determine whether students will work individually or in pairs and make the appropriate groupings.

# Monitoring/Facilitating the Task

Ask questions and prompt student thinking so that they:

- Carefully read the statements and questions and look for the verbal clues to determine when division should be used.
- Remember the steps required to perform long division with decimal numbers (moving the decimal place of the divisor and the dividend appropriately).
- Think about what kinds of answers are reasonable in the context of the question and why.
- Determine how to round (up or down) given the context of the problem statement.
- Consider other real-world examples where division is used.
- Use creativity when designing and creating their goal chart.

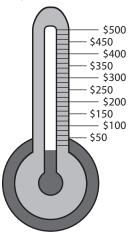
# **Debriefing the Task**

- Upon completion of the task, students should share their results.
- Encourage students to explain how they determined that division was required based on the verbal clues in the problem statement.
- Prompt students to describe the process of long division with decimal numbers.
- Ask students to describe how they decided what kinds of numbers were reasonable. How did they decide to round?

- Focus on students' comprehension of the solutions and their appropriateness, rather than on the procedures used for arriving at the solution.
- Probe for student understanding of quotients of rational numbers. As appropriate, ask students to explain why an answer is so big, or so small.
- Invite students to justify their solutions—to explain why they make sense.
- Ask students to share any difficulties they had with this task. Encourage them to describe how they addressed these difficulties.
- Urge students to share their other real-world examples that use division.
- Invite students to share their goal charts. How did they develop their design? What is being tracked (number of cartridges or dollars)?

# **Answer Key**

- 1. 22 students  $\div$  7 neighborhoods  $\approx$  3.14 students per neighborhood; division
- 2. The answer must be an integer, since you can't have 0.14 students. One option is to send 3 students each to 6 neighborhoods and 4 students to the remaining neighborhood. Students should be encouraged to make recommendations about which group would have the extra student—one covering a larger neighborhood, one with more businesses, etc.
- 3, \$500 ÷ \$0.80 = 625 used printer cartridges; division
- 4. The answer must be an integer. The group needs to collect 625 cartridges to reach the goal.
- 5, 625 used printer cartridges  $\div$  7 groups of students  $\approx$  89.29; division. The answer must be an integer, so each group must collect 90 cartridges.
- 6. Answers will vary, depending on whether students used dollar amounts or cartridges collected as goal markers. Sample goal chart using dollars as goal markers:



- 7. 625 cartridges ÷ 25 cartridges per bag = 25 bags; division. An integer is a reasonable number.
- 8. At 20 bags per box, you will need to buy 2 full boxes, even though you will only use 5 out of the second box.  $2 \times $4.70 = $9.40$  for bags
- 9. 12 miles ÷ 19 miles per gallon ≈ 0.63 gallons; division. A decimal number is a reasonable answer since you can pump portions of a gallon of gas. (Some students may argue that they need to pump integer amounts, and will round up to 1 gallon. Accept this answer as long as the argument is reasonable and the rest of the answers correspond).
- 10. 0.63 gallons × 7 cars × \$3.90 per gallon ≈ \$17.20 for gas (students who rounded to 1 gallon should calculate 1 gallon × 7 cars × \$3.90 per gallon = \$27.30); multiplication
- \$9.40 (for bags) + \$17.20 (for gas) = \$26.60 in expenses; \$26.60 ÷ \$0.80 = 33.25; addition and division. An additional 34 used printer cartridges will need to be collected to cover the cost of the plastic bags and gas.
- 12. Answers will vary. Sample answers: When planning food for a party—if each pizza has 6 slices and each of the 25 people you invite will eat 2 slices, how many pizzas should you order?; When going shopping—if you have \$150 to spend and each game costs \$26.75 including tax, how many games can you buy?

# Differentiation

Some students may benefit from the use of calculators during this task. Some students may find it helpful to use one of the online tools for instruction or review prior to completing the task.

Students who complete the task early could create a spreadsheet (for example, using Excel or Google Spreadsheet) that could be used to track and chart progress toward the goal.

# **Technology Connection**

Students could create a spreadsheet (for example, using Excel or Google Spreadsheet) that could be used to track and chart progress toward the goal.

# **Choices for Students**

Following the introduction, offer students the opportunity to use their own experiences with fund-raising. They could use information from their own experiences, rather than the information provided, to work through this task.

# **Meaningful Context**

This task uses a real-world example of fund-raising. It uses an example that supports recycling and increases environmental awareness among students and among those from whom they collect the used cartridges—individual citizens and businesses. Most students will be involved in some aspect of fund-raising during their school years or beyond. Many students will find themselves in a position where they need to help facilitate or coordinate the activities. This task also addresses the issue that many calculations in real-world applications involve numbers that are not integers.

# **Recommended Resources**

- Decimals Word Problems
   <u>www.walch.com/rr/CCTTG7DivideDecimals</u>
   This interactive site contains word problems involving division of decimal numbers.
- Division of Decimals by Decimals <u>www.walch.com/rr/CCTTG7Divide2Decimals</u> This site provides directions to divide two decimal numbers, as well as an applet with practice problems and answers.
- Goal Thermometer for Charting Progress <u>www.walch.com/rr/CCTTG7GoalThermometer</u> Download a sample goal thermometer for charting progress.
- Too Big or Too Small? <u>www.walch.com/rr/CCTTG7NumberSense</u> This activity reinforces number sense for large numbers, fractions, and decimal numbers.

# Part 1

You have been elected to organize a collection for used printer cartridges. Your class is trying to raise money for a class trip. As a group, you've chosen a recycling project because you can earn money AND raise environmental awareness at the same time. There are 22 students who have volunteered to help collect used printer cartridges. Your goal is to raise \$500.

1. There are 7 neighborhoods that you will visit to collect printer cartridges. You'll be collecting from homes and from businesses. How many students should go to each neighborhood? What operation is used to find this number?

2. Think about what kind of answer (fraction, decimal, integer) is reasonable in the context of this task. Describe fully how you would make the groups of students.

3. Most used printer cartridges are worth \$0.80 apiece. How many used printer cartridges must be collected to reach the goal of \$500? What operation is used to find this number?

4. Think about what kind of answer (fraction, decimal, integer) is reasonable in the context of this task. Describe fully how many used printer cartridges need to be collected to reach the goal.

# continued

5. How many used printer cartridges does each group of students need to collect? What operation is used to find this number? What kind of answer is reasonable in this context? Describe fully.

6. Create a goal chart for your fund-raiser that could be colored in as used printer cartridges are collected. Think about what scale you want to use and whether you will track number of cartridges collected or dollars earned.



# Part 2

You now need to make plans for the other expenses involved in this project.

First, you must coordinate transportation for the printer cartridge collection. You have adult volunteers to drive students, but your class will need to pay for the gas used.

Second, you have to purchase necessary supplies. You will use gallon-size reclosable plastic bags to collect and ship the cartridges. One gallon-size bag will hold 25 used printer cartridges.

7. How many bags of cartridges will you need to collect to reach your goal? What operation is used to find this number? What kind of number is reasonable in this context?

8. If you can buy a box of 20 gallon-size bags for \$4.70, how much will you need to spend to get enough bags?

9. You estimate that each car will travel 12 miles during the cartridge collection. The average mileage for each car is 19 miles per gallon. How much gasoline will each car need? What operation is used to find this number? What kind of number is reasonable in this context?

# continued

10. Gasoline costs \$3.90 per gallon. How much will your fund-raising group need to spend on gas if 7 adults volunteer to drive? What operation is used to find this number?

11. Remember that your goal is to earn \$500 for your class trip. How many additional used printer cartridges will you need to collect to cover the cost of the gasoline and reclosable bags? Show your work. What operation is used to find this number?

12. Can you think of another example in which these operations are used in a real-world context? Describe fully.

# MCC7.NS.3 Task • The Number System Making Cookies

Instruction

# **Common Core Georgia Performance Standard**

MCC7.NS.3

# Task Overview

# Background

The task is designed to help students conceptually understand what happens when you add, subtract, multiply, or divide rational numbers. It will help students see what happens in the context of modifying a recipe for cookies.

Prior to this task, students should have experience with adding, subtracting, multiplying, and dividing fractions. Students should also have experience with ordering rational numbers. The task is designed to expand on, and apply this knowledge to working with operations with rational numbers. Rational numbers include integers and any number that can be written as a fraction.

Students may struggle when multiplying and dividing rational numbers. The first part of the task asks them to multiply rational numbers by other rational numbers. The second part asks students to divide rational numbers by other rational numbers. Even if students have difficulty with these procedures, the real-world context should help them to reason through the processes.

The task also provides practice with:

- measurements
- comparing and ordering rational numbers
- reading a recipe

# **Implementation Suggestions**

- Consider allowing students to work with fraction tiles to help with operations with rational numbers, especially with division. You may wish to use overhead fraction tiles as well.
- Students may use actual measuring cups with rice to model the steps in the task.
- Students may work individually, in pairs, or in small groups to complete one or both parts of the task.
- Alternatively, students may meet in groups to share their results and reflect after individually completing the task, before a class-wide discussion.

# Introduction

Introduce the task by asking students if they have ever followed a recipe to make a dish. Have they ever adjusted the recipe to make more or less of the recipe? Would you know how to change the recipe? What mathematical operations would need to be used?

You might also want to have a discussion about what, in general, happens when you multiply and divide fractions. What is happening to the numbers? Are they becoming larger or smaller? Why? Does this make sense?

# Monitoring/Facilitating the Task

Ask questions and prompt student thinking so that they:

- Use the fraction tiles to help with operations with rational numbers, if helpful.
- Use the measuring cups and rice to model the steps in the task, if helpful.
- Think about how the answer should relate to the question before performing the actual mathematical operation. If a recipe is doubled, then you need to use more of each ingredient, but if a recipe is halved, you need less of each ingredient.
- Defend their responses. Make sure that students articulate how they used their calculations to answer a question. Have them describe their arithmetic and explain their responses, describing verbally how they determined their answers. Prompt for and encourage the use of proper mathematical terms.
- Understand why they should write their numbers in fraction form rather than converting to decimals in this task. Students should not use calculators for this task.
- Understand why they need to convert mixed numbers to improper fractions.
- Realize there are several ways to adjust a recipe. Students could double a recipe by adding the ingredients twice or by multiplying the ingredient amounts by 2. Reinforce that multiplication is repeated addition and that division is repeated subtraction.

