Shelby County Schools

Extended Learning Packet

Math + − × ÷

Grade 6

## Analyze Numerical Data

Name: _____

Prerequisite: Use a Dot Plot

Study the example showing how to display data in a dot plot. Then solve problems 1–8.

> ### Example
>
> Fishermen recorded the lengths, in inches, of fifteen fish that they caught on a fishing trip: 22, 14, 15, 3, 9, 20, 20, 11, 9, 10, 18, 19, 10, 12, 15. Make a dot plot of the data.
>
> Place a dot over each fish length on a number line.
>
> **Length of Fish**
>
>
>
> **Inches**
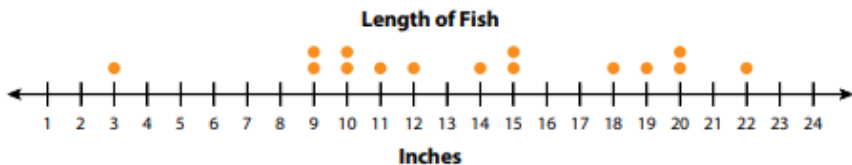
1 Describe the shape of the data in the example above. What does it tell you about the length of the fish?

_____

_____

2 What is the range of fish lengths, and what does this tell you about the fish lengths?

_____

_____

3 What is the median fish length? What is the mean fish length?

_____

4 How are the mean and the median similar? How are they different? Explain.

_____

_____

Solve.

5 Each morning, Mrs. Maki's class gets together before school to run laps around the track. Here are the numbers of miles each student ran by the end of the first month:

12, 14, 15, 17, 14, 18, 11, 8, 15, 20, 16, 35, 11, 13, 17, 17, 19

Make a dot plot of the data.

Mrs. Maki's Running Club

Miles

6 What is the median number of miles students ran? What is the mean?

_____

7 Which is a more accurate representation of this data: the mean or the median? Explain.

_____

_____

8 Identify the range of the data and explain how the outlier makes the range misleading.

_____

_____

_____

Name: _____

Explore Range with Box Plots

Study the example showing how the range measures variability in a box plot. Then solve problems 1–7.
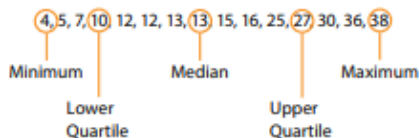
Example

A group of 13 birdwatchers in New York City has been recording the number of red-tailed hawks they have sighted in the past year. These are the results:
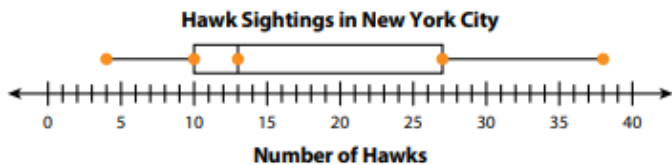
5, 10, 12, 13, 36, 30, 38, 27, 25, 13, 15, 4, 12, 16, 7

What does the range tell you about the variability of the sightings?

Arrange the data in order from least to greatest, and then find the median and the lower and upper quartile values.

④, 5, 7, ⑩, 12, 12, 13, ⑬, 15, 16, 25, ㉗, 30, 36, ㊳

Minimum   Median   Maximum

Lower   Upper
Quartile   Quartile

Draw a box plot to understand the problem.

Hawk Sightings in New York City

0   5   10   15   20   25   30   35   40

Number of Hawks

The range is 34. This means the numbers of red-tailed hawks seen by the birdwatchers are within 34 of each other, between 4 and 38.

Vocabulary

box plot a five-number summary that includes the minimum, the lower quartile, the median, the upper quartile, and the maximum.

lower quartile the middle number between the minimum and the median in an ordered set of numbers.

upper quartile the middle number between the median and the maximum in an ordered set of numbers.

1 Does data in the example above include outliers?

_____

2 Look at the box plot. What does the box represent? What does it mean in this context?

_____

_____

_____

Solve.

3 The number of pounds that each kitten or cat at an animal rescue center weighs is listed below. Display the data in a box plot.

1, 1, 2, 3, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 6, 8, 8, 10, 12

Weight (pounds)
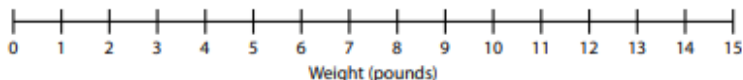
4 What are the lower quartile, the upper quartile, and the range for the data in problem 3?

_____

5 Explain what the box that you drew in the box plot in problem 3 represents in terms of the weights.

_____

_____

6 Tomas listed the amounts that he deposited in his savings account each week for 15 weeks. Explain how the range is calculated and is affected by the outlier in this data set.
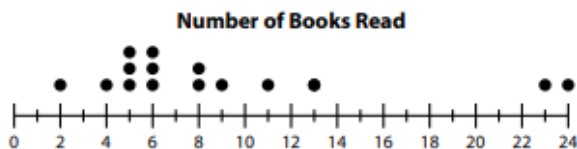
$25, $32, $32, $35, $35, $35, $35, $38, $40, $40, $43, $43, $44, $45, $85

_____

_____

_____

7 In problem 6, identify and describe which measure of center is the most accurate for the data set.

_____

_____

_____

Name: _____

Analyze Numerical Data

Solve the problems.

1 The dot plot shows the number of books that some sixth graders read last month. Find the mean and the median.

What does each dot on the plot represent?

Number of Books Read

Show your work.

Solution: _____

2 In problem 1, is the mean or the median a better measure of center? Explain.

How do outliers affect the measures of center?

_____

_____

3 The data represents how many text messages students send on a given day.

What is an outlier?

13, 14, 14, 15, 15, 15, 17, 17, 19, 20, 20, 20, 20, 21, 21, 21, 22, 22, 24, 25, 46

Explain how the range would change if the outlier was replaced with 22, and explain why the results would be more accurate without an outlier.

_____

_____

_____

_____

Solve.

4 The box plot shows the amount of money raised by some students for a school fund-raiser. Tell whether each statement is *True* or *False*.

What are the five numbers represented in a box plot?
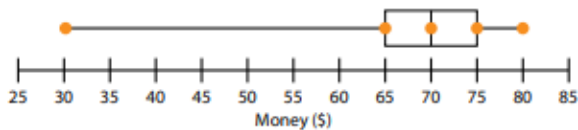
Money ($)

a. The variability of the middle 50% of the money raised is less than the variability of the lower 25%. ☐ True ☐ False

b. The box plot shows that 80 students raised money. ☐ True ☐ False

c. The range of the data could be misleading. ☐ True ☐ False

5 What is the range of the data in problem 4? Can you identify the median if you only know the range? Explain.

In the box plot, what does the rectangular box represent?

_____

_____

_____

_____

6 Josh recorded the amount of time, in minutes, that he exercised during the first 10 days of the month.

What is the difference between the median and the range?

30, 45, 25, 25, 35, 5, 40, 35, 30, 20

Identify the median and the mean of the data set. How are these two values related?

_____

_____

_____

Name: _____

Data Remix

What you need: Recording Sheet, Data Cards

Directions

● Your goal is to collect a set of data that best fits the Data Summary for each round on your Recording Sheet. Your cards are your data set.

● To start a round, shuffle and deal 7 cards to each player. Place the rest in a stack and turn one card faceup to start a discard pile.

● On your turn, you may draw 1 card from the stack, take the top card from the discard pile, or pass. If you take a card, you must discard one so that you end with 7 cards.

● In a round, each player gets to pick from the stack or discard pile 3 times. The resulting 7 cards make up the data set.

● To complete a round, write your data set on your Recording Sheet. Calculate and record the values listed in the Data Summary. The player whose data best matches the Data Summary gets 1 point.

● Play 5 rounds. The player with the most points wins.

Name: __Carrie__

Data Remix Recording Sheet

Data Summary	Data	Mean	Median	Mode	Range
Round 1: The closest mean and median					
Round 2: The most different mean and median					

3 5 5 8 10
7 9

I have to think about what numbers will help me meet the goal for each round. Sometimes I want numbers that are close together. At other times I need numbers that are more spread out.

Data Summary	Data	Mean	Median	Mode	Range
Round 1: The closest mean and median					
Round 2: The most different mean and median					
Round 3: The least range					
Round 4: The greatest range					
Round 5: No mode					

Points: _____ + _____ + _____ + _____ + _____ = _____

　　　　　　Round 1　　　　Round 2　　　　Round 3　　　　Round 4　　　　Round 5　　　　Total

1	2	3	4
5	6	7	8
9	10	1	2
3	4	5	6
7	8	9	10

1	2	3	4
5	6	7	8
9	10	1	2
3	4	5	6
7	8	9	10

Statistics and Probability

In this unit you learned to:	Lesson
recognize what makes a question a statistical question.	26
calculate measures of center such as mean and median.	27
calculate measures of spread such as range.	27
display data accurately with a dot plot, stem plot, box plot, or pie chart.	28
describe data using measures of center and measures of spread.	29

Use these skills to solve problems 1–6.

1 You ask 20 students from the same class one of the survey questions below. Which question is a statistical question? Select all that apply.

   A What was the highest score on the last math test?

   B How long did you spend studying for the test?

   C How many problems were on the last math test?

   D What is your favorite subject?

2 Which of these can be determined from a stem plot? Select all that apply.

   A mean

   B median

   C mode

   D range

   E minimum value

3 Create a set of six data values such that the mode is 6, the median is 6.5, and the mean is 7.

   _____

4 For which data set is the median a better measure of center than the mean?

   A 13, 14, 16, 16, 17, 18, 20

   B 4, 12, 15, 16, 17, 19, 21

   C 11, 13, 14, 15, 18, 19, 20

   D 8, 10, 12, 14, 16, 18, 20

Solve.

5 Answer the questions about a box plot. Select *Yes* or *No*.

a. Is a box plot more useful than a dot plot to find the number of data values in a set? ☐ Yes ☐ No

b. Should each interval included in the box plot be the same size? ☐ Yes ☐ No

c. Is it always possible to find the mean using a box plot? ☐ Yes ☐ No

d. Is it always possible to find the median using a box plot? ☐ Yes ☐ No

e. Is it always possible to find the mode using a box plot? ☐ Yes ☐ No

f. Is it always possible to find the maximum and minimum values using a box plot? ☐ Yes ☐ No

6 The dot plot shows the number of students in different classrooms in East School.
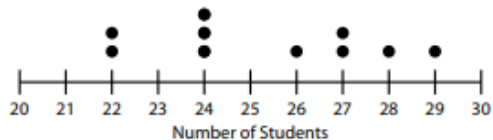
Select whether each statement is *True* or *False*.

a. There are 9 data values in the set. ☐ True ☐ False

b. The median is 25. ☐ True ☐ False

c. The mode is 24. ☐ True ☐ False

d. The mean is 25.3. ☐ True ☐ False

e. There is a low outlier. ☐ True ☐ False

f. The range is 7. ☐ True ☐ False

Graph Equivalent Ratios

Study the example problem showing how to graph equivalent ratios. Then solve problems 1–10.

Example

The graph compares how far Jorge walks to how many steps he takes. How many feet does he walk in 6 steps? How many steps does Jorge take to walk 30 feet?

Each point on the graph can be represented by an ordered pair. The point represented by (6, 18) shows that Jorge takes 6 steps to walk 18 feet.

The ordered pair for 30 feet is (10, 30), which means that Jorge walks 30 feet in 10 steps.
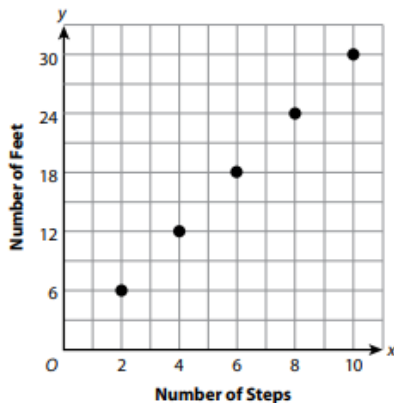
1 What ordered pair represents the number of steps Jorge takes to walk 24 feet?