# **Debriefing the Task**

- Encourage students to:
  - Explain the steps they used in determining the new amounts of ingredients required.
  - Use appropriate mathematical terminology.
  - Share where they had difficulty and how they resolved their questions.

- Some students might have approached Part 1 by multiplying the values by <sup>3</sup>/<sub>2</sub>. Others might have multiplied by 3 first and then divided the results by 2. Encourage discussion around why both of these approaches result in the same values.
- Some students might have approached Part 2 by dividing the values by 3 and some might have multiplied by <sup>1</sup>/<sub>3</sub>. Explore the relationship between dividing by a number and multiplying by the reciprocal.
- Assess understanding by expanding the parameters used in the task. Encourage students to develop alternate responses for making a different number of cookies.
- Encourage students to share their own experiences with cooking and following recipes. Ask students to explore how this task can relate to their life outside of the math classroom.

# Answer Key

- 1. Multiply by  $\frac{3}{2}$ .
- 2. Each ingredient needs to be multiplied by  $\frac{3}{2}$ .
- 3. Revised ingredient list:
  - 3 cups of white sugar
  - <sup>3</sup>/<sub>4</sub> (12 fluid ounce) can of evaporated milk
  - 1<sup>1</sup>/<sub>8</sub> cups of butter
  - 5<sup>1</sup>/<sub>4</sub> cups of quick-cooking oats
  - $1\frac{1}{2}$  (3.4 ounce) packages of instant butterscotch pudding mix
- 4. The revised ingredient amounts should be greater than in the original recipe. You can confirm this by writing the two amounts using a common denominator. Then confirm that the numerator of the revised ingredient amount is greater. You could also convert the rational numbers to decimal form and confirm that the revised ingredient amount is greater.
- 5. You can multiply each ingredient by 3 and then divide by 2.
- 6. The  $\frac{2}{3}$  cup of sugar is  $\frac{1}{3}$  of the 2 cups in the original recipe. The  $\frac{2}{3}$  cup of sugar will make  $\frac{1}{3}$  of 24 cookies (8 cookies).
- 7. All of the remaining ingredients will need to be multiplied by  $\frac{1}{3}$ .

- 8. Revised ingredient list:
  - <sup>2</sup>/<sub>3</sub> cup of white sugar
  - <sup>1</sup>/<sub>6</sub> (12 fluid ounce) can of evaporated milk
  - <sup>1</sup>/<sub>4</sub> cup of butter
  - $1^{1/_{6}}$  cups of quick-cooking oats
  - <sup>1</sup>/<sub>3</sub> (3.4 ounce) package of instant butterscotch pudding mix
- 9. The revised ingredient amounts should be less than in the original recipe. You can confirm this by writing the two amounts using a common denominator. Then confirm that the numerator of the revised ingredient amount is smaller. You could also convert the rational numbers to decimal form and confirm that the revised ingredient amount is smaller.
- 10. You could divide each ingredient by 3.

#### Differentiation

Some students may benefit through the use of fraction tiles in this task. Some students may benefit through the use of measuring cups and rice to model the steps in this task.

#### **Technology Connection**

Students could create a spreadsheet that shows the original recipe along with revised recipes for greater or smaller amounts.

#### **Choices for Students**

Following the introduction, offer students the opportunity to use their own favorite recipe to complete the task.

# **Meaningful Context**

This task may be expanded by allowing students to hold a bake-off. The students would work in pairs or small groups to create a dish using a recipe that has been increased or decreased in some way, and the best-tasting dish would win. The groups would submit their recipes and then randomly draw how the recipe would be modified.

# **Recommended Resources**

- AllRecipes.com—Scaling Recipes <u>www.walch.com/rr/CCTTG7ScalingRecipes</u> This link to the popular recipe site provides an explanation of the site's automatic scaling feature that you can display and/or students can explore to model the concepts in this task.
- Disney Family Fun—Recipes <u>www.walch.com/rr/CCTTG7MathRecipes</u> Search for kid-friendly recipes that encourage students to use math skills in the kitchen.
- Moms Who Think—Food and Recipes <u>www.walch.com/rr/CCTTG7KidRecipes</u> This helpful site provides a number of kid-friendly recipes.
- TLC Cooking—Recipe Tips <u>www.walch.com/rr/CCTTG7RecipeTips</u> This link leads to tips for increasing or decreasing recipes.
- YouTube—Multiplying and Dividing Rational Numbers <u>www.walch.com/rr/CCTTG7MultiplyDivideRationalNumbers</u> This video features a straightforward review of the procedures for multiplying and dividing rational numbers.

# MCC7.NS.3 Task • The Number System Making Cookies

# Part 1

You want to make cookies for a bake sale at school. You choose the following recipe.

No-Bake Butterscotch Oatmeal Cookies		
Ingredients	Directions	
2 cups of white sugar $\frac{1}{2}$ (12 fluid ounce) can of evaporated milk	1. In a 3-quart microwaveable bowl, combine the sugar, butter, and evaporated milk.	
$\frac{2}{\frac{3}{4}}$ cup of butter	<ol> <li>Cook on high power for 2 to 5 minutes, stirring occasionally until the mixture comes to a rapid boil.</li> </ol>	
$3\frac{1}{2}$ cups of quick-cooking oats 1 (3.4 ounce) package of instant butterscotch pudding mix	<ol> <li>Allow the mixture to boil undisturbed for 20 to 30 seconds. Remove from heat and stir in the instant pudding and oatmeal.</li> <li>Spoon onto cookie sheets lined with waxed paper. Allow to sit at least 15 minutes or until firm.</li> </ol>	

The recipe will make 24 cookies. You have been asked to bring 36 cookies to the bake sale. How can you change this recipe to end up with 36 cookies? Answer the following questions.

1. How do you need to adjust the recipe in order to make 36 cookies?

2. How do you need to adjust the amount of each ingredient?

# continued

# MCC7.NS.3 Task • The Number System Making Cookies

3. Write a revised ingredient list, showing the amounts of each ingredient needed to make 36 cookies.

4. Should the revised ingredient amounts be greater or less than the amounts in the original recipe? How can you confirm your answer?

5. Can you think of another way to determine the amounts needed to make 36 cookies? Explain fully.



23

# MCC7.NS.3 Task • The Number System Making Cookies

# Part 2

You want to make some more cookies to keep at home. You measure the sugar you have left over and realize that you have only  $\frac{2}{3}$  cup. You need to determine how many cookies you can make. You also need to figure out how much of each of the remaining ingredients you need to use for this batch.

- 6. How can you determine how many cookies you can make with the  $\frac{2}{3}$  cup of sugar?
- 7. How do you need to adjust the amount of the remaining ingredients?

8. Write a revised ingredient list. Show the amounts of each ingredient needed for the new batch.

9. Should the revised ingredient amounts be greater or less than the amounts in the original recipe? How can you confirm your answer?

10. Can you think of another way to determine the amounts needed to make this batch of cookies? Explain fully.

# Saving Money

Instruction

# **Common Core Georgia Performance Standard**

MCC7.EE.2

# **Task Overview**

# Background

Understanding that different representations of expressions can result in equivalent expressions is an important building block in algebra. Sometimes rewriting an expression in an equivalent form can result in a problem that is easier to solve. This task helps students discover relationships between different forms of an expression in real-world examples using money. In the first part, students will investigate interest rates (which result in increase by a percent) and in the second part, students will investigate sales rates (which result in decrease by a percent).

Prior to this task, students should have experience with converting percentages to decimal numbers, multiplying by decimal numbers, and writing algebraic expressions from verbal descriptions.

The task also provides practice with:

- multiplying with decimal values
- calculating percentages
- writing algebraic expressions from verbal descriptions

# **Implementation Suggestions**

Students may work individually, in pairs, or in small groups to complete one or both parts of the task. Some students may benefit from the use of calculators for the evaluation portion of this task.

# Introduction

Introduce the task by asking students if they have a savings account to help them save the money they make or receive as gifts. Do they earn interest on the account? Are they involved in the decision of where the money is invested (in an account, a CD, or other savings plan)? Do they keep track of the account balance? What do they know about interest rates?

When introducing Part 2, ask students if anyone has a family business. How are the prices set? Are there ever sales designed to help move merchandise? Is anyone involved with any aspects of the business?

Begin the task by having students recall the difference between an algebraic expression and an algebraic equation (an equation has an equal sign). If necessary, discuss how the students might determine if two expressions are equivalent (substitute a value into each expression and see if the result is the same). Review how verbal expressions translate into mathematical operations (for example, "increased by" means *add to* and "reduction amount" means *subtract from*). Determine whether students will work individually or in pairs and make the appropriate groupings.

# Monitoring/Facilitating the Task

Ask questions and prompt student thinking so that they:

- Use variables to represent unknown amounts.
- Accurately rewrite percentages as decimal numbers.
- Remember the rules of multiplying decimal numbers.
- Think about reasonable answers. In a savings account, should the amount at the end of the interest period be more or less than the initial amount? At a store, should the sale prices be more or less than the original price?
- Determine if two expressions are equivalent.
- Represent an algebraic expression in verbal form.

# **Debriefing the Task**

- Upon completion of the task, students should share their results.
- Ask students to explain how they determined if they should be adding or subtracting the percentages. Encourage a discussion around reasonableness of answers.
- Ask students to discuss how they determined that a variable should be used ("an amount") and how they decided which variable to use.
- Ask students to describe how they determined that two expressions are equivalent. How is that shown? (by the use of an equal sign between the two expressions)
- Ask students to share any difficulties they had with this task. Encourage them to describe how they addressed those difficulties.

# **Answer Key**

- 1. 0.02*x*
- 2. Multiply the amount invested by 0.02 and add the result to the original amount invested.
- 3. x + 0.02x
- 4. 1.02*x*
- 5. The expressions are equivalent. Sample answer: Suppose you invest \$200.

200 + 0.02(200) = 204

1.02(200) = 204

The amounts are equal.

- 6. x + 0.2x = 1.02x; Sample answer: A savings account with a 2% interest rate yields the same amount as multiplying the amount invested by 1.02.
- 7. Use the expression 0.10*x*, for which *x* is the original price of any item. Multiply the price by 0.10, and the result is the amount by which the price will be reduced.
- 8. Multiply the original price by 0.10 and subtract the result from the original price.
- 9. x 0.10x
- 10. 0.90*x*
- 11. Evaluate each expression using the same original price and confirm that the results are the same.

# Differentiation

Some students may benefit from the use of calculators during this task. Some students may find it helpful to use one of the online instruction/review tools prior to completing the task.

Students who complete the task early could research various interest rates and calculate earnings of a certain amount invested.

# **Technology Connection**

Students could use one of the online tools prior to completing the task.

# **Choices for Students**

Following the introduction, offer students the opportunity to research and use their own interest rates for the first part of the task. Similarly, they could set their own sales rate for the second part of the task. You might ask them to find advertised interest rates or sales in the newspaper.

# **Meaningful Context**

•

This task uses the real-world example of a savings account. Many students have a college fund or other savings account and could track their own interest earnings. The second part of the task uses the real-world example of sale prices. Students should be aware of various ways to represent sale prices or interest rates.

# **Recommended Resources**

- Algebra Tiles
   <u>www.walch.com/rr/CCTTG7AlgebraTiles</u>
   Students can use this interactive tool to determine which expressions are equivalent.
- Equivalent Expressions
   <u>www.walch.com/rr/CCTTG7EquivExpressions</u>
   This applet uses algebra tiles to explore equivalent expressions.
  - Money Rates <u>www.walch.com/rr/CCTTG7InterestRates</u> Look up current daily interest rates for various types of accounts.

# MCC7.EE.2 Task • Expressions and Equations Saving Money

# Part 1

You are planning to open a savings account. You want to shop around to get the best interest rate so you can earn the most money.

1. The first bank you visit has an interest rate of 2% per year. Write an algebraic expression to represent the amount of interest you would earn on an amount of money after one year.

2. How would you determine the total amount of money you would have in your savings account at the end of one year? Describe fully.

3. Write an algebraic expression to represent the amount you described in problem 2, your total savings after one year in the account.

4. You visit a different bank that advertises that they will multiply the amount of money that you have in your savings account by 1.02. Write an algebraic expression to represent this scenario.

# continued

5. Compare the two algebraic expressions you wrote. Describe their relationship. Select an amount of money you think you will have to open your account. Use that amount to evaluate the two expressions you wrote. How do the amounts compare?

6. Using algebraic notation, develop an equation from the above information. Write a verbal statement that corresponds to the algebraic notation.



#### MCC7.EE.2 Task • Expressions and Equations Saving Money

#### Part 2

Your mother owns a skateboard shop. You are helping to price items for an upcoming sale. All prices will be reduced by 10% for the sale.

7. How can you determine the amount by which the original price will be reduced for any item? Describe fully.

8. How can you determine the sale price of an item? Describe fully.

9. Write an algebraic expression to represent the sale price of an item.

10. Think about the expression you wrote. Write another algebraic expression to represent the sale price.

11. How can you determine that the two expressions are equivalent? Describe your reasoning.

## How Much Money Do You Need?

Instruction

#### **Common Core Georgia Performance Standard**

MCC7.EE.3\*

#### Task Overview

#### Background

This task presents a real-world application of problem solving with positive and negative rational numbers. It engages students with scenarios for earning, saving, and spending money.

Students often struggle when working with decimal values in mathematics despite their ability to work accurately with dollar amounts. This task helps students internalize decimal operations and rounding skills, while focusing on earning money and saving to meet goals.

The task also provides practice with:

- the four operations with decimal values
- calculating percents
- reading data tables

#### **Implementation Suggestions**

Students may work individually, in pairs, or in small groups to complete one or both parts of the task. Alternatively, students may meet in groups to share their results and reflect after individually completing the task, before a class-wide discussion.

#### Introduction

Introduce the task by asking students about how they earn money and keep track of the money they earn. Ask for student strategies for saving money. Question students as to how they determine if they have made enough money to meet a goal. Ask students if they have ever worked to earn money for a charitable organization.

\*partially addressed

#### Monitoring/Facilitating the Task

Ask questions and prompt student thinking so that they:

- Recognize the mathematics that they are doing, and can articulate their strategies and justify their solutions. Make sure that students articulate where and when they are using addition, subtraction, multiplication, and division during each calculation.
- Use proper mathematical terms in their explanations.
- Realize there are several ways to calculate percentages and that the calculation often requires multiple steps.

#### **Debriefing the Task**

- Students will be calculating monetary values. They must recognize how to round their decimal values appropriately. Problem 1 requires that students recognize calculations must be rounded to indicate a dollar amount. Problem 4 requires that students realize they must work longer to earn slightly more than they need to reach their goal.
- Problem 6 can be calculated using multiple techniques. Encourage students to share how they calculated their answer. Ask students to explain why calculating percentage week-to-week or monthly yields the same result.
- Students may have multiple explanations for their response to problem 8. Encourage students to share their arithmetic based on logical reasoning methods.
- Problems 9 and 10 require students to remember that they will still be donating 10% of their income when they are earning money.
- Assess understanding by expanding the parameters used in the task. Encourage students to develop alternate responses for an increase (or decrease) in pay or charitable donation.
- Encourage discussion about student experiences with earning money and keeping track of their earnings. Ask students to explore how this task can relate to their life outside of the math classroom.

#### **Answer Key**

Answers may vary depending on whether students round the amounts for each job separately, or after adding them together. Alternate answers are listed in parentheses.

- 1. Week 1 = \$20.51 (or \$20.50), Week 2 = \$21.50, Week 3 = \$31.07 (or \$31.06), Week 4 = \$14.88
- 2. \$87.96
- 3. No; explanations will vary depending on the student's method. Students may multiply \$15 by 8 (\$120) to determine that the total earned is less than that amount, or they may divide the total earned by 8 to determine that the amount per week is less than \$15. Check student reasoning. Sample answer: No, because when I divide the amount earned by 8, the result is less than \$15.
- 4. 8 hours. I'm 32.04 short;  $32.04 \div 4.25 = 7.54$ , which rounds up to 8 full hours. (Some students may arrive at a shortfall of 32.06;  $32.06 \div 4.25 = 7.54$ , which rounds to 8 full hours.)
- 5. I need to do 5 hours of yard work. 5 hours of homework help 4.25 = 21.25; the \$32.04 I still need 21.25 = 10.79;  $10.79 \div 2.5 = 4.32$ , which rounds up to 5 full hours. (For students who determined a shortfall of \$32.06: 32.06 21.25 = 10.81;  $10.81 \div 2.5 = 4.32$ , which rounds to 5 full hours.)
- 6. \$8.80 (\$8.79)
- 7. \$79.16 (\$79.15)
- 8. No, because I didn't have enough before, and now I have 10% less money.
- 9. 11 hours (Students may arrive at an answer of 10 hours, but if they plan to donate 10% of additional income to the animal shelter, they need to work an extra hour to accommodate the donation and still have at least \$120.)
- 10. 9 hours (again, accounting for donations to the animal shelter)

#### Differentiation

Some students may benefit from the use of calculators during this task.

#### **Technology Connection**

Students could create a spreadsheet to track their earnings and donations each week.

#### **Choices for Students**

Following the introduction, offer students the opportunity to set their own earning goals, using their own allowance or job earnings. Have them develop their own plan for reaching a goal, while also saving or donating a portion of their income.

#### **Meaningful Context**

You may expand this task by having students set a goal to earn money for a charitable donation. Have students research and select a worthwhile organization. Students should then develop a time line for earning money to reach their goal. Each week, have students calculate the money they have earned and the money they need to earn, then ask them to explain if they are on track to meet their goal and to reassess their goal if necessary.

#### **Recommended Resources**

- Scholastic—Kids' Tips for Earning and Saving Money <u>www.walch.com/rr/CCTTG7EarningSaving</u> This article provides background information and suggestions for earning and saving money.
- Federal Reserve of San Francisco, Fedville <u>www.walch.com/rr/CCTTG7Fedville</u> This game teaches kids about earning, spending, and saving money.

Instruction

#### Part 1

You can earn \$4.25 an hour helping a younger neighbor with his homework. You can earn \$2.50 an hour doing yard work on the weekend. During the month of May, you worked hard to earn money for your summer vacation. You used a table to keep track of the number of hours you worked each week. How much money did you save? Is it enough spending money for the summer? Will you have to work more? If so, how much? Round to the nearest hundredth.

	Homework help (hours)	Yard work (hours)	
Week 1	3.5	2.25	
Week 2	3	3.5	
Week 3	2.75	7.75	
Week 4	3.5	0	

1. Calculate the amount of money you earned each week. Show your work and explain your reasoning.

2. Calculate the amount of money you made in the month of May. Show your work and explain your reasoning.

3. Your summer vacation is 8 weeks long. You want to have at least \$15 to spend each week. Have you met your goal? Explain how you know.

#### continued

4. How many more full hours do you need to help your neighbor with homework to meet your goal? Explain how you know.

5. You were only able to help your neighbor with homework for 5 more hours. How many more full hours will you need to work in the yard to reach your goal? Explain how you know.



#### Part 2

You would like to donate money to help the animal shelter in your neighborhood. You have decided to donate 10% of your income to the shelter. For each problem, round to the nearest hundredth.

- 6. Calculate the amount of money you donated in the month of May. Show your work and explain your reasoning.
- 7. Calculate the amount of money you saved in the month of May. Show your work and explain your reasoning.
- 8. Have you met the goal you set for your summer spending money? Explain how you know.
- 9. How many more full hours do you need to help your neighbor with homework to meet your goal? Explain how you know.
- 10. You were only able to help your neighbor with homework for 5 more hours. How many more full hours will you need to work in the yard to reach your goal? Explain how you know.

# **Home Improvement Project**

Instruction

#### **Common Core Georgia Performance Standard**

MCC7.EE.4a\*

#### Task Overview

#### Background

Solving equations is one of the fundamental learning goals in algebra. Prior to this task, students should have experience with writing algebraic expressions and equations from verbal statements. Students should also have experience with the distributive property.