_____

2 Choose another point on the graph. Write the ordered pair and tell what it represents.

_____

3 What ordered pair represents the number of feet Jorge walks in 3 steps?

_____

4 Joan looks at the graph and says the number of steps is always 3 times the number of feet. Is she correct? Explain your answer.

_____

_____

Solve.

Use the following situation for problems 5–8.

> To make a scarf, Jenny uses blue yarn and white yarn. The number of yards of blue yarn she uses is 4 times the number of yards of white yarn in each scarf.

5 Write four ratios to show the number of yards of white yarn to blue yarn for each scarf.

6 Are the ratios in problem 5 equivalent? Explain how you know.

_____

_____

7 Jenny wants to make a scarf that uses 24 yards of blue yarn. How many yards of white yarn will she need?

_____

8 If Jenny wants to keep the ratio of blue yarn to white yarn the same, can she make a scarf using 42 yards of blue yarn? If so, how much white yarn will she need? If not, why not?

_____

_____

_____

9 Adrianna can read 7 pages in 10 minutes. At this rate, how many pages can she read in 25 minutes?

_____

10 Max calculated that he could read at a rate of 2 pages per minute. Is he reading at a faster rate than Adrianna? Explain.

_____

_____

_____

_____

Equivalent Ratios

Solve the problems.

1 Kate, Mario, Sato, and Den each use a different recipe to make trail mix. Which recipe uses a different ratio of cups of raisins to cereal than the rest?

To find one ratio that's different, I need to find some that are equal to each other.

A Kate uses 3 cups of raisins for every 8 cups of cereal.

B Mario uses 4 cups of raisins for every 12 cups of cereal.

C Sato uses 6 cups of raisins for every 16 cups of cereal.

D Den uses 9 cups of raisins for every 24 cups of cereal.

2 The graph shows the number of teaspoons of lemon juice in cups of lemonade.
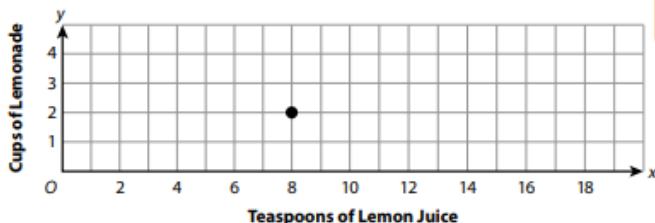
Which number is first in an ordered pair?

Which ordered pair represents a ratio equivalent to the ratio of teaspoons of lemon juice to cups of lemonade shown by the point on the graph?

A (4, 16)   **C** (9, 3)

B (6, 1)   **D** (16, 4)

Oscar chose **A** as the correct answer. How did he get that answer?

_____

_____

Solve.

3 Rey buys 4 cards for $10. He plots the point (4, 10) on a graph. All cards are the same price. He wants to see how much it would cost to buy more cards. Tell whether each statement is *True* or *False*.

> Be sure that you understand what Rey's ordered pair means.

a. The point (6, 15) will be on the graph. ☐ True ☐ False

b. Rey buys 1 card for $3.50. ☐ True ☐ False

c. Rey buys 100 cards for less than $40. ☐ True ☐ False

d. The point (14, 35) will be on the graph. ☐ True ☐ False

4 Each table shows four ratios of boys to girls at different sporting events. Which tables show four equivalent ratios of boys to girls? Select all that apply.

> What makes two ratios equivalent?

A

3	5	9	12
5	7	15	20

C

45	25	10	5
18	10	4	2

B

3	4	7	11
12	16	28	44

D

200	150	100	50
50	40	30	20

5 Rosa earns $10 for every 3 hours that she works. Ralph earns $7 for every 2 hours that he works. Who earns more per hour? How much *more* does this person earn after 12 hours of work?

Show your work.

> Be careful not to compare $10 to $7—these represent earnings for different numbers of hours.

Solution: _____

_____

Solve Problems with Unit Rate

Name: _____

Prerequisite: Equivalent Ratios

Study the example problem showing how to find equivalent ratios. Then solve problems 1–6.

Example

Ramon needs 12 oranges to make 3 glasses of juice. How many oranges does he need to make 5 glasses? How many oranges does he need to make 8 glasses?

You can make a table to show ratios of the number of oranges to the number of glasses of juice.

Number of Oranges	4	8	12	16	20	24	28	32
Number of Glasses	1	2	3	4	5	6	7	8

Ramon needs 20 oranges to make 5 glasses of juice.

Ramon needs 32 oranges to make 8 glasses of juice.

---

1 What ratio is given in the problem for the number of oranges to the number of glasses of juice?

_____

2 What is the unit rate? Explain what it means in this situation.

_____

_____

3 Explain how you can write equivalent ratios.

_____

_____

_____

_____

_____

_____

Vocabulary

equivalent ratios two or more ratios that are equal to one another.

24 : 2, 36 : 3, 48 : 4

Solve.

4 Nathan does push-ups for the same amount of time every day. He does 9 minutes of push-ups in 3 days. How many minutes of push-ups does Nathan do in 7 days? Make a table to show the relationship between the number of minutes and the number of days.

Show your work.

Solution: _____

5 Students are knitting scarves for a fund-raiser. Elaine can knit 4 scarves in 20 days. Mario can knit 2 more scarves than Elaine can in 40 days. What is the difference in the time it takes each of them to knit a scarf? Explain your answers.

Show your work.

Solution: _____

6 There are 24 total customers seated at 4 tables in a restaurant. Each table has the same number of customers. Tell whether each statement is *True* or *False*.

a. Multiply 24 by 4 to find the number of customers per table. ☐ True ☐ False

b. The unit rate for the number of customers per table is 6. ☐ True ☐ False

c. The ratio of customers to tables is 24 : 4. ☐ True ☐ False

d. If all the tables are the same size, a maximum of 30 customers can sit at 6 tables. ☐ True ☐ False

Lesson 4 Solve Problems with Unit Rate

Name: _____

Unit Price

Study the example problem showing how to solve a problem about unit price. Then solve problems 1–7.

Example

All the comic books in a store are the same price. Vera buys 3 comic books for $7.50. How much do 5 comic books cost? How much do 8 comic books cost?

Divide 7.50 by 3 to find the unit price.

$7.50 \div 3 = 2.50$

The price per book is $2.50. You can use the unit price to make a table of equivalent ratios.

Cost ($)	2.50	5.00	7.50	10.00	12.50	15.00	17.50	20.00
Comic Books	1	2	3	4	5	6	7	8

The cost of 5 comic books is $12.50.

The cost of 8 comic books is $20.00.

1 How can you use multiplication to find the cost of 5 comic books?

_____

2 How can you use addition to find the cost of 8 comic books?

_____

3 Explain how to find the number of comic books you could buy with $25.00.

_____

_____

_____

_____

Solve.

Use the following situation to solve problems 4–7.

All of the used hardcover books at a yard sale are the same price. Hugo paid $4.50 for 6 books.

4 Explain how to find the unit price of the books.

_____

5 Hugo's friends bought used books at the yard sale. Sonia paid $2.25, John paid $6.00, and Keisha paid $3.75. How many books did each friend buy?

Show your work.

Solution: _____

6 Kim bought 10 used books at the yard sale. How much did she pay? Did you use addition or multiplication to solve this problem? Why?

_____

7 The price for the used paperback books at the yard sale was $0.25 less than for the hardcover books. How many more paperback books than hardcover books could someone buy with $3.00?

Show your work.

Solution: _____

Name: _____

Constant Speed

Study the example problem showing how to solve a problem about constant speed. Then solve problems 1–7.

Example

Kenja traveled 120 miles in 3 hours on a train. At this speed, how long will it take her to travel 200 miles?

The unit rate for miles per hour is 120 ÷ 3, or 40. Use the unit rate to make a double number line.

Miles 0   40   80   120   160   200   240   280

Hours 0   1   2   3   4   5   6   7

Divide 200 by 40.
200 ÷ 40 = 5

It will take Kenja 5 hours to travel 200 miles.

---

1 How many miles could Kenja travel in 1 hour. Is this the same number of hours it takes Kenja to travel 1 mile? Explain your answer.

_____

_____

_____

2 Explain how to use the unit rate for miles per hour to find how many miles Kenja can travel in 8 hours.

_____

3 Explain how to use the double number line to find how many hours it will take Kenja to travel 220 miles.

_____

_____

_____

Solve.

Use the following situation to solve problems 4–6.

Zachary exercises by jogging at a constant speed.
During one week, he jogged 36 miles in 6 hours.

4 Complete the double number line to show the
relationship between the number of miles and the
hours that Zachary jogs.

Miles 0 ☐ 12 18 ☐☐ 36 42 ☐

Hours 0 1 ☐☐ 4 5 6 ☐ 8
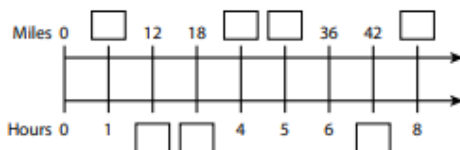
5 Explain how you found the number of hours it takes
Zachary to jog 18 miles.

_____

_____

6 How many miles does Zachary jog in 4.5 hours? Explain
how to use the double number line to find the answer.

_____

_____

7 Alyssa and Caleb both drove 210 miles to the beach in
separate cars. They left at the same time. They both
drove at a constant speed. Alyssa drove 105 miles in
3.5 hours. Caleb drove 168 miles in 4 hours. Who
arrived earlier? How much earlier?

Show your work.

Solution: _____

Converting Measurement Units

Study the example problem showing how to solve a problem involving conversion of measurement units. Then solve problems 1–6.

Example

Hannah needs 78 inches of ribbon to make a picture frame. She knows that there are 60 inches in 5 feet. How many feet of ribbon are in 78 inches?

You can find the unit rate and make a double number line. There are 60 inches in 5 feet, so there are $60 \div 5 = 12$ inches in 1 foot. The unit rate is 12.

Feet 0	1	2	3	4	5	6 6½ 7	8

Inches 0	12	24	36	48	60	72 78 84	96
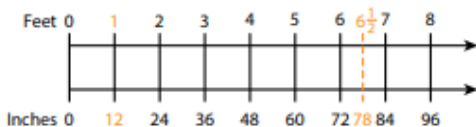
Because the number of inches, 78, is halfway between 72 and 84, the number of feet must be halfway between 6 and 7 feet. There are $6\frac{1}{2}$ feet of ribbon in 78 inches.

---

1 Explain how to use the unit rate without the number lines to find how many feet of ribbon are in 48 inches.

_____

2 How many inches of ribbon are in 3 feet? Explain how to find the answer without using the number lines.

_____

3 What is the difference between using the unit rate to find how many feet are in a given number of inches and using the unit rate to find how many inches are in a given number of feet?

_____

_____

_____

Solve.
Use the following situation to solve problems 4–5.

Antonio measures items in his pocket. He knows there
are 50 millimeters in 5 centimeters. His key chain is
3.5 centimeters long. His library card is 80 millimeters long.

4 How many centimeters long is his library card? Explain
how to use the unit rate to find the answer.

_____

_____

_____

5 How many millimeters long is his key chain? Draw a
double number line to find the answer.

Show your work.

Solution: _____

6 Claire is measuring ingredients for recipes. She knows
that there are 12 cups in 6 pints. She also knows that
4 quarts equals 16 cups. Which has more cups, 5 pints
or 3 quarts? How many more cups?

Show your work.

Solution: _____

Solve the problems.

1 The double number line shows the relationship between the number of minutes and the number of pages that a printer prints. How many pages does the printer print in $4\frac{1}{2}$ minutes?

Where is $4\frac{1}{2}$ minutes located on the number line?