This task is designed to give students practice with solving equations in the real-world context of planning how to finish a basement. Students will write expressions from a verbal description of the problem statement, write an equation using additional given information, and then solve the equation. They will be asked to write the equations in two ways—one using the distributive property and one without using the distributive property. Students will also make a drawing of the plan for the basement.

The task also provides practice with:

- finding the area of rectangles
- adding and subtracting numbers
- multiplying and dividing numbers
- writing expressions and equations from verbal expressions
- making a drawing of a plan

#### **Implementation Suggestions**

Students may work individually, in pairs, or in small groups to complete one or both parts of the task. Students may use one of the virtual manipulative tools as they complete the task.

\*partially addressed

#### MCC7.EE.4a Task • Expressions and Equations Home Improvement Project

Instruction

#### Introduction

Introduce the task by asking students if they have ever helped with a home improvement project. What was the project? In what ways did they help?

As necessary:

- Have students recall the difference between an algebraic expression and an algebraic equation (an equation contains an equal sign).
- Review how to write an algebraic equation given a verbal description of a problem statement.
- Make sure students remember how to find the area of a rectangular figure ( $A = l \times w$ ).
- Ask students to describe how two algebraic expressions are equivalent.
- Review the distributive property and how to write equivalent expressions or equations with and without using it.

Determine whether students will work individually or in pairs and make the appropriate groupings.

#### **Monitoring/Facilitating the Task**

Ask questions and prompt student thinking so that they:

- Make a sketch of the basement if they are having trouble getting started.
- Think about the use of a variable here. What does the variable represent?
- Understand why is it important to properly label the diagram, including units of measure.
- Realize that there is more than one way to write an algebraic expression to represent the problem statement.
- Read the questions carefully and make the appropriate changes to their expressions and equations.

#### **Debriefing the Task**

- Upon completion of the task, ask students to share their results.
- Have them explain the steps they used to solve their equations using appropriate mathematics terminology (for example, subtract from both sides of the equation, apply the distributive property, divide both sides of the equation by...).

- Encourage students to share their drawings of the plan for the basement. Discuss why it is important to properly label drawings and use units of measure (so another person looking at the plan can follow it).
- Ask students to share any difficulties they had with this task. Encourage them to describe how they addressed these difficulties.

#### **Answer Key**

- 1. 2x + 28
- 2. 28(2x + 28) = 1,680
- 3. 28(2x + 28) = 1,680 or 56x + 784 = 1,680

2x + 28 = 60	56x = 896

= 16

x = 16

The lengths of the family room and home office are 16 feet each.

Home	Family	Laundry	28 ft
office	room	room	
16 ft	16 ft	28 ft	

- 4. The answer depends on the equation used to solve problem 3. Students may write either 28(2x + 28) = 1,680 or 56x + 784 = 1,680.
- 5. See the answer to problem 3. The result is the same, 16 feet.
- 6. 3x + 18
- 7. 28(3x + 18) = 1,680

```
8. 28(3x + 18) = 1,680 or 84x + 504 = 1,680

3x + 18 = 60 84x = 1,176

3x = 42 x = 14

x = 14
```

The lengths of the family room, the workshop, and the home office are 14 feet each.

Workshop	Home office	Family room	Laundry room	28 ft
14 ft	14 ft	14 ft	18 ft	

- 9. The answer depends on the equation used to solve problem 8: either 28(3x + 18) = 1,680 or 84x + 504 = 1,680.
- 10. See the answer to problem 8. The result is the same, 14 feet.

#### Differentiation

Some students may benefit from the use of calculators during this task. Some may find it helpful to use one of the online demonstration/modeling tools prior to or while completing this task.

Students who complete the task early could modify the task by adding a hallway or another feature to the basement plan, and then complete the task with the change.

#### **Technology Connection**

Students could use one of the online demonstration/modeling tools prior to or while completing the task. Students could use a drawing tool (for example, GeoGebra or Geometer's Sketchpad) to make the drawing of the basement.

#### **Choices for Students**

Following the introduction, offer students the opportunity to use dimensions from their own house to complete the task. Alternatively, they could add an extra room or hallway and then complete the task.

#### MCC7.EE.4a Task • Expressions and Equations Home Improvement Project

**Meaningful Context** 

This task uses a real-world example of a do-it-yourself home improvement project. Many students will be involved in a home improvement project at some point in their lives. This task shows how formal algebraic equations can be used to represent real-world scenarios and help solve the problems.

#### **Recommended Resources**

- GeoGebra <u>www.walch.com/rr/CCTTG7GeoGebra</u> This free geometry software tool can be used for a variety of geometry and algebra applications.
- Geology Rocks Equations

   www.walch.com/rr/CCTTG7GeologyEquations
   In this sample lesson, students use blocks and counters to represent and solve algebraic equations.
- Solving Equations with Balance-Strategy
  Demo: <u>www.walch.com/rr/CCTTG7BalanceDemo</u>
  Game: <u>www.walch.com/rr/CCTTG7BalanceGame</u>
  These applets let students use the balance strategy to solve equations. In the demo
  applet, the computation is completed automatically; in the game applet, students

complete the computation themselves.

Instruction

#### MCC7.EE.4a Task • Expressions and Equations Home Improvement Project

#### Part 1

You are going to help finish the basement in your house. You will build three separate rooms: a family room, a home office, and a laundry room. First you must develop a plan for the project. You don't have to worry about windows and doors now—just how to divide the space. You know the following:

- The area of the basement is 1,680 square feet.
- The width of the basement is 28 feet.
- Your mom wants the laundry room to be square.
- The other two rooms will have equal lengths.

Use expressions and equations to help plan the new spaces.

- 1. Write an algebraic expression to represent the length of the basement in terms of the lengths of the rooms.
- 2. Using the area of the basement, write an equation that could be used to solve for the equal lengths of the family room and the home office.
- 3. Use this equation to determine the lengths of the family room and the home office. Show all of your steps. Sketch a drawing of the plan for the basement. Be sure to label the rooms and their lengths and widths.

#### continued

4. There are two ways to write the equation—one way uses the distributive property and the other does not. Look at your own equation. If you used the distributive property, write an equivalent equation that does not use the distributive property. If you did not use the distributive property, write an equivalent equation that uses it.

5. Use your revised equation to determine the lengths of the family room and the home office. Show all of your steps. How does this answer compare to your answer in problem 3?



#### MCC7.EE.4a Task • Expressions and Equations Home Improvement Project

#### Part 2

Your dad has decided that he wants a workshop for his model building. Your mom agrees that she can use a smaller space for the laundry room. The laundry room will now be 18 feet long. The other three rooms (the workshop, family room, and home office) will have equal lengths.

Use expressions and equations to help plan the new spaces.

6. Write an algebraic expression that represents the length of the basement in terms of the lengths of the rooms.

7. Using the area of the basement, write an equation that can be used to solve for the lengths of the family room, the workshop, and the home office.

8. Use this equation to determine the lengths of the family room, the workshop, and the home office. Show all of your steps. Sketch a drawing of the revised plan for the basement. Be sure to label the rooms and their lengths and widths.

#### continued

#### NAME: MCC7.EE.4a Task • Expressions and Equations Home Improvement Project

9. Again, there are two ways to write the equation—one with the distributive property and one without. Look at your own equation. If you wrote it with the distributive property, write an equivalent equation without it. If you did not use the distributive property, write an equivalent equation that does use it.

10. Use your revised equation to determine the lengths of the workshop, the family room, and the home office. Show all of your steps. How does the answer compare to your answer in problem 8?

#### Instruction

#### **Common Core Georgia Performance Standard**

MCC7.EE.4b\*

#### Task Overview

#### Background

Solving inequalities is a skill that is often used in the real world but is one that students can find difficult to accomplish in a formal manner. Prior to this task, students should have shown mastery of solving equations. Student should also have had experience determining if a certain value is a solution to an inequality. In this task, students will perform operations with decimal numbers and will have to determine what kinds of answers (integer, decimal, or fraction) are reasonable.

This task gives students practice with solving inequalities in a real-world context of planning for purchases based on a specific budget. Students will write inequalities from a verbal representation of the problem statement. They will need to formulate inequalities employing  $\geq$  or  $\leq$  to accurately reflect the situations. Then they will determine how many items they can purchase or if it is possible to purchase a given amount of items.

The task also provides practice with:

- operations with decimal values
- writing algebraic expressions from verbal clues
- working with money

#### **Implementation Suggestions**

Students may work individually, in pairs, or in small groups to complete one or both parts of the task. It may be helpful to use one of the online resources for instruction or review prior to completing the task.

You may choose to assign only Part 1 of the task. Part 2 requires similar mathematics and can be considered optional—for extra practice, homework, or review.

#### Introduction

Introduce the task by asking students if they have ever had to plan purchases based on a certain spending limit. How did they decide what to buy? Did they ever need to reconsider what to buy based on the prices? How did they do that?

\*partially addressed

If needed, begin the task by having students describe how to tell the difference between an equation and an inequality. Answer: An equation uses an equal sign (=), and an inequality uses a less than (<) or greater than (>) sign.

As appropriate, make sure the students are familiar with how to perform operations with decimal numbers.

If desired, discuss the terms that indicate that a situation needs an inequality in order to solve it (for example, "more than," "less than," and "at most"). Keep the list on the board for the duration of the task.

As necessary, review reasonableness of answers. How do regular rounding rules apply to a realworld inequality problem? For example, if the problem is "at most" and the answer should be an integer, you should truncate the decimal number rather than round up.

#### Monitoring/Facilitating the Task

Ask questions and prompt student thinking so that they:

- Recognize words that indicate an inequality is required to solve the problem.
- Think about what kinds of answers (integer, decimal, or fraction) are appropriate for the problem.
- Check to see if an answer is correct (by plugging it in and checking the truth of the inequality).
- Monitor their accuracy with operations with decimal numbers.
- Keep track of some of their calculations so that they do not have to repeat them each time.
- Realize that there are infinite answers to an inequality.

#### **Debriefing the Task**

- Ask students to share their results and explain which verbal clues they used to determine that they needed an inequality to solve the problem.
- Encourage students to share how they decided when to round and how they rounded to correctly address the inequality statement.
- Some students may have used an estimation approach to determine if a certain number of items could be purchased. Ask them to share how they did this.
- Encourage students to think about other real-life situations that use inequalities. (Examples: If I have \$20, how many gallons of gas can I buy? What is the least number of 6-slice pizzas I need to buy if 10 people will each eat 2 slices of pizza?)
- Ask students to share any difficulties they had with this task. Encourage them to describe how they addressed these difficulties.

49

#### **Answer Key**

- 1.  $12.55x + 27.95 \le 300$
- 2.  $12.55x \le 272.05$

 $x \le 21.68$ 

You can buy at most 21 binders. The answer should be an integer. You cannot round up in this case; you must round down because you can only buy what you have enough money for.

- 3.  $4.98x + 27.95 + 5(12.55) \le 300$
- 4.  $4.98x + 90.70 \le 300$

 $4.98x \leq 209.30$ 

 $x \leq 42.03$ 

You can buy at most 42 book covers. The answer should be an integer.

- 5.  $4.98x + 183.70 + 27.95 + 5(12.55) \le 300$
- 6. Plug 7 into the inequality and determine if it is true.

 $4.98(7) + 183.70 + 27.95 + 5(12.55) \leq 300$ 

 $309.26 \le 300$  is not true, so you cannot purchase all of these items.

- 7.  $15.29x + 2(36.79) + 27.95 + 5(12.55) + 7(4.98) \le 300$
- 8.  $15.29x + 199.14 \le 300$

 $15.29x \leq 100.86$ 

 $x \leq 6.60$ 

You can buy at most 6 shirts. The answer needs to be an integer and needs to be rounded down.

- 9.  $12.52x + 3(27.89) + 27.95 + 5(12.55) + 7(4.98) \le 300$
- 10. Plug 5 into the inequality and check the truth of the inequality.

 $12.52(5) + 3(27.89) + 27.95 + 5(12.55) + 7(4.98) \le 300$ 

 $271.83 \le 300$  is true, so your friend is correct. You can buy 5 shirts.

11.  $12.52x + 209.23 \le 300$ 

 $12.52x \leq 90.77$ 

 $x \le 7.25$ 

You can buy at most 7 shirts.

#### Differentiation

Some students may benefit from the use of calculators during this task. Some students may find it helpful to use one of the online resources for instruction or review prior to the task.

Students who complete the task early could obtain actual prices of items using store Web sites, and then complete the task using the real prices.

#### **Technology Connection**

Students could use one of the online resources for instruction/review prior to completing the task.

#### **Choices for Students**

Following the introduction, offer students the opportunity to select alternative items to purchase using prices obtained from store Web sites.

#### **Meaningful Context**

This task uses a real-world example of planning purchases. Many students will have had an experience of shopping based on a budgeted amount. Being able to plan purchases is an important real-life skill that all students should have.

#### **Recommended Resources**

- Algebra Cruncher Game: Solving Inequalities <u>www.walch.com/rr/CCTTG7AlgebraCruncher</u> Students click to generate inequalities and then solve them.
- Linear Inequalities Activities
   <u>www.walch.com/rr/CCTTG7InequalityGames</u>
   This Web page contains links to inequality activities, including a quiz-show game in
   the style of *Jeopardy*!

Instruction

#### Part 1

You are going shopping for school supplies. Between what you have saved and what you have been given, you have \$300 to use to buy back-to-school items. You decide to plan ahead to figure out how to best spend your money. There are many items you would like to purchase, and you want to make sure that you can get everything you need as well as some of the things you want.

1. You know that you must buy a calculator. The calculator you need costs \$27.95 including tax. You also need some binders. Each binder costs \$12.55 including tax. If you buy only the calculator and binders, write an inequality that you could use to determine how many binders you could buy.

2. How many binders could you buy along with the calculator? What kind of number should your answer be—an integer, a fraction, or a decimal? Why? Do regular rounding rules apply here? Why or why not?

3. You decide you need 5 binders and you also need some book covers. Each book cover costs \$4.98 including tax. If you buy the calculator, 5 binders, and some book covers, write an inequality that you could use to determine how many book covers you could buy.

#### continued

4. How many book covers could you buy along with the calculator and binders? What kind of number should your answer be?

5. You have decided that you would like to buy a new MP3 player. The one you like the best costs \$183.70 including tax. Write an inequality that you could use to determine how many book covers you could buy along with the MP3 player, if you are still buying the calculator and the binders.

6. You receive a letter from school saying that you need 7 book covers. How could you determine if you have enough money to buy the calculator, the 5 binders, the MP3 player, and the 7 book covers? Describe fully.



#### Part 2

You have decided that you really don't need to buy the MP3 player. Instead, you would like to buy a few new clothing items. However, you still only have \$300 and you still need to buy the calculator, the 5 binders, and the 7 book covers.

7. You decide that you would like to buy 2 pairs of jeans, which cost \$36.79 each including tax. You also want to buy some shirts that cost \$15.29 each. Write an inequality that you could use to determine how many shirts you could buy.

8. How many shirts can you buy? Determine what kind of number the answer should be (integer, fraction, or decimal) and whether regular rounding rules apply.

9. You discover that the items you want to buy are on sale. Each pair of jeans costs \$27.89 and each shirt costs \$12.52. You decide to buy 3 pairs of jeans. Write an inequality that you could use to determine how many shirts you could buy.

#### continued

10. Your friend thinks you can buy 5 shirts. Is she correct? How do you know? Describe fully.

11. Can you determine how many shirts you could buy at most? Show your work to justify your answer.

#### Instruction

#### **Common Core Georgia Performance Standard**

MCC7.G.1

#### Task Overview

#### Background

Scale drawings are often used to depict items that are too large to fit on paper or too small to see key details. Understanding the use of scales, as well as how to convert measurements, is an important skill that is frequently needed in real-life applications. Scale drawings are often used in mapmaking, architecture, interior design, and marketing. This task allows students to work with scale drawings and make their own.

The task also provides practice with:

- finding proportions
- performing area calculations
- using scale factors

#### **Implementation Suggestions**

Students may work individually or in pairs to complete one or both parts of the task. Alternatively, students may meet in groups to share their results and reflect after individually completing the task, before a class-wide discussion.

It may be beneficial to review the Green Flight Challenge site with students prior to the start of the task so they are familiar with the scenario. (See the URL listed in the Recommended Resources for more information.) Students can explore the planes that entered the competition as well as the criteria for competing.

Students will need rulers and paper to complete this task. It may be helpful to provide graph paper to ensure accuracy of drawings.

#### Introduction

Introduce the task by asking students if they have ever seen a scale drawing. Encourage students to describe the situations in which they saw the drawings, and then ask students if they have ever used scale drawings. Students may be familiar with car, train, or airplane models, as well as maps and designs. Ask students to describe the way in which the scale is often represented. A model car may have a scale of 1:16, whereas a map may include text such as "1 inch = 100 miles." Encourage students to see that both representations are ratios and may or may not include units.

#### Monitoring/Facilitating the Task

Ask questions and prompt student thinking so that they:

- Recognize the mathematical operations being used. Make sure that students articulate where and when they are using addition, subtraction, multiplication, and division during each calculation.
- Articulate how they used their calculations to answer the questions. If students are having difficulty describing their arithmetic or explaining their responses on paper, ask them to explain aloud.
- Use proper mathematical terms.
- Write the area formulas for a rectangle and triangle prior to their calculations and consider how these formulas could help them find the area of the airport.
- Measure the dimensions on the map in order to calculate the area. Students should realize that there are several ways to find the actual lengths and areas of the airport, but, regardless of these differences, the resulting answers should be the same.
- Use the correct scale when multiplying. While circulating, check that students are multiplying each measured length by 1,200 before calculating the area, or that students are finding the area using the measured lengths and then multiplying by the square of 1,200. Use this time to discuss with students area and square units. Explore this idea with smaller numbers and a sheet of graph paper to ensure understanding.
- Identify that a triangle is formed between the runways when calculating the area enclosed by Runway 14 and Runway 19.

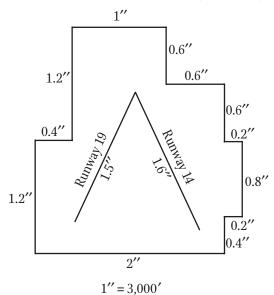
#### **Debriefing the Task**

- Though the questions in the task asked only for area calculations, students should have realized that, in order to find the answers, they needed to calculate the dimensions of the airport and the lengths of the runways. Choose volunteers to share their processes for finding the unknown lengths.
- Students should have recognized that, because the airport is not shaped like a rectangle, a single formula cannot be used to calculate the overall area. Students may have divided the airport into smaller, more manageable rectangles and then found the sum of the areas. Choose students who used different processes for calculating the total area to share their steps with the class.
- Assess understanding by giving students various scale ratios and asking students how each ratio would change the size of the drawing. Follow up by giving scaled lengths and having students calculate the actual length, and, alternately, by giving actual lengths and having students calculate the scaled lengths. Have volunteers share their answers.
- Encourage discussion about how scale drawings can be used outside of the classroom.

#### **Answer Key**

- 1. 1 inch = 1,200 feet
- 2. 4,200 feet; answers may vary. Students may have determined the answer by creating a proportion and solving for the unknown length, or they may have multiplied 3.5 by 1,200.
- 3. 3.75 inches
- 4. 4,500 feet
- 5. 37,080,000 square feet
- 6. Answers may vary. Students may have divided the airport up into smaller rectangles and then added the areas together, or they may have created a larger rectangle, determined its area, and then subtracted the areas of the smaller rectangles that are not part of the airport.
- 7. 8,820,000 square feet; since the area enclosed by the airport is a triangle, students should have used the formula  $area = \frac{1}{2}(base)(height)$ .

8. Review students' work to verify accuracy of drawings.



- 9. 8,820,000 square feet
- 10. The area of the airport should be the same for each drawing. Students could recalculate the area of the smaller scale drawing to verify the accuracy, or explain that each drawing is a representation of the actual airport and, therefore, each drawing should have the same area as the actual airport.
- 11. Answers may vary, but student scales should show that 1 inch is equal to a value less than 1,200 feet.

#### Differentiation

Depending on available resources, students may create scale drawings using paper, overhead transparencies, or computer software.

Some students may benefit from the use of calculators during this task.

Students struggling with determining lengths may want to take a strip of paper to use with the drawing and mark each inch. Above each mark, students could write the equivalent length based on the given scale.