Pages 0   20   40   60   80   100

Minutes 0   1   2   3   4   5
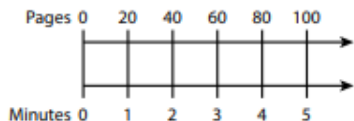
A   80 pages   **B**   85 pages   **C**   90 pages   **D**   100 pages

2 A carpenter uses 65 shelves to make 13 bookcases. She uses the same number of shelves for each bookcase. Are 32 shelves enough to build 6 more bookcases?

What is the unit rate?

Show your work.

Solution: _____

3 The price of 6 pretzels is $5.10. Simon and Sofia bought 8 pretzels and shared the cost equally. How much did each person pay?

One calculation is not enough to solve this problem.

A $0.85   **C** $6.80

B $3.40   **D** $20.40

Jacob chose **C** as the correct answer. How did he get that answer?

_____

_____

Solve.

4 Michael drove 350 miles in 7 hours at a constant speed. Tell whether each statement is *True* or *False*.

How can you find a unit rate?

a. The unit rate for miles to hours is 50. ☐ True ☐ False

b. Michael drove 250 miles in 4 hours. ☐ True ☐ False

c. To find the number of miles Michael drove in 3 hours, multiply 3 by 50. ☐ True ☐ False

d. To find the number of hours it took Michael to drive 300 miles, divide 300 by 50. ☐ True ☐ False

5 Jorge says there are 198 inches in 5.5 yards. Is he correct? Explain your answer.

Show your work.

Do you know the unit rate for inches per foot? Do you know the unit rate for feet per yard?

Solution: _____

6 At Teen Tops, a package of 5 T-shirts costs $38. At Bargain City, a package of 4 T-shirts costs $34. Which statement is the most accurate?

Finding unit prices will help you choose the correct answer.

A Bargain City is the better buy because it sells T-shirts at $8.50 per T-shirt.

B Teen Tops is the better buy because the package has more T-shirts.

C Bargain City is the better buy because $34 is less than $38.

D Teen Tops is the better buy because it sells T-shirts at $7.60 per T-shirt.

**Study the example showing how to find the unit rate.
Then solve problems 1–7.**

Example

A bus driver made 100 stops on his route in 5 days. The
double number line shows the relationship between the
number of stops and the number of days.

Stops 0   20   40   60   80   **100**   120   140   160

Days 0   1   2   3   4   **5**   6   7   8
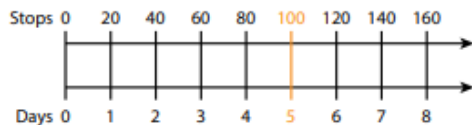
In the diagram, 100 and 5 represent the ratio of 100 stops
to 5 days.

You can write a multiplication equation to show how
5 days and 100 stops are related.

$5 \times 20 = 100$

1 Look at the corresponding pairs of numbers on the
number lines. Write a multiplication equation to show
how 3 days and 60 stops are related. Repeat for two
other corresponding pairs of numbers.

_____

2 What is the relationship between the number of stops
and each corresponding number of days?

_____

_____

3 What is the rate of stops per day? What is the unit rate?

rate: _____   unit rate: _____

Vocabulary

rate a ratio that
compares the first
quantity to only one of
the second quantity.

unit rate the number in
a rate that is being
compared to 1.

Solve.

Use the following situation to solve problems 4–5.

Caroline earns $54 babysitting for 6 hours.

4 Fill in the blanks on the double number line to show the relationship between the amount of money Caroline earns and the number of hours she works.

Money ($) 0 [ ] 18 [ ] 36 [ ] 54 63 [ ]

Hours 0 1 [ ] 3 [ ] 5 6 [ ] 8
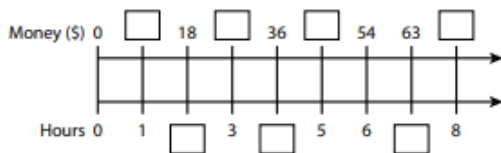
5 What is Caroline's rate, in dollars per hour? What is her unit rate?

_____

6 Ling uses 21 bananas to make 7 fruit smoothies. What is the rate that Ling uses for bananas per each smoothie? What is the unit rate? Explain how to use equivalent fractions to find the answer.

_____

_____

7 Kelly drove 440 miles in 8 hours. Alberto drove 468 miles in 9 hours. Both drove at a constant speed. Who drove farther in 1 hour? How many miles farther?

Show your work.

Solution: _____

Study the example problem showing how to find the percent of a number. Then solve problems 1–6.

Example

In an after-school sports program, 70% of 400 students play soccer. How many students play soccer?

You can use a model to find 70% of 400.

40	40	40	40	40	40	40	40	40	40

0% 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%
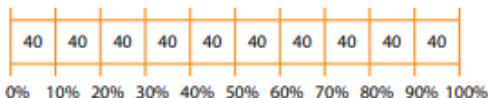
The model shows 400 divided into groups of 40. Each group of 40 represents 10% of 400, so 7 groups of 40 represent 70% of 400. This means that 70% of 400 is 7 · 40, or 280.

There are 280 students who play soccer.

1 What is 70% written as a fraction? _____

2 Use the fraction to write and evaluate a multiplication expression that represents 70% of 400. Compare the answer to the one you got using the model.

Show your work.

Solution: _____

3 What is 75% of 400? Write and evaluate an expression to find the answer. Then explain how to use the model to justify the answer.

_____

_____

Solve.

Use the following situation to solve problems 4–5.

The results of a survey show that 40% of 300 students chose conserving natural resources as the top priority for their generation.

4 How many students chose conserving natural resources? Make a model to find the answer.

Show your work.

Solution: _____

5 Suppose only 24% of 300 students chose conserving natural resources. How many students chose conserving natural resources? Explain how you found your answer. How can the model help you justify the answer?

_____

_____

_____

6 There are 50 puzzles in Maggie's puzzle book. Maggie finished 30% of the puzzles. How many puzzles does she have left to do?

Show your work.

Solution: _____

Study the example problem showing how to find the whole when a part and the percent are given. Then solve problems 1–6.

Example

Carmen saved $27, which was 30% of the money she earned. How much did Carmen earn?

You can use a double number line to find the whole when a part and the percent are given.
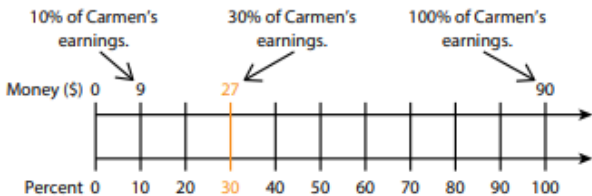
Carmen earned $90.

1 How can you find 10% of Carmen's earnings using the ratio 27 to 30? What is 10% of Carmen's earnings?

_____

_____

_____

2 How many times as great as 10% is 100%?

_____

3 How can you find 100% of Carmen's earnings using the ratio of her earnings to 10%? What is 100% of Carmen's earnings?

_____

_____

_____
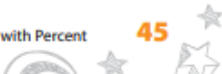
Solve.

4 Diane received 300 votes in the election for student council president. That was 60% of the students who voted in the election. How many students voted in the election? Use a double number line in your explanation.

_____

_____

5 Students sold 80% of the books donated to the used book sale. They sold 48 books in all. How many books were donated to the used book sale? Use a table in your explanation.

_____

_____

6 Omar spends $63 on souvenirs during his vacation. That is 35% of the money he brought with him. How much money does Omar have left to spend?

Show your work.

Solution: _____

Solve the problems.

1 Jamil traveled 210 miles, which is 70% of the total distance to his grandfather's house. How many more miles does he need to travel to reach his grandfather's house?

> You need to make two calculations to solve this problem.

A   90 miles   **C**   300 miles

B   147 miles   **D**   390 miles

Kate chose **B** as the correct answer. How did she get that answer?

_____

_____

2 Brandon plowed snow from 84 driveways in 7 days. He plowed the same number of driveways each day. Tell whether each statement is *True* or *False*.

> Use the ratio of driveways to days to help you.

a.   The rate is 84 driveways to 1 day.   ☐ True   ☐ False

b.   The unit rate for driveways per day is 12.   ☐ True   ☐ False

c.   The rate in fraction form is $\frac{12}{1}$.   ☐ True   ☐ False

d.   If Brandon continues at the same rate, he will plow 120 driveways in 12 days.   ☐ True   ☐ False

3 A meteorologist said that it rained during 20% of the past 60 days. On how many days did it not rain?

Show your work.

> What operation does the word "of" indicate?

Solution: _____

Solve.

4 At tryouts for the school talent show, 60% of 30 performers played a musical instrument. How many performers played a musical instrument? Use the model to find the answer.

How many groups of 3 are in 60%?

3	3	3	3	3	3	3	3	3	3

0% 10% 20% 30% 40% 50% 60% 70% 80% 90% 100%
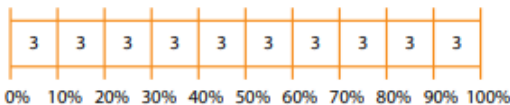
A 3 performers   **C** 30 performers

B 18 performers   **D** 60 performers

5 Students collected 600 cans for the canned food drive. That was 80% of their goal. How many more cans do they need to collect to reach their goal?

Show your work.

Do you need to find the part or the whole?

Solution: _____

6 Megan correctly spelled 45 out of 50 words in a spelling competition. Justin spelled 27 out of 30 words correctly. Fernando spelled 84 out of 120 words correctly. Which statements are true? Select all that apply.

How can you find a percent using a ratio?

A Fernando spelled the greatest percent of words correctly.

B Megan and Justin spelled the same percent of words correctly.

C Justin spelled the least percent of words correctly.

D The percent of words that Megan spelled correctly is greater than the percent of words that Fernando spelled correctly.

Step-by-Step Tape Diagram Lesson

Question: In a school, there are 250 students. The ratio of boys to girls is 3:2. What is the total number of boys in the school? What is the total number of girls in the school? Draw a Tape Diagram for a visual display of the problem.

Answer:

a) Start with what you know.

1. We know the total number of students is 250.

2. We know that for every 3 boys, there are 2 girls.

b) Start to draw your tape diagram for the ratio.

3 boys =

2 girls =

3. Combine the information to show the composition of the group. If the total number of students is equal to 250 and is composed of 5 groups, each group has 50 (250/5) students. The diagram will look like the following:

boys | 50 | 50 | 50 |
girls | 50 | 50 |

250
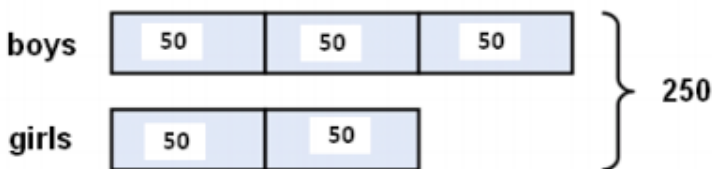
4. To find the total number of girls, add the segments 50 + 50 = 100 girls.

5. To find the total number of boys, add the segments 50 + 50 + 50 = 150 boys.

Answer: There are 150 boys and 100 girls in the school.

Understand
Division with Fractions

Name: _____

> **Prerequisite: How do you divide with unit fractions?**

Study the example problem showing division of a whole number by a unit fraction. Then solve problems 1–7.

Example Problem

Karl puts $\frac{1}{2}$ cup of chopped tomatoes into each salad he makes. How many salads can he make with 3 cups of tomatoes?

The model represents the problem. You can use the model to write a division equation and a multiplication equation.

3 cups of tomatoes

$$3 \div \frac{1}{2} = 6 \qquad 3 \times 2 = 6$$

$\frac{1}{2}\ \frac{1}{2} \qquad \frac{1}{2}\ \frac{1}{2} \qquad \frac{1}{2}\ \frac{1}{2}$

Both equations show that Karl can make 6 salads with 3 cups of tomatoes.

---
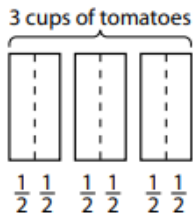
1 Explain how the model represents $3 \div \frac{1}{2} = 6$.