Groups that finish early should be encouraged to compare their scale drawings with another group. Use questions from the debrief to encourage students' inter-group discussion.

#### **Technology Connection**

Students could use a free online software program such as Gliffy to create their scale drawings. (For more information, see the Recommended Resources.)

#### **Choices for Students**

Following the introduction, offer students the opportunity to choose a different airport to "host" the Green Flight Challenge. Students can create a scale drawing of the airport and then calculate the length of the runways and the area of the airport.

Some students may benefit from being challenged to create a scale drawing using a different scale from the one given in problem 8.

#### **Meaningful Context**

Students will likely encounter scale drawings in many contexts throughout their lives. Familiarity with scales and calculating dimensions based on the given scale is an essential skill in many pursuits, such as construction, interior design, product design, landscaping, engineering, cartography, and in certain crafts and hobbies.

Have students do research online or contact a local airport, university, or business of their choosing to get its dimensions and then create a scale drawing of the structure. If a large group is doing representations of the same place, see if the airport/university/business will display students' work on site for a period of time.

#### **Recommended Resources**

- Gliffy
  - www.walch.com/rr/CCTTG7Gliffy

With this free online resource, users can create a floor plan of a room, house, or office, or make a scale drawing of another space using a blank document. Registration is not necessary.

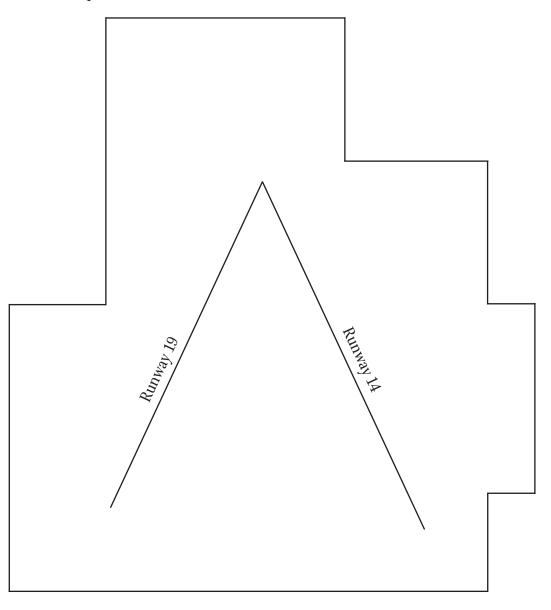
- NASA—Green Flight Challenge <u>www.walch.com/rr/CCTTG7GreenFlight</u> Learn about NASA's Green Flight Challenge sponsored by Google.
- Scale Drawings

www.walch.com/rr/CCTTG7ScalesAsRatios

This step-by-step guide on how to write scales as ratios also explains how to calculate actual lengths and scaled lengths.

#### Part 1

In 2011, NASA held the Green Flight Challenge at Charles M. Schulz Sonoma County Airport in Santa Rosa, California. This flight competition focused on efficiency, noise reduction, and safety. To host the competition, the airport needed a certain number of emergency responders on hand. They based this number on the area of the airport, as well as on the length of the runways used. The scale drawing below is a representation of the airport. Use the ruler provided and the diagram of the airport to answer the questions that follow.







61

1. What is the scale of the airport?

2. How many feet does 3.5 inches represent? How did you find your answer?

3. How many inches long is Runway 19 in the diagram?

4. What is the actual length of Runway 19?



5. What is the actual area of the airport?

6. What process did you use to calculate the actual area of the airport?

7. What is the actual area of the space enclosed by Runway 19 and Runway 14?



63

#### Part 2

You have been asked to create a smaller scale drawing of the airport for use in pamphlets and brochures. Use the information in the problems that follow to answer the questions.

- 8. On a separate sheet of paper, create a scale drawing of the airport using the following scale: 1 inch = 3,000 feet. Include both Runaway 19 and Runway 14 in your drawing as well as the scale. Make sure to label your drawing.
- 9. What is the actual area of the space enclosed by Runway 19 and Runway 14?

10. Is the actual area of the airport the same for each scale drawing? How do you know?

11. Determine an appropriate scale to make a drawing of the airport that is **larger** than the original scale drawing you were given in Part 1.

# A Slimmer Jewel Case

#### Instruction

#### **Common Core Georgia Performance Standard**

MCC7.G.6

#### Task Overview

#### Background

Students often struggle when working with volume and surface area, even when they can easily calculate the area of a two-dimensional figure. This task helps students internalize area, surface area, and volume calculations as they relate to the packaging and shipping of CD cases.

The task also provides practice with:

- performing operations with fractions
- simplifying fractions

#### **Implementation Suggestions**

Students may work individually or in pairs to complete one or both parts of the task. Alternatively, students may meet in groups to share their results and reflect after individually completing the task, before a class-wide discussion.

Students may want to sketch and label the jewel cases on a separate sheet of paper before making any calculations. Students may also want to sketch and label the shipping box on a separate sheet of paper prior to making calculations.

#### Introduction

Introduce the task by asking students if they have ever purchased a CD or received one as a gift. Ask them to identify the different components to a CD, such as the actual CD, the cover art, the jewel case, the shrink wrap, etc. Discuss how the CDs were packaged. Is the packaging always the same or is there a variety of packaging? Talk about why they think CDs would come in different packaging and if there is a benefit to one type of package over another.

#### Monitoring/Facilitating the Task

Ask questions and prompt student thinking so that they:

- Recognize the correct dimensions for calculating the area of the front cover of the CD.
- Recognize the mathematical operations they are using. Make sure that students articulate where and when they are using addition, subtraction, multiplication, and division during each calculation.
- Recognize that the dimensions given in the problem statement are mixed numbers. Make sure that students correctly change the mixed numbers to improper fractions.
- Correctly identify the formula needed for each calculation.
- Understand that when they are determining the amount of shrink-wrap needed, they are calculating surface area of the jewel case.
- Realize that the amount of plastic shrink-wrap needed to package the CD will likely be more than the calculated minimum amount. Make sure students understand that they are calculating the smallest amount possible to cover the CD, which can be used as a guideline for the CD packaging company.
- Understand that when they are determining the amount of space each jewel case takes up, they are calculating volume.
- Realize that, when determining how the CD cases will fit in the box, they cannot "break up" the cases to make them fit—each CD case takes up the same volume.
- Defend their responses. Make sure that students articulate how they used their calculations to answer the questions.

#### **Debriefing the Task**

- When calculating area, surface area, and volume, students should have recognized which dimensions were to be used in each calculation.
- Ask students to explain their process for determining the area of the front cover. Encourage students to describe how they determined which dimensions to use as well as which formula to use.
- Choose several students to volunteer and demonstrate their process for performing the multiplication and simplification of the fractions. Encourage students to use correct terms during their explanations.
- Have students share any difficulties they had completing each problem. Ask them to offer advice for overcoming those difficulties.

• To assess student understanding, give students the dimensions of another right prism and a shipping box with questions similar to those in the task. Observe the process each student follows and check for accuracy.

#### **Answer Key**

- 1. rectangles
- 2.  $26^{13}/_{16}$  square inches
- 3.  $61^{13}/32$  square inches
- 4.  $10^{7/128}$  cubic inches
- 5. Answers may vary depending on how students fit the CD cases into the box. Students could put up to 140 CDs into the box, but they should explain what configuration they used to fit them all in. Possible answer: To ensure the CDs fit comfortably, you can create 3 rows of 28 CDs for a total of 112 CDs.
- 6. The company saves money on packaging and shipping costs and the consumer has more space for their CD collection. Costs saved by the company often result in lower costs for the consumer. Also, a smaller case implies less waste.

#### Differentiation

Some students may benefit from the use of calculators during this task. Some students may benefit from the use of graph paper or isometric dot paper when drawing the jewel case and shipping box.

It may be helpful for students to look at actual CD cases prior to performing calculations or when creating their drawings.

Students who complete their work early may attempt to design a new, more efficient package for CDs, demonstrating that it has a smaller surface area and volume.

#### **Technology Connection**

Students could use the Interactivate online resource to practice computations of surface area and volume of right prisms. See the URL listed in the Recommended Resources.

#### **Choices for Students**

Following the introduction, offer students the opportunity to research the size of shipping boxes to determine how many jewel cases could be packaged in each one. Students could research the dimensions of other types of CD packaging or DVD packaging and then complete the task. Students could also create their own packaging design and complete the task.

#### **Meaningful Context**

This task may be expanded by having students create their own CD case design. Have students create a labeled drawing of the CD case. Have students research and select a box for shipping at least 100 CDs. Students should then calculate the amount of space their CD case design allows for cover art, the minimum amount of shrink-wrap needed, and the amount of space each CD takes up.

#### **Recommended Resources**

- Illuminations: Isometric Drawing Tool <u>www.walch.com/rr/CCTTG7IsometricDrawing</u> Students can use this site to explore three-dimensional objects by first creating an object on virtual isometric dot paper. Students can then manipulate the object to better understand area, surface area, and volume.
- Interactivate: Surface Area and Volume <u>www.walch.com/rr/CCTTG7InteractiveDimensions</u> Users can manipulate a rectangular or triangular prism and adjust each dimension with this interactive tool. Both volume and surface area of the solid are calculated and can be compared as the dimensions are changed.
- Surface Area Formulas <u>www.walch.com/rr/CCTTG7SurfaceAreas</u> This site provides an overview of surface area and how to calculate it.
- Volume Formulas
   <u>www.walch.com/rr/CCTTG7VolumeFormulas</u>
   Review volume formulas for various figures.

## MCC7.G.6 Task • Geometry A Slimmer Jewel Case

#### Part 1

A great deal of planning and preparation goes into the design, production, and shipping of many products, such as CDs. After each CD is burned, it is often placed in a jewel case. A standard jewel case used to ship, store, and protect CDs measures  $4\frac{7}{8}$  "long  $\times 5\frac{1}{2}$ " wide  $\times \frac{3}{8}$ " deep. How does the size of the case impact the art, packaging, and shipping of the CDs?

1. Think about a jewel case. What shapes is it composed of?

2. Jewel cases typically contain art on the front cover. What is the area of the cover art?

3. Once CDs are placed in the jewel cases, they are then shrink-wrapped in plastic. Determine the minimum amount of shrink-wrap needed for each jewel case. Assume there is no overlap in shrink-wrap.

#### continued

## MCC7.G.6 Task • Geometry A Slimmer Jewel Case

#### Part 2

After jewel cases are shrink-wrapped, they are placed into a box for shipping.

4. How much space does each jewel case take up?

5. How many shrink-wrapped jewel cases will fit in a box that measures  $10\frac{3}{4}$  "×  $6\frac{1}{2}$ "×  $20\frac{1}{4}$ "? Explain your answer and describe how you would fit the CDs into the box.

6. From their creation in 1982, the style of jewel cases remained virtually unchanged until the production of the slim jewel case, which is half the thickness of a regular jewel case. What impact does the use of slim jewel cases have on the company and the consumer?

# **Blood Groups**

Instruction

#### **Common Core Georgia Performance Standard**

MCC7.SP.6

#### **Task Overview**

#### Background

Students often struggle when working with probability models. This task helps students internalize the meaning of probability models and predicting outcomes based on theoretical versus experimental probability.

The task also provides practice with:

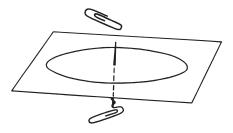
- calculating relative frequencies
- comparing theoretical and experimental probabilities
- predicting outcomes

#### **Implementation Suggestions**

For Part 1, students should work in small groups with one spinner per group. Assign a certain number of spins to each group so that the total number of spins in the class is 200. For Part 2, students come together as a whole class to compile their data.

Decide if you want the groups to assemble their spinners, or if you will make them ahead of time. Supplies needed for each spinner: two paper clips, a printout of the spinner template, and a sheet of stiff paper or thin cardboard for the back (such as from a cereal box).

Use the following illustration as a guide to create your spinner. The spinner template is provided following the Recommended Resources.



#### Introduction

Introduce the task by asking students if they know their blood type. Ask students if anyone in their family has ever donated blood. Question students about their knowledge of different blood types and the implications of blood type on receiving a transfusion. Point out the distinction between the four blood *groups*—A, B, AB, and O—and the eight major blood *types*—A+, A–, B+, B–, AB+, AB–, O+, and O–. For the purposes of this task, we will focus on the probabilities associated with the four blood groups.

There are four general blood groups: A, B, AB, and O. Doctors need to know what type of blood you have before giving you a blood transfusion because not all blood types mix safely. Type O blood is said to be the universal donor because anyone can receive type O. Type AB is said to be the universal recipient because someone with type AB blood can receive any other type of blood.

#### Monitoring/Facilitating the Task

Ask questions and prompt student thinking so that they:

- Understand what spinning the spinner represents and what each section of the spinner represents.
- Recognize that each group is creating data that will be combined to create a large sample of data.
- Recognize the mathematical operations they are using in calculating relative frequencies. Make sure that students articulate where and when they are using addition, multiplication, and division during each calculation.
- Defend their responses. Make sure that students articulate how they used their calculations to answer a question.
- Recognize the arithmetic they are doing, particularly in reference to converting percents to decimals and vice versa. If students are having difficulty writing descriptions of their arithmetic or explaining their responses, ask them to discuss how they determined their answers with you. Prompt for and encourage the use of proper mathematic terms.
- Recognize the difference between experimental and theoretical probability models.
- Realize that collecting more data means the experimental results will be closer to the theoretical results.

#### **Debriefing the Task**

- Students will be calculating relative frequencies. They must know the procedure for calculating relative frequencies, as well as the relationship between percentages in a population and relative frequencies. Ask students to explain how they calculated their relative frequencies.
- Ask students about their observations of the class data as compared to the theoretical data. Students should notice that the experimental data typically does not result in the exact theoretical data. Encourage students to think about why this is and to explain their thinking.
- Ask students about the benefits of using a simulation. Students should report that simulations are easier and usually faster to conduct.
- Encourage discussion about student experiences with games of chance, such as flipping a coin to make a decision, and if they ever chose a side of a coin based on the results of previous flips. Ask them to explain their thinking and to evaluate their method of choosing a side. Students might think that evaluating the previous flips will give some insight into the results of a subsequent flip, when in reality there is always a 50% chance of flipping heads.
- To extend the task, ask students to look at their small-group data and compare their relative frequencies to the class data and then to the theoretical data. Small-group data could be vastly different from the theoretical data. Encourage students to discuss why this is and to examine what happened to the data as compared to the theoretical data as they began combining the class and small-group data.
- Assess understanding by expanding the parameters used in the task. Encourage students to develop a new probability model based on a different geographical region.

#### **Answer Key**

- 1. Answers will vary. Check to see that students spun the spinner the appropriate number of times and that the frequency column agrees with the tally column.
- 2. You need the frequency of each blood type and the total number of spins to calculate the relative frequency.
- 3. Answers will vary depending on experimental results. Make sure students report relative frequencies as whole percentages and that the total of the relative frequency column adds to 100%.
- 4. Answers will vary according to experimental results.
- 5. Since 44% of the population reportedly has type O blood, find 44% of 800: 0.44 800 = 352; 352 people are predicted to have type O blood.

- 6. Answers may vary. It's unlikely that the experimental distribution was exactly the same as the theoretical distribution, but it is possible that it is similar. Students should recognize that chance events are more predictable with long-run frequencies, and that the more data that is collected, the closer the results are to the theoretical distribution.
- 7. 10 people; 10 people would theoretically result in 4.4 people having type O blood.

There are multiple ways to think about this problem. One way is to set up a proportion:  $\frac{44}{100} = \frac{4}{x}; 44x = 400; x \approx 9.09.$  Since you can't have 0.09 of a person, round up to 10.

8. Theoretical: 0.44 + 0.10 = 0.54 or 54%

Experimental: Answers will vary. Student explanations should echo the rationale used in the answer to problem 6.

- 9. 0.31 + 0.19 = 0.50 or 50%
- 10. Yes, it is possible but unlikely. Just as with the spinner, it is possible for the spinner to land on the AB section on the first spin, but there is only a 4% chance of that.

## Differentiation

Some students may benefit from the use of calculators during this task. Students who finish early might want to explore the blood types of differing regions and create a new spinner with the appropriate percentages. Then, these students could generate more experimental data and run through the task a second time, calculating relative frequencies with the new data. Additional data can be found in a table at <a href="https://www.walch.com/rr/CCTTG7WikiBlood">www.walch.com/rr/CCTTG7WikiBlood</a>. You may also direct advanced students to research the percentages for all eight primary blood types (Rh positive and negative), create a spinner that reflects these percentages, and redo the task using the new spinner.

#### **Technology Connection**

As an alternative to creating a physical spinner, students could use one of the virtual manipulative spinners provided in the Recommended Resources list. Additionally, students could use a spreadsheet to record their results and make calculations of relative frequencies.

#### **Choices for Students**

The task focuses on blood group O. Students could choose a different blood group.

#### Meaningful Context

Agencies use the percentages of each blood group in a population to generate probability models and estimate how many donors they will need to ensure they have enough blood of a specific type. Students may contact their local Red Cross or blood donation clinic to obtain blood group population numbers for their community.

#### **Recommended Resources**

- American Red Cross: Blood Types
   www.walch.com/rr/CCTTG7BloodTypes
   Along with an excellent explanation of the blood groups and types, this site from
   the American Red Cross includes an interactive chart depicting the ease of donation
   between different types, as well as a breakdown of the percentage of each blood type in
   the U.S. population.
- National Library of Virtual Manipulatives—Spinners
   <u>www.walch.com/rr/CCTTG7CustomSpinners</u>
   This virtual manipulative allows students to customize a spinner. The application is
   also able to record the results of multiple spins and displays the results as a histogram.
   Requires Java.
- Shodor Interactivate: Adjustable Spinner
   <u>www.walch.com/rr/CCTTG7AdjustableSpinner</u>

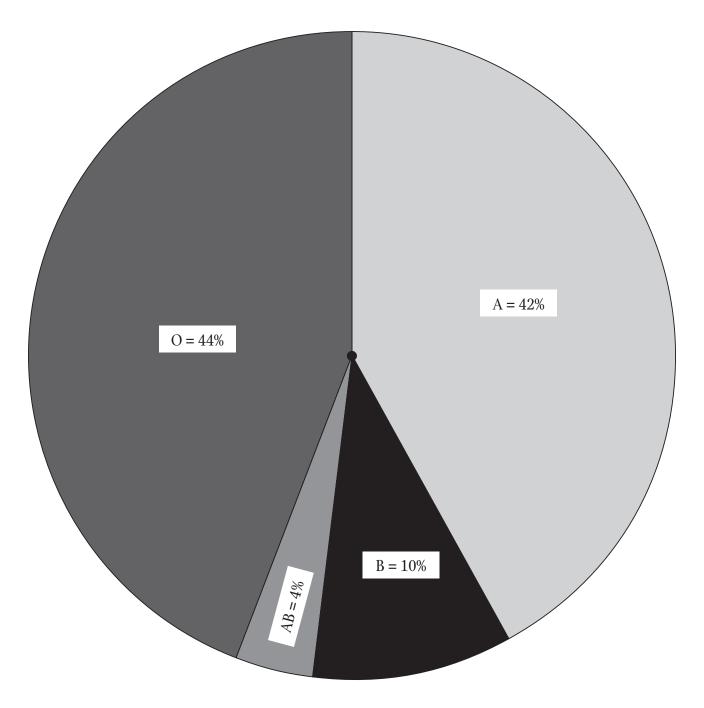
This virtual spinner can be customized. The application allows you to set a number of spins and it then keeps track of the count and the experimental probabilities. The table also has a column for the theoretical probabilities for easy comparison. When the simulation is complete, students can use the program to display a pie chart of the experimental results.

 Wikipedia article: "Blood type" <u>www.walch.com/rr/CCTTG7WikiBlood</u> This article offers further explanation of blood types, and includes a table listing blood type distribution by country.

Instruction

## MCC7.SP.6 Task • Statistics and Probability Blood Groups





#### Introduction

Imagine if a train collided with a city bus and 20 people were injured. In a situation like this, Red Cross agencies would ask for blood donations to help victims in need. How many donors would be needed to guarantee that enough blood of a specific type was received to help the injured?