_____

_____

2 Explain how the model represents $3 \times 2 = 6$.

_____

_____

3 Suppose Karl uses 5 cups of tomatoes. How many salads can he make? Write both a division equation and a multiplication equation to show your solution.

_____

_____

Vocabulary

unit fraction a fraction with a numerator of 1.

$\frac{1}{3}, \frac{1}{8}$, and $\frac{1}{12}$ are unit fractions.

Solve.

4 Four students are sharing $\frac{1}{3}$ carton of yogurt equally. Complete the steps to find what fraction of the carton each student gets.
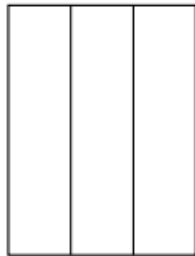
   a. The model at the right represents 1 carton. Shade the model to show $\frac{1}{3}$ carton.

   b. Divide the model into 4 equal parts by drawing horizontal lines to represent sharing among 4 students. Shade one row to show $\frac{1}{4}$.

   c. Complete the equation to show what fraction of the carton of yogurt each student gets.

   $\frac{1}{3} \div 4 =$ _____ carton of yogurt

5 Use the model in problem 4 to write a multiplication equation that can be used to solve the problem.

_____

6 Find $2 \div \frac{1}{3}$. Explain how to use the number line to find the answer.

_____
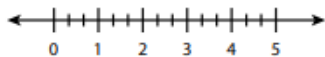
7 Ana has $\frac{1}{2}$ hour of free time. She divides the time equally between walking her dog and playing her favorite song on the piano. If she plays the song 3 times, how long is the song? Give your answer as a fraction of an hour. Write division equations to represent the problem.

Show your work.

Solution: _____

Name: _____

Divide by a Fraction

Study the example problem showing division of a fraction by a fraction. Then solve problems 1–10.

Example

Mr. Garcia has $\frac{3}{4}$ yard of ribbon to make badges for winners of the science fair. He uses $\frac{1}{8}$ yard of ribbon for each badge. How many badges can Mr. Garcia make?
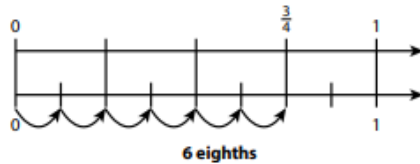
6 eighths

Find the number of eighths in $\frac{3}{4}$. Use the number lines.

$\frac{3}{4} \div \frac{1}{8} = 6$

Mr. Garcia can make 6 badges.

---

1 What does $\frac{3}{4}$ on the top number line represent?

_____

2 What does each equal part on the bottom number line represent?

_____

3 How many eighths are in $\frac{3}{4}$? _____

4 Suppose Mr. Garcia is making badges using $\frac{3}{8}$ yard of ribbon for each badge. He starts with the same amount of ribbon, $\frac{3}{4}$ yard. How many badges can he make? Write a division equation that supports your answer.

_____

Solve.

Use the following situation to solve problems 5–9.

Rosa puts $\frac{2}{3}$ cup of vegetable mixture in 1 tortilla. She has 8 cups of vegetable mixture.

5 Rosa says that to find how many tortillas she can fill, she first finds find how many $\frac{1}{3}$ cups are in 8 cups. What else must Rosa do to find how many tortillas she can fill?

_____

_____

6 Do you expect the number of tortillas Rosa can fill to be less than or greater than 8? Explain.

_____

_____

7 The rectangles represent 8 cups of vegetable mixture. Draw lines to divide each rectangle into thirds.

8 Circle groups of $\frac{2}{3}$ rectangle. How many groups are there? _____

9 Complete the division equation to show how many tortillas Rosa can fill.

$8 \div \frac{2}{3} =$ _____ tortillas

10 Mike pours $\frac{12}{8}$ cups of orange juice into serving glasses. Each glass holds $\frac{3}{4}$ cup. How many glasses can he fill? Use a common denominator to divide.

Show your work.

Solution: _____

Reason and Write

Study the example. Underline two parts that you think make it a particularly good answer and a helpful example.

Example

Steve said that $\frac{4}{3} \div \frac{1}{6}$ equals $\frac{4}{6}$. How do you know without dividing whether Steve's statement is reasonable? Justify your answer by showing how to find the quotient.

Show your work. Use numbers, words, and models to explain your answer.

Steve's statement is not reasonable. The division $\frac{4}{3} \div \frac{1}{6}$ asks how many sixths are in $\frac{4}{3}$. $\frac{4}{3}$ is greater than 1, and there are 6 sixths in 1. So I know there are more than 6 sixths in $\frac{4}{3}$. That means the quotient must be greater than 1. It could not be a fraction less than 1, such as $\frac{4}{6}$.

I drew a number line model to find the quotient. The top number line is divided into thirds and shows $\frac{4}{3}$. The bottom number line is divided into sixths and shows that there are 8 sixths in $\frac{4}{3}$. So $\frac{4}{3} \div \frac{1}{6} = 8$.
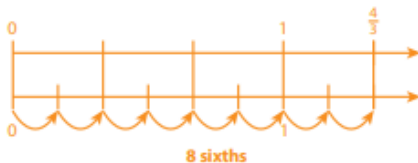
8 sixths

Where does the example . . .
- use numbers to explain?
- use words to explain?
- use models to explain?
- give details?

Solve the problem. Use what you learned from the model.

Brenda said that $\frac{5}{2} \div \frac{1}{4}$ equals 10. How do you know without dividing whether Brenda's statement is reasonable? Justify your answer by showing how to find the quotient.

Show your work. Use numbers, words, and models to explain your answer.

▶ On-Level Activity

Make a fraction division handbook or poster.

Guide students to create a three-page "handbook" or three-part poster highlighting the steps to solving a fraction division problem.

Examples should include dividing a fraction by a whole number, dividing a whole number by a fraction, and dividing a fraction by a fraction. Each example should include an expression, a model, and an explanation for how to solve the problem.

Prerequisite: Divide with Unit Fractions

Study the example problem showing how to solve a word problem that involves dividing with unit fractions. Then solve problems 1–6.

Example

The students in Mrs. Marco's art class use 5 jars of paint altogether. Each student uses $\frac{1}{3}$ jar of paint. How many students are in the class?

To answer this question, you need to find how many $\frac{1}{3}$s are in 5. You can draw a model to understand the problem.

$5 \div \frac{1}{3} = 15$

There are 15 students in Mrs. Marco's art class.

---

1 Explain how the model represents $5 \div \frac{1}{3} = 15$.

_____

_____

2 Complete the equation. Explain how the model also shows this equation.

_____ × _____ = 15

_____

3 Andy divided his $\frac{1}{3}$ jar of paint equally between 2 projects. What fraction of a jar of paint did Andy use for each project? Explain how to draw a model to find the answer.

_____

_____

_____

_____

Solve.

4 Judi is making a rope ladder. She uses $\frac{1}{2}$ yard of rope for each step. How many steps can she make with 6 yards of rope?
Show your work.

Solution: _____

5 Harry has $\frac{1}{4}$ of an apple pie that he wants to cut into 3 equal slices. What fraction of the whole pie is each slice?
Show your work.

Solution: _____

6 Ryan wants to plant $\frac{1}{5}$ packet of seeds in each row of his garden. He has 4 packets of seeds. Ryan used the expression $\frac{1}{5} \div 4$ to find the number of rows he can plant. Explain what is wrong with his expression. Then write an equation to show the correct number of rows.

_____

_____

_____

Name: _____

Divide a Whole Number by a Fraction

Study the example problem showing how to divide a whole number by a fraction. Then solve problems 1–6.

Example

On a field trip, students ate $\frac{3}{10}$ of a box of oranges. Altogether they ate 6 pounds of oranges. How many pounds of oranges were in the full box?

You can draw a model to represent the problem.
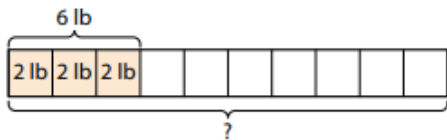
You can also use an equation to represent that $\frac{3}{10}$ of the full box is 6 pounds: $\frac{3}{10} \times ? = 6$. To solve a missing factor problem, divide: $6 \div \frac{3}{10} = ?$.

To divide by a fraction, multiply by its reciproal.

$6 \div \frac{3}{10} = 6 \times \frac{10}{3} = 20$

There were 20 pounds of oranges in the full box.

1 Look at the model. Explain why each tenth of the model is 2 pounds.

_____

_____

2 How can you use the model in the example to find how many pounds of oranges were in the box?

_____

_____

_____

_____

3 Suppose $\frac{4}{5}$ of a different box of oranges weighs 8 pounds. How many pounds of oranges are in the full box?

_____

Vocabulary

multiplicative inverse a number is the multiplicative inverse of another number if their product is 1.

$9 \times \frac{1}{9} = 1$

The fraction $\frac{1}{9}$ is the multiplicative inverse of 9.

reciprocal the multiplicative inverse of a number; with fractions, the numerator and denominator are switched.

$\frac{8}{5}$ is the reciprocal of $\frac{5}{8}$.

Solve.

4 Ling walks $\frac{3}{8}$ of the distance home from school in 9 minutes. She wants to know how long it will take her to walk the entire distance at the same speed. Ling uses the expressions $9 \div \frac{3}{8}$ and $9 \times \frac{3}{8}$ to find the answer. Explain what is wrong with Ling's expressions and then write the correct solution.

_____

_____

_____

5 Daniel has 20 quarts of water. How many $2\frac{1}{2}$-quart containers can he fill?

Show your work.

Solution: _____

6 Write a word problem that you can represent with the expression $8 \div \frac{2}{3}$. Draw a model and use equations to show the solution.

_____

_____

_____

Study the example problem showing how to divide a fraction by a fraction. Then solve problems 1–7.

Example

A construction company is building a fence that is $\frac{2}{3}$ mile long. They can build $\frac{1}{6}$ mile of the fence every hour. How many hours will it take them to complete the fence?

You can draw a picture to represent the problem.
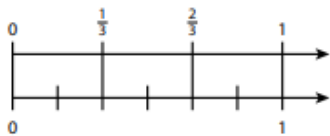
You can also use a division equation:

$\frac{2}{3} \div \frac{1}{6}$. Think: How many $\frac{1}{6}$s are in $\frac{2}{3}$?

$\frac{2}{3} \div \frac{1}{6} = \frac{2}{3} \times \frac{6}{1} = 4$

It will take 4 hours for the company to build the fence.

1 How can you use the number lines to find how many hours it will take the company to build the fence?

_____

_____

2 Suppose you were told that the company built the fence in 4 hours and that they completed $\frac{1}{6}$ mile of the fence each hour. How would you use the double number line to help you find the length of the fence?

_____

_____

3 Suppose the length of the fence was $1\frac{1}{3}$ mile. How would you change the number lines to solve the problem?

_____

_____

Solve.

4 A chef cooks $\frac{5}{6}$ of a pound of pasta. She plans to serve $\frac{1}{12}$ of a pound to each customer. How many customers can she serve? Explain.

_____

5 Carla wants to know how many batches of birdseed she can make with $\frac{1}{2}$ cup of sunflower seeds. She puts $\frac{1}{6}$ cup of sunflower seeds in every batch. Carla divides $\frac{1}{2}$ by $\frac{1}{6}$ to find the answer. She says this is the same as multiplying $\frac{1}{6}$ by 2. Explain what Carla did wrong and show the correct solution.

_____

_____

_____

6 Jared ate $\frac{1}{4}$ of a loaf of bread. He cut the rest of the loaf into $\frac{1}{8}$-loaf slices. How many slices of bread did he cut?
Show your work.

Solution: _____

7 A running track at a school is shaped like oval. The track is $\frac{1}{2}$ mile long. Mr. Perez puts a marker down every $\frac{1}{8}$ mile. How many markers does he need? Show how you found your answer.

_____

Name: _____

Divide a Mixed Number by a Fraction

Study the example problem showing how to divide a mixed number by a fraction. Then solve problems 1–6.

Example

Mali has $2\frac{1}{3}$ cups of fruit to make smoothies. She puts $\frac{2}{3}$ cup of fruit in each smoothie. How many smoothies can she make?

You can draw a model to represent the problem.

You can also use equations. Think about how many $\frac{2}{3}$s are in $2\frac{1}{3}$.

$2\frac{1}{3}$ cups

$\frac{2}{3}$ cup in 1 smoothie $\frac{1}{3}$ cup in $\frac{1}{2}$ smoothie

$$2\frac{1}{3} \div \frac{2}{3} = \frac{7}{3} \div \frac{2}{3}$$

$$\frac{7}{3} \div \frac{2}{3} = \frac{7}{3} \times \frac{3}{2} = 3\frac{1}{2}$$

Mali can make $3\frac{1}{2}$ smoothies.
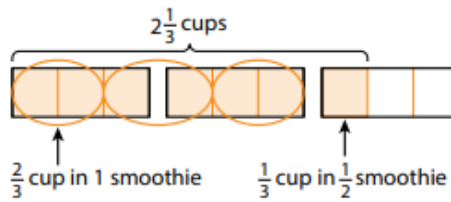
1 Explain why there is $\frac{1}{3}$ cup of fruit in half a smoothie.

_____

_____

2 Look at the equations. Explain how you know that $2\frac{1}{3}$ is equal to $\frac{7}{3}$.

_____

3 Explain how the model shows that $2\frac{1}{3}$ is equal to $\frac{7}{3}$.

_____

_____

Solve.

4 Otis made $1\frac{3}{5}$ cups of oatmeal. He put $\frac{2}{5}$ cup of oatmeal into each bowl. How many bowls of oatmeal did Otis make? Use equations to solve the problem.
Show your work.

Solution: _____

5 Juan wants to know how many $\frac{1}{4}$-cup servings are in $1\frac{3}{8}$ cups of juice. She uses the expression $\frac{12}{8} \div \frac{1}{4}$ to find the answer. Explain what is wrong with Juan's expression and find the correct solution.

_____

_____

_____

_____

6 Carmela mixes $\frac{3}{4}$ kilogram of walnuts, $\frac{1}{2}$ kilogram of almonds, and $\frac{1}{4}$ kilogram of pecans together. She divides the mixed nuts into $\frac{3}{10}$-kilogram bags. How many bags of mixed nuts does she have?
Show your work.

Solution: _____

Solve the problems.

1 Dario divides $\frac{4}{6}$ yard of rope equally into $\frac{1}{12}$-yard pieces for a craft project. How many pieces of rope does Dario have? Use the number lines to solve the problem.

> What fraction is each number line divided into?

A 4 pieces

B 6 pieces

C 8 pieces

D 12 pieces

2 Check a box in each row to show whether the quotient of each expression is less than, greater than, or equal to 1.

> Read each expression as a question. For example: How many $\frac{3}{10}$s are in $\frac{3}{5}$?

	quotient is less than 1	quotient is equal to than 1	quotient is greater than 1
$\frac{3}{5} \div \frac{3}{10}$			
$\frac{3}{10} \div \frac{3}{5}$			
$6 \div \frac{2}{5}$			
$2\frac{3}{4} \div \frac{3}{4}$			
$3\frac{2}{3} \div \frac{11}{3}$			
$\frac{9}{8} \div \frac{3}{2}$			

Solve.

3 Omar jogged a total of $3\frac{3}{5}$ miles last week. Each day that he jogged, he went $\frac{9}{10}$ mile. On how many days did he jog?

Show your work.

> Think: How many $\frac{9}{10}$s are in $3\frac{3}{5}$?

Solution: _____

4 Choose all of the expressions that are equivalent to $2\frac{1}{2} \div 1\frac{2}{6}$.

A $\frac{5}{2} \times \frac{6}{8}$

B $\frac{2}{5} \times \frac{6}{8}$

C $1\frac{2}{6} \div 2\frac{1}{2}$

D $\frac{5}{2} \div \frac{8}{6}$

> Can you draw a model to help you find $2\frac{1}{2} \div 1\frac{2}{6}$?

5 What is the value of the expression $3 \div \frac{3}{4}$?

A $\frac{1}{4}$

B $2\frac{1}{4}$

C 4

D 12

> How can you use multiplication to divide a number by $\frac{3}{4}$?

Jane chose **B** as the correct answer. How did she get that answer?

_____

_____

_____

The Coordinate Plane

Prerequisite: Graph Points

Study the example showing how to plot points on a coordinate grid. Then solve problems 1–11.

Example

The location of a point is named with an *x*-coordinate and a *y*-coordinate. The coordinates are written as an ordered pair, (*x*-coordinate, *y*-coordinate). Follow these steps to plot point *A* at (3, 4).

- Start at the origin.

- Move 3 units to the right.

- Move 4 units up.

- Label the point *A*.
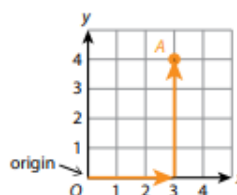
1 What ordered pair describes the origin? _____

2 What are the coordinates of point *A*?

x-coordinate: _____   y-coordinate: _____

3 Along which axis do you count each number of units in order to plot point *A*?

3 units to the right: ___-axis   4 units up: ___-axis

4 Plot a new point at (4, 3). Label the point *C*.

5 Zachary says that point *C* has the same location as point *A* because both points have the same coordinates. Is Zachary right? Explain why or why not.

_____

_____

_____

Vocabulary

x-coordinate a point's horizontal distance from the origin along the *x*-axis.

y-coordinate a point's vertical distance from the origin along the *y*-axis.

Solve.

Use the coordinate plane at the right to solve problems 6–9.

6 Plot and label the following points.

    Q(5, 5)   R(7, 3)   S(2, 8)

7 Choose one point from problem 6. Complete the following statements to describe how you plotted the point.

    a. Start at (_____, _____).

    b. Move _____ units to the right. Move _____ units up.

    c. Label the point _____.

8 Plot points at (0, 3), (0, 1), and (0, 5). What is true about all points with an x-coordinate of 0?

_____

9 Plot points at (2, 0), (4, 0), and (3, 0). What is true about all points with a y-coordinate of 0?

_____

Use the coordinate plane at the right to solve problems 10–11.

10 Write ordered pairs for four points that you can plot on the coordinate plane. Each ordered pair must have a y-coordinate that is 2 units less than its x-coordinate. Plot the points.

_____

_____

11 Describe a pattern for the points you plotted in problem 10.

_____

_____

_____

Name: _____
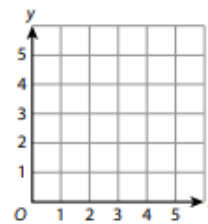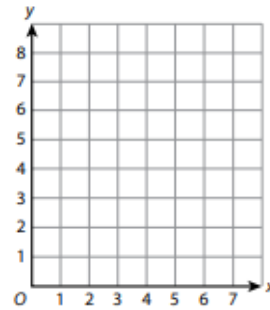
Graphing on the Coordinate Plane

Study the example showing how to graph on the coordinate plane. Then solve problems 1–7.

Example

The table shows the locations of exhibits at a science museum. Graph each exhibit on the coordinate plane.

Exhibit	Fossils	Birds	Planets	Energy
Coordinates	(3, 2)	(−1, −3)	(2, −2)	(−3, 1)
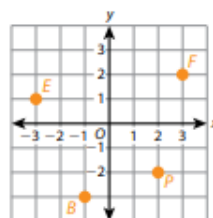
For each ordered pair in the table, start at the origin, move left or right according to the x-coordinate, and then move up or down according to the y-coordinate.

1. Which exhibit is located at point *E* on the coordinate plane?

_____

2. What are the *x*- and the *y*-coordinates of point *E*?

_____

3. How are the *x*-coordinate and the *y*-coordinate in an ordered pair related to the origin?

_____

_____

4. Complete the table below to describe the location of each exhibit.

Exhibit	Location from the Origin
Fossils	
Birds	
Planets	
Energy	

Solve.

Use this information for problems 5–6.

You can use a coordinate plane to represent the locations of different activities at a summer camp. The ordered pairs in the table show the location of each activity.

Activity	Canoeing	Swimming	Hiking	Art	Fishing
Coordinates	(−6, 5)	(2, −2)	(−3, −3)	(4, 6)	(−4, 0)

5 Graph each activity as a point on the coordinate plane. Label each point with the first letter of the activity.
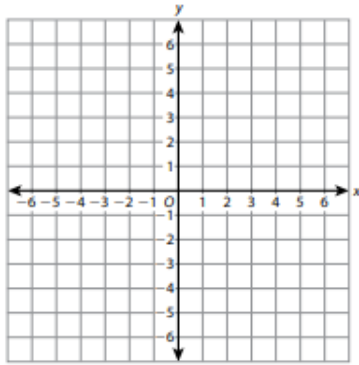
6 Describe the location from the origin of each point in problem 5.

_____

_____

_____

7 What are the signs of the coordinates of a point in each of the four quadrants?

_____

_____

_____

_____

Name: _____

Reflect Points

Study the example problem showing how to reflect points across the *x*-axis. Then solve problems 1–9.

Example

Rectangle *LMNK* is reflected across the *x*-axis to get rectangle *PQRS*. How do the coordinates of point *L* change when it is reflected across the *x*-axis?

The coordinates of point *L* are (2, 6). The reflection of point *L* across the *x*-axis is point *P*. The coordinates of point *P* are (2, −6).

The *x*-coordinate of the reflection of point *L* is the same as point *L*, but the *y*-coordinate has the opposite sign.
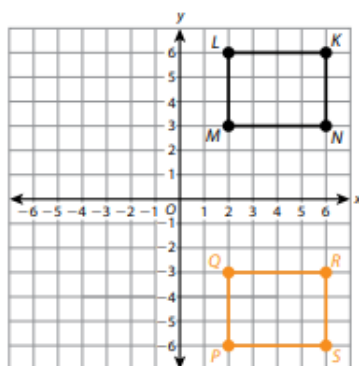
1 What points are the reflections of points *M*, *N*, and *K* across the *x*-axis?

_____

_____

2 List the coordinates of the other points of rectangle *LMNK* and rectangle *PQRS*.

Point *M*: (_____)   Point *Q*: (_____)

Point *N*: (_____)   Point *R*: (_____)

Point *K*: (_____)   Point *S*: (_____)

3 How are the coordinates of points *M* and *Q*, points *N* and *R*, and points *K* and *S* the same? How are they different?

_____

4 How do the coordinates of a point compare with the coordinates of its reflection across the *x*-axis?

_____

Solve.

Use the information for problems 5–7.

The points *L*, *M*, and *N* are reflected across the *y*-axis to get the points *F*, *G*, and *H* on the coordinate plane at the right.

5 List the coordinates of the points shown in the graph.

Point *L*: (_____)   Point *F*: (_____)

Point *M*: (_____)   Point *G*: (_____)

Point *N*: (_____)   Point *H*: (_____)
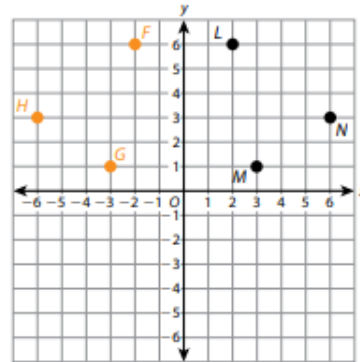
6 How are the coordinates of each point and its reflection the same? How are they different?

_____

_____

7 How do the coordinates of a point compare with the coordinates of its reflection across the *y*-axis?

_____

_____

8 Becky reflects point *Q* at (−5, −4) across the *x*-axis to get point *Z*. What are the coordinates of point *Z*? Explain how you know.