The table below shows the four blood groups—A, B, AB, and O—and the percentage of Americans with each blood type. Your group will use a spinner that reflects these percentages to answer questions. Your teacher will provide the spinner or guide you in putting it together yourself.

Blood group/type	А	В	AB	0
Percent of U.S. population:	42	10	4	44

#### Part 1

1. In your group, spin the spinner the number of times assigned to you by your teacher. (Each group might have a different number of spins, but the class total will be 200 spins.) Each time you finish a spin, make a tally mark in the appropriate column of the following table. When you are done spinning, tally the marks and write the total of each blood type in the "Group frequency" column. You will use this data in Part 2 of the task.

Туре	Tally	Group frequency
А		
В		
AB		
0		

2. In the next part of the task you will be calculating relative frequencies. What information do you need to know in order to calculate relative frequencies?



## MCC7.SP.6 Task • Statistics and Probability Blood Groups

#### Part 2

The Red Cross needs to collect blood from enough people so that they can have a minimum number of universal blood donors. People with type O blood are considered to be "universal donors" because anybody with any blood type can receive type O blood. Determine how many people need to donate blood to ensure that enough donors are type O.

3. Combine your class data from the "Group frequency" column in the table from Part 1 and record the results in the table below. When you have all the data, calculate the relative frequencies of each blood type and enter that percentage in the "Relative frequency" column. Round your relative frequencies to the nearest whole percentage.

Туре	Frequency	Relative frequency
А		
В		
AB		
0		
		Total:

4. Based on your **experimental** results (collected data), if 800 people donate blood, how many people would you predict have type O blood? Explain your reasoning.

5. Based on the **theoretical** probabilities (the table provided in Part 1), if 800 people donate blood, how many people would you predict have type O blood? Explain your reasoning.

#### continued

## MCC7.SP.6 Task • Statistics and Probability Blood Groups

6. Are your predictions in numbers 5 and 6 identical? Explain why or why not.

7. A local hospital needs four people to donate type O blood for transfusions. How many people would you predict need to donate blood to ensure that four of the donors have type O blood? Explain how you determined your answer.

8. An accident victim has blood type B. This person can receive type O or type B blood. What is the probability that one person walking into the Red Cross has either of these types based on the **theoretical** probability?

Based on the **experimental** probability?

If your answers are different, explain why this might be.



9. The relative frequencies of blood types are different in other countries. For example, in Hungary, 31% of the people have type O blood and 19% have type B blood. If a person in Hungary were to donate blood, what is the **theoretical** probability that this person would have either of these types of blood? Show your work.

10. Type AB is the most rare blood type in the United States. Is it possible that, on any given day, the first person to come in to the Red Cross at random has type AB blood? Explain your reasoning.

Instruction

#### **Common Core Georgia Performance Standard**

MCC7.SP.8a

#### Task Overview

#### Background

Students often struggle when working with probabilities, especially those involving compound events. This task helps students internalize this concept through the context of having multiple sets of identical twins. Students will use tree diagrams and the Fundamental Counting Principle of Multiplication to make conclusions.

The task also provides practice with:

- converting fractions to decimals
- multiplying fractions

#### **Implementation Suggestions**

Students may work individually to complete the task and then meet in small groups to share their results, before a class-wide discussion. As an implementation aid, make blank tree diagrams available for students who request or need them.

#### Introduction

Introduce the task by asking students if there are twins in their families or if any of their friends are twins. Continue the discussion to include the difference between fraternal and identical twins. Students may not be aware of the differences between the two types. Fraternal twins are twins originating from two different fertilized eggs, whereas identical twins are twins originating from one fertilized egg that divides and develops into two babies. Ask students if they are aware of the differences in the number of occurrences of fraternal twins versus identical twins, and if they can account for the differences. Such differences in the number of fraternal twins born include race, age of mother, family history, and geographic location. The incidences of identical twins remain relatively the same across race, age, history, and location. Students have the option to further explore this information.

#### Monitoring/Facilitating the Task

Ask questions and prompt student thinking so that they:

- Recognize the correct probability of having identical twins. Students will need to translate the verbal expression of 1 out of 250 into the fraction 1/250. Prompt students to articulate the meaning of the words in the verbal expression if they are having difficulty.
- Realize that the task deals strictly with the probability of either having identical twins or not having identical twins. Singles, fraternal twins, triplets, etc., are included in the "Not identical twins" category.
- Understand that the calculation of the probability of having two sets of identical twins is a compound event where the outcome of each event is independent. Therefore, this situation is very much like a combination problem that students may be more familiar with (flavors at an ice cream shop or number of outfits from a given number of clothing articles).

#### **Debriefing the Task**

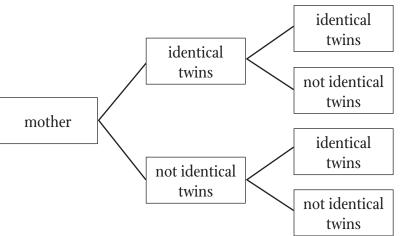
- Students will be calculating probabilities. They must recognize both the probability that an event will happen and the probability that an event will not happen.
- To ensure that students recognize what the probability of having identical twins is, have students convert this number to a decimal. Ask students to describe what the decimal means in terms of each pregnancy.
- In question 6, students are asked to calculate the probability of having two sets of identical twins. Encourage students to share their process for calculating this answer. Ask students why we must multiply the two probabilities rather than add them. Be sure students are able to explain that if the probability of having identical twins is 1/250, then that is similar to thinking of 250 pregnant women with 250 different outcomes. For instance, the first woman could have identical twins and the rest would not; or, the second woman would have identical twins and the first woman as well as the 248 women after the second woman would not. The same is true for the second pregnancy. Therefore, the events are independent from one another and are a permutation of results.
- In question 8, students are to calculate the probability of having a third set of identical twins. Ask students to share how they were able to find this probability. Students should recognize this is just a continuation of question 6 and that they need to multiply by 1/250 again.
- Be sure students defend their responses to question 9 and include a mathematical justification in their response rather than fall back on personal events and thoughts on having identical twins. It is very unlikely that a couple with one set of identical twins would have a second set of identical twins, although it is possible.

Instruction

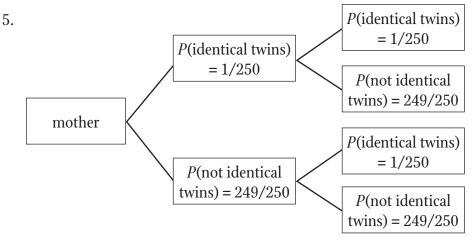
#### **Answer Key**

- 1.  $\frac{1}{250} = 0.004$
- 2.  $^{249}/_{250} = 0.996$





4. (identical twins, no twins), (identical twins, identical twins), (no twins, identical twins), (no twins, no twins)



- 6.  $\frac{1}{62,500} = 0.000016$
- 7. Multiplication was used to find the probability of having two sets of identical twins. This is because if there are m ways for the first event to occur and n ways for the second event to occur, then there are  $m \times n$  number of ways that both can occur.
- 8.  $(\frac{1}{250})(\frac{1}{250}) = \frac{1}{15,625,000} = 0.00000064$
- 9. Answers may vary, but should include that the probability of having a second set of identical twins is  $\frac{1}{62,500}$ . This is a very small percentage, so it is highly unlikely that the couple will have another set of identical twins.

#### Differentiation

Students struggling to create a tree diagram would benefit from having a blank diagram to fill in. Ask students who finish early to simulate the event by using statistical software or a graphing calculator.

#### **Technology Connection**

Students could use statistical software or a graphing calculator to simulate the event.

#### **Choices for Students**

Following the introduction, offer students the opportunity to research the probability of having identical triplets. Students could complete the task for identical triplets rather than identical twins.

Students could also research the probability of fraternal twins and the factors that affect that outcome. Students will see that there are many factors that influence that outcome (such as age, race, family history of fraternal twins, and location). Students may also wish to research why there is so much variance in the outcome of fraternal twins versus identical twins.

#### **Meaningful Context**

With the world's population now past 7 billion people, concerns about overpopulation have grown. Just how many children should a family have in order to be socially and environmentally responsible? As the debate continues, couples still experience many emotions surrounding the possibility of having twins, but few people actually understand the probability of having identical twins, or even fraternal twins.

## **Recommended Resources**

- Basic Principles of Counting <u>www.walch.com/rr/CCTTG7CountingPrinciples</u> This site provides an overview of the addition and multiplication rules of probability.
- Probability Lessons: Probability of Compound Events <u>www.walch.com/rr/CCTTG7CompoundProbability</u> This site includes a review of compound probability with examples.
- Tree Applet <u>www.walch.com/rr/CCTTG7TreeApplet</u> This applet creates a tree diagram and computes the probability of two events.
- Working with Tree Diagrams <u>www.walch.com/rr/CCTTG7TreeDiagramReview</u> Access these multiple-choice questions to review tree diagrams and probability.

#### Introduction

According to the U.S. Census Bureau, the world population estimate for July 2012 was roughly 7 billion people. This large number has led to many discussions about how many people is too many people, as well as how many people the Earth will be able to sustain. Countries, states, and families have questioned the number of children each family should have.

Annie and her husband, Paul, are aware of the problem of overpopulation, and they are concerned about the impact the size of their family would have on the world population. If Annie and Paul carry two pregnancies to term, what is the probability that they will have two sets of identical twins? One out of 250 pregnancies is expected to result in identical twins.

#### **Task Questions**

- 1. What is the probability of Annie and Paul having a set of identical twins?
- 2. What is the probability of not having identical twins?
- 3. Draw a tree diagram to represent the possible outcomes of having 2 sets of identical twins.

#### continued

4. What is the sample space of this situation?

- 5. Label the probability of each event in your tree diagram for problem 3.
- 6. What is the probability of Annie and Paul having 2 sets of identical twins?

7. What operation did you use to answer question 6? Explain why.

8. If a couple has 2 sets of identical twins, what is the probability that they will have a third set of identical twins with their next pregnancy?

#### continued

9. Your aunt, Mari, has already given birth to a set of identical twins. She is hoping for another set of identical twins with her next pregnancy. Explain what the probability of having a second set of identical twins is, and why Mari should or should not prepare for a second set of identical twins.