_____

_____

9 Kanika plots point *A* at (1, −2). Next she plots a reflection of point *A* at point *W*. Finally, Kanika plots a reflection of point *W* at point *T*, which is located at (−1, 2). Describe how Kanika could have reflected each of the points to arrive at point *T*.

_____

_____

_____

Name: _____

Distance Between Points

Study the example showing how to find the distance between points in different quadrants. Then solve problems 1–9.

Example

The locations of different stores are shown on the map. There is a sports store at point S and a clothing store at point C. Each unit on the coordinate plane represents 1 mile. How many miles is the clothing store from the sports store?

Notice that the stores have the same x-coordinates, but they are in different quadrants. To find the distance between them, find the distances of both points from the x-axis and add them.

$$|5| + |-6| = 5 + 6 = 11$$

The clothing store is 11 miles from the sports store.
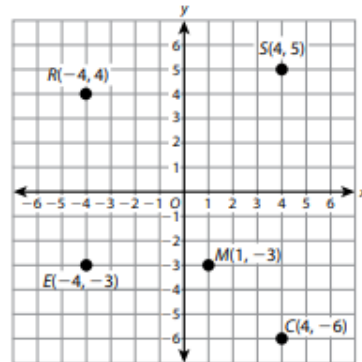
1 What are the y-coordinates of the sports store and the clothing store?

_____

2 What do $|5|$ and $|-6|$ represent in the example?

_____

_____

3 What is the relationship between the distance of the sports store and the clothing store from the x-axis and the y-coordinate of each point?

_____

_____

4 Explain how to count units to check the answer.

_____

_____

Solve.

Use the situation below and the coordinate plane to solve problems 5–8.

A music store is at point *M* on the coordinate plane. An electronics store is at point *E,* and a restaurant is located at point *R.* Each unit represents 1 mile.

Coordinate plane with points: $R(-4, 4)$, $S(4, 5)$, $E(-4, -3)$, $M(1, -3)$, $C(4, -6)$
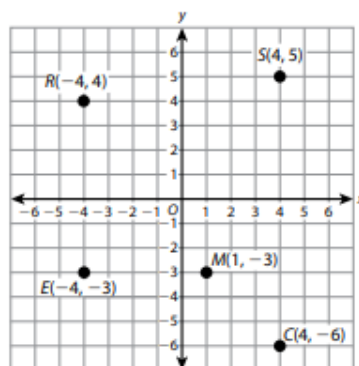
5 What are two ways you can find the distance between the electronics store and the music store?

_____

_____

6 What is the distance between the electronics store and the music store? Count units to find the answer.

_____

7 Use absolute value to find the distance between the music store and the electronics store.

Show your work.

Solution: _____

8 Helen drove from the restaurant to the toy store, which is not shown on the map. She made a right turn at the toy store and drove to the music store. She drove a total distance of 12 miles. What are the coordinates of the toy store?

_____

9 Point *A* is located at $(-3, y)$, and point *B* is located at $(-6, y)$. How can you find the distance between these points using absolute values?

_____

_____

Name: _____

The Coordinate Plane

Solve the problems.

1 Which statements are true about the coordinate plane? Choose all that apply.

Which axis are the triangles reflected across?

A $\triangle KLM$ is reflected across the y-axis to get $\triangle RTS$.

B The x-coordinates of points K and R are the same.

C The y-coordinates of points K and R have opposite signs.

D The y-coordinates of points L and T are the same.

2 In the coordinate plane in problem 1, how can you find the distance from point R to the x-axis? Choose all that apply.

What are the x- and y-coordinates of point R?

A Find the absolute value of the x-coordinate in $(-4, -5)$.

B Add the absolute values of the x- and y-coordinates.

C Count the number of units down from the x-axis.

D Find the absolute value of the y-coordinate in $(-4, -5)$.

Julio chose **A** as a correct answer. How did he get that answer?

_____

_____

Solve.

3 Four points representing the corners of a square are $D(-4, 5)$, $E(3, 5)$, $F(3, -2)$, and $G(-4, -2)$. Graph and label the points on the coordinate plane.
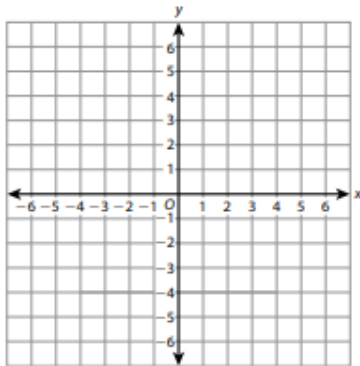
What do the coordinates of a point tell you about its location from the origin?

4 How could you move the square in problem 3 so that each corner point is a reflection of another corner point across the x- and y-axes? Explain your answer and graph the new square.

Show your work.

How do you find the distance between points?

Solution: _____

_____

_____
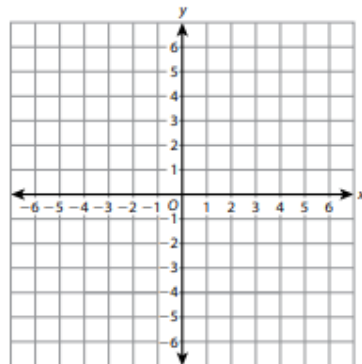
Algebraic Expressions

Name: _____

Study the example showing how to write numerical expressions. Then solve problems 1–6.

Example

Write a numerical expression for this phrase: 12 minus the product of 3 and 2.

Think about what the words mean.

12 minus   the product of 3 and 2
   ↑   ↑
Minus means   A product is the result
 to subtract.   of multiplication.

Before you can subtract the product from 12, you need to multiply 3 by 2 to find the product. Use parentheses to show that first you need to multiply.

The numerical expression is $12 - (3 \times 2)$.

1 Jennifer says that you can also write $(12 - 3) \times 2$ for the phrase in the example. Is Jennifer correct? Explain why or why not.

_____

_____

_____

2 Write a numerical expression for the phrase "16 times the difference of 9 and 3." What operation should you perform first? Explain.

_____

_____

_____

_____

Vocabulary

parentheses the symbols ( ) that can be used to group numbers and operations in an expression.

$24 - (3 \times 5)$

$(5 + 7) \times 3$
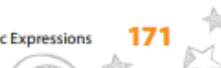
Solve.

3 To evaluate the expression "10 minus the sum of 2 and 3," should you subtract or add first? Explain how you know.

_____

_____

4 Write a numerical expression for each word phrase. Then evaluate the expression.

a. 5 times the sum of 3 and 4

_____

b. 24 divided by the sum of 6 and 2

_____

c. Divide the difference of 18 and 3 by the sum of 1 and 2.

_____

d. the sum of 4 and 3 multiplied by the quotient of 4 and 2

_____

5 Write a word phrase for the expression $12 \div (7 - 3)$.

_____

6 Marisa made a fruit salad. She used 1 cup of green grapes and 3 cups of red grapes. She used twice as many cups of blueberries as cups of grapes.

Write an expression for the number of cups of blueberries that Marisa used. Then evaluate the expression. Explain your reasoning.

_____

_____

_____

_____

Study the example showing how to write an expression from words. Then solve problems 1–10.

Example

Write an expression with the same meaning as "add a number times 2 to 5."

Find operation words to help you write the expression. *Add* a number times 2 to 5. This expression will be an addition of two terms.

First term	+	Second term

The first term is 5. The second term is 2*x*. So the expression is 5 + 2*x*.

1 What does the variable *x* in the example represent?

2 The number 2 in the expression 5 + 2*x* is called the coefficient of *x*. How does changing the coefficient to 6 change the meaning of the expression?

3 In the expression, 5 + 2*x*, how is the first term different from the second term?

Vocabulary

variable a letter that stands for an unknown number.

constant a term that is a known number without variables.

coefficient a factor of a variable term that is a known number. The coefficient of the term 4*x* is 4.

4 Write an expression for each word phrase.

 a. Multiply 4 by a number and then subtract 5.

 b. 15 more than half a number

Solve.

5 Connie says an expression for the phrase "10 more than the square of a number" is $x^2 + 10$. Sharon says it is $10x^2$. Who is correct? Explain.

_____

_____

6 Write an expression for each word phrase.

   a. 5 less than the quotient of a number and 2

   _____

   b. 5 minus the quotient of a number and 2

   _____

7 How are the expressions that you wrote in problem 6 similar? How are they different?

_____

_____

_____

8 Write a word phrase for the expression $16 \div (x + 4)$.

_____

9 Write an expression with two terms. One term should have a coefficient with a variable and the other term should be a constant. Name the coefficient, the variable, and the constant in the expression. Then write a word phrase for your expression.

_____

_____

10 Mario says that the expression $4 + 3n^2$ has four terms: 4, 3, n, and 2. Is he correct? Explain.

_____

_____

_____

▶ Challenge Activity

Use a table to evaluate different values.

Present students with the following: *A store has CDs on sale for $8 each and DVDs on sale for $15 each.*

1. Create a box model to translate the expression.

2. Write an expression that gives the total cost for CDs and DVDs.

3. Provide a table to evaluate the expression for different values of CDs and DVDs.

4. If Shannon has $150 dollars to spend, what are the possible combinations of CDs and DVDs she can purchase? Explain your possibilities.

Write and Evaluate Expressions

Study the example showing how to write and evaluate expressions. Then solve problems 1–7.

Example

Lina is making jewelry. She has 7 beads and buys 4 additional packets of beads that each have the same number of beads. Write an expression to show the total number of beads that Lina uses.

Draw the beads she starts with and the packets she buys, and label the number of beads in each. You don't know how many beads are in each packet, so use a variable like b to label the number of beads in each packet.

Amount Lina Starts With	Additional Amount Lina Buys
●●●●●●●	☐ ☐ ☐ ☐
7 beads	b beads b beads b beads b beads
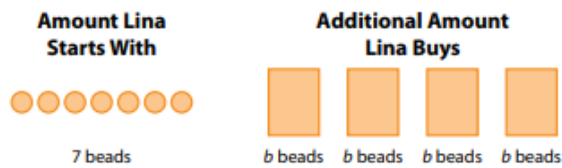
1 Write an expression for each word or phrase.

a. the number of beads Lina starts with

_____

b. the total number of beads in the four packets

_____

c. the total number of beads Lina has

_____

2 Laura wrote and solved the following expression to find the total number of beads Lina has if there are 6 beads in each packet. Find and correct Laura's mistake.

$$7 + 4b = 11b$$
$$= 11(6)$$
$$= 66$$

_____

_____

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Solve.

3 Blake and three friends meet for lunch. His friends all get the same thing, but Blake gets a different lunch that costs $6. Write an expression to show the total amount that Blake and his friends spend. Then find the total amount that Blake and his friends spend if each friend spends $8.

_____

4 Ana's age is 8 years less than 4 times her sister's age. Write an expression for Ana's age. How old is Ana if her sister is 5 years old?

_____

5 Belle put the muffins she baked on six plates, four of which are red and two of which are yellow. The four red plates each have 5 muffins. The two yellow plates each have the same number of muffins. Write an expression for the total number of muffins Belle baked. If each yellow plate has 8 muffins, find how many muffins Belle baked in all. Explain.

_____

_____

6 Adam says that the expression $52 - 3y$ is equal to 20 when $y = 2$. Explain why Adam's answer is incorrect.

_____

_____

7 A blue suitcase weighs 10 pounds less than three-fourths the weight of a green suitcase. Write an expression that you can use to find the weight of the blue suitcase. Then explain how you can find the total weight of both suitcases if the green suitcase weighs 36 pounds.

_____

_____

_____

_____

Study the example showing how to write and evaluate more expressions. Then solve problems 1–5.

Example

Last week Juan mowed lawns and walked his neighbor's dog to earn money. For mowing lawns, he earned $6 less than twice as much as he did for walking dogs. Juan saves one-third of the money he earns and spends the rest.

Write an expression to show how much money Juan earned last week.

Draw a picture to help you understand the problem.

walking dogs   mowing lawns

money saved
money spent
money spent

Let w be the amount Juan earned walking dogs. Then $(2w - 6)$ is the amount Juan earned mowing lawns. The total amount Juan earned is $w + (2w - 6)$, or $3w - 6$.

---

1 Emma wrote the expression $2(3w - 6)$ to represent the amount of money that Juan spent. Is she correct? Explain.

_____

_____

2 Explain how you can find the amount of money Juan saved if he earned $12 walking dogs.

_____

_____

_____

_____

Solve.

3 The price *p* of a gallon of gas goes up $0.05 cents on Friday. On Saturday the price goes down $0.03. Write an expression with three terms to show the price of a gallon of gas on Saturday.

_____

4 Look at problem 3. If the price of a gallon of gas was $2.59 on Friday morning before the change in price, what was the price of a gallon of gas on Saturday? Explain how you know.

_____

_____

5 Katie gives Maggie half of her pencils. Maggie keeps 5 pencils and gives the rest to Jamil.

a. Write an expression for the number of pencils Maggie gives to Jamil.

_____

b. If Katie had 16 pencils, how many pencils does Maggie give to Jamil?

Show your work.

Solution: _____

c. How many pencils did Katie have if Maggie gave Jamil 1 pencil? Explain how you can use the expression to help you answer the question.

Show your work.

Solution: _____

_____

Algebraic Expressions

Solve the problems.

1 Lewa's hiking backpack weighs 5 pounds less than $\frac{1}{2}$ the weight of Alani's hiking backpack. Write an expression to describe the weight of Lewa's backpack. How many pounds does Lewa's backpack weigh if Alani's backpack weighs 36 pounds?

Show your work.

Finding $\frac{1}{2}$ of an amount is the same as dividing that amount by 2.

Solution: _____

2 A bookcase has two shelves. The top shelf has 10 more than $\frac{1}{3}$ the number of books on the bottom shelf. There are 12 books on the bottom shelf. How many books are on the top shelf?

A 4   **C** 40

B 14   **D** 46

Cohen chose **D** as the correct answer. How did he get that answer?

_____

_____

Which operations will you use to solve this problem?

3 Which expression equals 6 when $a = 5$ and $b = \frac{1}{3}$? Circle all that apply.

A $9b^2 + 3a - 10$

B $a^2 - 20 - 3b$

C $3(a - 2) - a + 6b$

D $9b + ab$

Remember to use the order of operations when evaluating expressions.

Solve.

4 Martin used some apples to make muffins. Omar used some apples to make applesauce. Omar used 5 fewer than half as many apples as Martin used.

After you find the solution, read the problem again and check to be sure that your solution makes sense.

a. Write an expression to show the number of apples that Martin and Omar used in all. What does your variable represent?

_____

_____

b. Could Martin have used 10 apples? Why or why not? Use the expression to help you decide.

Show your work.

Solution: _____

_____

_____

_____

5 Lilla read $\frac{1}{5}$ of her book last week. This week she read 3 times as much as she read last week.

What should the variable in your expression represent?

a. Write an expression to show how much of her book Lilla has left to read. Then simplify the expression.

_____

_____

b. There are 75 pages in Lilla's book. How many pages does she have left to read?

Show your work.

Solution: _____

Understand
Solutions to Equations

Name: _____

> Prerequisite: **How can you use the properties of operations to write equivalent expressions?**

Study the example problem showing how to write equivalent expressions. Then solve problems 1–8.

Example

Gail plants 3 pots of roses and 2 pots of tulips. The number of flowers in each pot is the same. Write an expression for the total number of flowers. Simplify the expression to create an equivalent expression.

You can use math tiles to represent the problem.

Roses | f | f | f |

Tulips | f | f |

Add to find the total number of flowers. An expression for the total number of flowers is $3f + 2f$. Then simplify.

$$3f + 2f = f(3 + 2) = 5f$$

1 Look at the example. What does f represent?

_____

2 Tell what each expression below represents.

   a. $3f$ _____

   b. $2f$ _____

   c. $3f + 2f$ _____

3 How was the distributive property used to create an expression that is equivalent to $3f + 2f$?

_____

Vocabulary

like terms terms in an expression that have the same variable raised to the same power. Constants are like terms.

x and $-4x$

1 and 1.5

x^2 and $8x^2$

Solve.

4 David says that he can apply the commutative and distributive properties to $7s + 8 + 5s$ to get $12s + 8$. Is he correct? Explain.

_____

_____

_____

5 Use three of the terms below to fill in the two expressions. Each term may be used only once. Both of your expressions must be equivalent to $0.5x + 1.5$.

| 0.5 | 2 | x | 0.25x | 3 | 0.75 |

_____ (_____ + _____)

_____ (_____ + _____)

6 Write a story that you could represent with the expression $8b + 4b - 2$. Then write an expression that is equivalent to $8b + 4b - 2$.

_____

_____

_____

_____

7 Is $d(10 + 20)$ equivalent to $d \times 10 + 20 \times d$? Use a property, or properties, to explain.

_____

_____

8 Use the distributive property to write an expression that is equivalent to $45 + 30x$.

_____

**Study the example problem showing how to write and
solve equations. Then solve problems 1–9.**

Example

Larry mows 4 lawns and earns $24. He is paid the same
amount of money for each lawn. Write and solve an
equation to find how much Larry is paid to mow one lawn.

You can draw a bar model to help you
write and solve an equation that
represents the problem. The equation
$4p = 24$ represents the problem.

24			
p	p	p	p
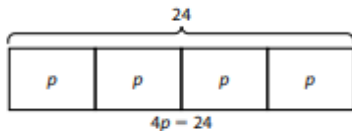
$4p = 24$

The equation is asking: What number
could you multiply by 4 to get 24?

$4 \times 6 = 24$

Larry is paid $6 to mow one lawn.

1 What does p represent in the example?

_____

2 What does the expression $4p$ represent?

_____

3 What is the solution to the equation $4p = 24$?

$p =$ _____

4 Bev went to the grocery store with $45. She spent
d dollars and came home with $21. Write and solve
an equation to find how much Bev spent at the store.

Show your work.

Solution: _____

Solve.

Use this situation for problems 5–9.

Yaro buys a baseball cap for $9.50. He also buys a new baseball. Yaro spends $13.50 altogether.

5 Write an equation to represent how much Yaro pays for the baseball.

_____

6 Do you expect the solution to your equation to be less than or greater than $13.50? Explain.

_____

_____

7 What is the solution to the equation you wrote in problem 5? Draw a number line. What increments did you use to label your number line? How can you use it to help you find the solution?

_____

_____

_____

8 How much does Yaro pay for the baseball? _____

9 Write an equation using a different operation to represent how much Yaro pays for the baseball. Explain why you can use equations with different operations to represent the same problem.

_____

_____

_____

Study the example. Underline two parts that you think make it a particularly good answer and a helpful example.

Example

Ling says that the solution to $8s = 2$ is that s must be greater than 1. Does Ling's solution make sense? Explain how you know whether or not Ling's solution makes sense without solving the equation. Then draw a model of the problem and solve the equation.

Show your work. Use numbers, words, and models to explain your answer.

Ling's solution does not make sense. The expression $8s$ means to multiply 8 by s. If I multiply 8 by 1, I get 8, which is greater than 2. So the solution must be less than 1.

I can draw a bar model to help me solve the problem.

The bar model shows that 8 times s gives me 2, so I can ask myself what number I could multiply by 8 to get 2. I know that the number is less than 1, so it must be a fraction.

The model shows that 8 bars represent 2, so 4 bars must represent 1. Therefore, each bar represents $\frac{1}{4}$.

The solution to $8s = 2$ is $s = \frac{1}{4}$.

Where does the example . . .

• use numbers to explain?

• use words to explain?

• use models to explain?

• give details?

Solve the problem. Use what you learned from the model.

Jake says that the solution to $8.5 - a = 5$ is that a equals 13.5 because addition and subtraction are inverse operations and $8.5 + 5 = 13.5$. Does Jake's solution make sense? Explain how you know whether or not Jake's solution makes sense without solving the equation. Then draw a model of the problem and solve the equation.

Show your work. Use numbers, words, and models to explain your answer.

Prerequisite: Solve Equations

Study the example problem showing how to solve an equation. Then solve problems 1–7.

Example

Taryn planted 91 tulip bulbs in 7 rows. She planted the same number of bulbs in each row. How many bulbs did she plant in each row?

Create a bar model to represent the 7 rows and the total number of bulbs that Taryn planted.

91

| b | b | b | b | b | b | b |
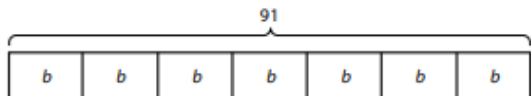
1 What does *b* represent in the bar model?

_____

2 What does each part of the bar model represent?

_____

_____

3 Explain how the model shows that $7b = 91$.

_____

4 What operation will you use to solve the equation $7b = 91$? Solve the equation to find the number of bulbs, and justify each step you take to solve the equation.

Show your work.

Solution: _____

Solve.

5 Milo and Audrey sold tickets to the school concert. Milo sold 14 fewer tickets than Audrey. If Milo sold 32 tickets, how many tickets did Audrey sell?

Show your work.

Solution: _____

6 Carmen buys 4 daisies and some roses to make a flower arrangement. The number of daisies is $\frac{1}{3}$ of the number of roses that she buys. How many roses does she buy?

Show your work.

Solution: _____

7 Write a multiplication equation and a subtraction equation that both involve a fraction and have the same solution. Solve your equations to show that the solutions are the same.

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Study the example problem showing how to write and solve an inequality for a real-world problem. Then solve problems 1–9.

Example

Mr. Gomez gets a notice from the bank when the amount in his checking account drops below $20. For what amounts will Mr. Gomez receive a notice from the bank?

Use words and symbols to represent the situation. Let x represent the amount in Mr. Gomez's account. When x is less than $20, the bank will send a notice.

$x < 20$

Graph the inequality to show all of the solutions. Use an open circle shows that 20 is NOT a solution.
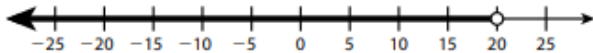
1 Why is the graph in the example shaded to the left?

_____

2 Is −$10 a solution? Substitute −10 into the inequality in the example to check.

_____

3 Name an amount that is NOT a solution to the inequality. Explain how you know.

_____

_____

4 Suppose the bank sent Mr. Gomez a notice whenever the amount in his account dropped to $15 or less. How would the graph in the example change?

_____

_____

Vocabulary

inequality a statement that contains the symbol $<, >, \leq,$ or $\geq$. For any inequality, there are many possible solutions. These are inequalities.

$x < 5$
$x > -2$
$x \leq 15$
$x \geq 0$

Solve.

5 Write the inequality shown on each graph.

a.

_____

b.

_____
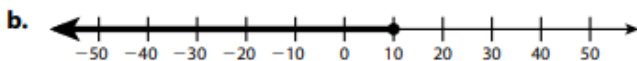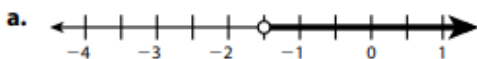
6 The children at Lincoln School go outside for recess if the temperature is 3°C or higher. For what temperatures will the students go outside? Write an inequality to represent this situation. Then graph the solution.

_____
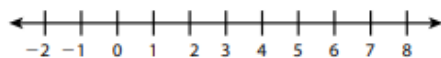
7 If your score on a computer game is less than 0, you lose your next turn. For what scores will you lose your turn? Write an inequality to represent this situation. Then graph the solution.

_____
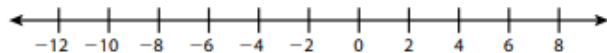
8 Write a real-world situation for this inequality: $x \geq 9$

_____

_____

9 Explain how an equation and an inequality are different. Give an example of each.

_____

_____

_____

Study the example problem showing how to write and solve an inequality. Then solve problems 1–9.

Example

You need to be at least 40 inches tall to ride on the roller coaster at the amusement park. What are some possible heights for riders? Write an inequality to represent the heights, and graph the solution on a number line.

Use symbols to represent the situation. Let x be the possible heights in inches for riders.

$x \geq 40$

Graph the inequality to show all of the solutions.
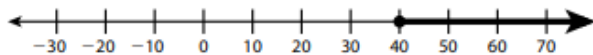
1. Leigh is 40 inches tall. Can she ride on the roller coaster?

_____

2. Brennon is 38 inches tall. Can he ride on the roller coaster? Use the graph to explain your answer.

_____

3. Joy wrote the inequality $40 \leq x$ to represent the situation. Is her inequality correct?

_____

4. Suppose this graph represents a problem about the height of people riding the roller coaster. How would the situation have changed?

_____

_____
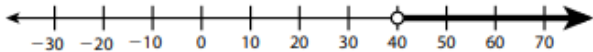
Solve.

Use this situation for problems 5–6.

You must spend at least $10 at the grocery store to get a free greeting card.

5 Write an inequality to represent the amount you need to spend to get a free greeting card. Then graph the solution on the number line.

_____

```
←─┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼─→
   4  6  8  10 12 14 16 18 20 22 24
```
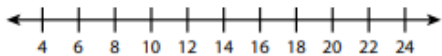
6 If you spend $9.50, will you get a free greeting card? Use the graph to explain how you know.

_____

7 Zarina is scuba diving. She will not dive below −30 meters relative to the surface of the water. Write an inequality that represents this situation. Is −20 a solution to the inequality? Explain how you know.

_____

8 Write an inequality that has the solution shown on the graph. Then write a real-world situation for the inequality.

```
←━━━━━━━○──┼──┼──┼──┼─→
  1  2  3  4  5  6  7  8  9  10 11
```

_____

_____
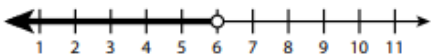
9 Markim looks at the graph below and says that −1 is the only possible negative solution. Do you agree or disagree? Explain.

```
←─┼──┼──┼──●━━┼━━┼━━┼━━┼━━┼━━┼→
  −4 −3 −2 −1  0  1  2  3  4  5  6
```

_____

_____

_____

Solve the problems.

1 Write an inequality for each graph.

a.

_____
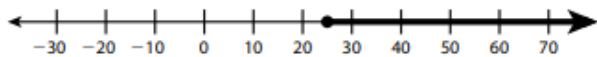
b.

_____
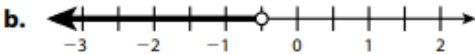
> What does the direction of the arrow on the shaded line tell you about the inequality?

---

2 Samuel wants to eat at least 15 grams of protein each day. Let x represent the amount of protein he should eat each day to meet his goal. Which inequality represents this situation?

A $x < 15$

C $x \le 15$

B $x > 15$

D $x \ge 15$

Karli chose **B** as the correct answer. How did she get that answer?

_____

> Would a graph for this situation have an open circle or a closed circle?

---

3 The graph shows information about the low temperature in a particular city in degrees Celsius each day during one week in January. Write an inequality for this situation. Then write in words what the graph shows about the temperature readings.

_____

_____
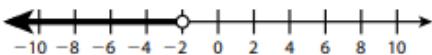
> What are some words that describe a situation in which the shaded line on the graph points left?

Solve.

4 Kalista practices the piano for at least 8 hours each week. Write an inequality for this situation. Then graph the solution on the number line. _____
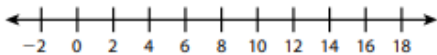
Does "at least" include 8 as a solution?

5 Which of these values is a solution to the inequality $x - 1 \geq 6$?

7    $3\frac{1}{2}$ 2.5   10

Show your work.

Does a given value for x make the inequality true or false?

Solution: _____

6 Consider the inequality $x > -0.75$. Tell whether each statement is *True* or *False*.

How do you know whether a given value is a solution to the inequality?

a. -0.75 is a solution to the inequality.
☐ True   ☐ False

b. There are many solutions to this inequality.
☐ True   ☐ False

c. All of the solutions to the inequality are negative.
☐ True   ☐ False

d. The inequality $-0.75 < x$ is equivalent to the given inequality.
☐ True   ☐ False

e. -4.5 is a solution to the inequality.
☐ True   ☐ False

Create word problems from a given inequality.

Ask students to write a real-world situation that is represented by the inequality $x > 35$, or for more difficulty, $-5 \leq x \leq 30$. Have them graph their solutions.

Dependent and Independent Variables

Name: _____

Study the example showing how to compare two patterns on a graph. Then solve problems 1–9.

Example

Suppose two patterns both start at 0. The rule for one pattern is "add 2," and the rule for the other pattern is "add 4." Compare the patterns using a graph.

Write the first four numbers of each pattern.

add 2: 0, 2, 4, 6
add 4: 0, 4, 8, 12

Use the numbers in each pattern to write ordered pairs and plot them on a graph.

(0, 0)   (2, 4)   (4, 8)   (6, 12)
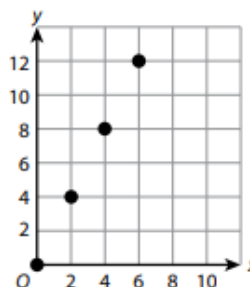
1 Explain how the terms in each pattern are related to the coordinates in the ordered pairs.

_____

_____

_____

_____

2 Use the ordered pairs to describe how the terms in the second pattern are related to the corresponding terms in the first pattern.

_____

_____

3 How would the graph look if you connected the points?

_____

_____

Vocabulary

corresponding terms
the numbers that are in the same position in two or more related patterns.
Pattern 1: 0, **2**, 4, 6
Pattern 2: 0, **4**, 8, 12
The numbers 2 and 4 are corresponding terms.

ordered pairs a pair of numbers that locate a point on the coordinate plane.

Solve.

Use the following patterns for problems 4–8.

Suppose two patterns both start at 0. The rule for one pattern is "add 6," and the rule for the other pattern is "add 2."

4 Complete the table to show the first four numbers in each pattern. Use the corresponding terms in each pattern to write ordered pairs.

Add 6	Add 2	Ordered Pairs
0	0	

5 Graph the ordered pairs.

6 Describe the relationship between the corresponding terms of the two patterns.

_____

_____

_____
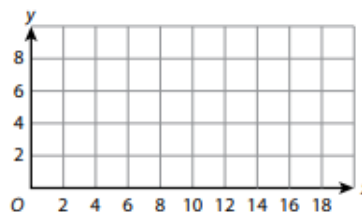
7 What directions would you give someone to get from one point to the next on the graph?

_____

8 How do the directions that you would give in problem 7 relate to the rules for the patterns?

_____

_____

9 Consider the ordered pairs.

(0, 0)   (1.5, 4.5)   (3, 9)   (4.5, 13.5)

Write two rules, one for the x-terms of the given ordered pairs and one for the y-terms. Describe the relationship between corresponding terms.

_____

_____

Name: _____

Relationship Between Variables

Study the example showing the relationship between variables with a table and an equation. Then solve problems 1–7.

Example

A music store sells sets of headphones for $6. The table shows the relationship between the number of headphones the store sells, h, and the amount of money, m, the store earns from headphone sales. Write an equation that represents the amount of money the store earns from headphone sales.

Number of Headphones, h	0	1	2	3	4	5
Amount of Money, m ($)	0	6	12	18	24	30

Use the table to write an equation.

amount of money	equals	price of each set of headphones	times	number of headphones
m	$=$	6	$\cdot$	h

The equation is $m = 6h$.

1 Which is the dependent variable and which is the independent variable in the example? Explain.

_____

_____

_____

2 How much money does the store earn if the store sells 8 sets of headphones? Explain how to use the equation to find the answer.

_____

3 One week, the store earned $60 in headphone sales. How many sets of headphones did the store sell? Can you use the equation to find the answer? Explain.

_____

Solve.

Use the example problem to solve problems 4–7.

In the example, you explored how to represent a relationship with a table and an equation.

Number of Headphones, h	0	1	2	3	4	5
Amount of Money, m ($)	0	6	12	18	24	30

$m = 6h$

You can also represent the same situation with a graph.

4 Think of h and m as x- and y-coordinates, and use the values from the table to write ordered pairs (h, m).

_____

5 Graph the ordered pairs. How do they show solutions to the equation $m = 6h$?

_____

_____

_____

_____

6 Which point represents the amount of money the store earns if the store sells 4 headphones? How do you know?

_____

_____

_____

_____

7 Sonia paid $18 for headphones. How many sets of headphones did she buy? Explain how to use the graph to find the answer.

_____

_____

_____

_____

Name: _____
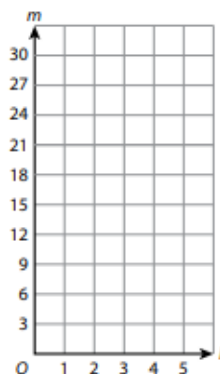
Representing a Problem

Study the example showing how to represent a problem with an equation and a table. Then solve problems 1–8.

Example

Jamil pays $10.00 for a swim club membership and $1.50 for each day that he goes to the pool. Write an equation and make a table to represent the total cost c that Jamil will pay the swim club if he goes to the pool d days.

You can use the information given in the problem to write an equation.

total cost	equals	price per day	times	number of days	plus	membership fee
c	$=$	1.5	$\cdot$	d	$+$	10

The equation is $c = 1.5d + 10$. You can use the equation to make a table to find Jamil's total cost.

Days, d	$1.5d + 10$	Total Cost, c ($)
0	1.5(0) + 10	10
1	1.5(1) + 10	11.5
2	1.5(2) + 10	13
3	1.5(3) + 10	14.5
4	1.5(4) + 10	16

---

1 Name the dependent and independent variables in the problem. Describe the relationship between them.

_____

_____

2 What is the total cost if Jamil uses the pool on 6 days? Explain how to use the equation to find the cost.

_____

3 Lee joins the club but does not go to the pool. Is the total cost $0? Use the equation to explain your answer.

_____

_____

Solve.

Use the example problem to solve problems 4–8.

In the example, you explored how to represent a relationship with an equation and a table.

$c = 1.5d + 10$

Days, d	1.5d + 10	Total Cost, c ($)
0	1.5(0) + 10	10
1	1.5(1) + 10	11.5
2	1.5(2) + 10	13
3	1.5(3) + 10	14.5
4	1.5(4) + 10	16

4 Use the values from the table to write ordered pairs.

_____

5 Graph the ordered pairs on the coordinate plane.

6 What is the total cost of using the pool for 6 days? Explain how to use the equation and the graph.

_____

_____

_____

_____
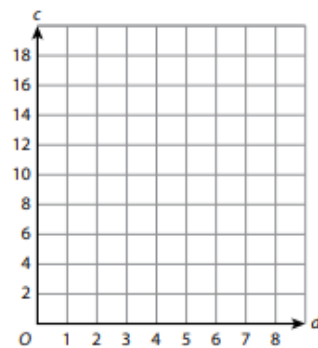
7 If a member paid $25, how many times did he or she use the pool? Explain how you found your answer. Did you use the table, equation, or graph to find your answer? Why?

_____

_____

8 Why is only the first quadrant shown in the graph of this situation?

_____

_____

Name: _____

Dependent and Independent Variables

Solve the problems.

1 Admission to an amusement park costs $5.00 and each ride ticket costs $1.50. The equation $c = 1.5t + 5$ represents the total cost, c, for admission with a certain numbers of ride tickets, t. Which statement about the equation is true? Select all that apply.

How can you use the equation to find the total cost?

A The variable t is the dependent variable.

B The total cost for admission with 5 ride tickets is $7.50.

C The total cost for admission with 6 ride tickets is $14.00.

D The total cost, c, depends on the number of ride tickets, t.

Colin chose **A** as a correct answer. Why did he choose that answer?

_____

_____

2 Which ordered pair is NOT included in the graph of $p = 3m + 6$? Select all that apply.

Which coordinate in the ordered pairs represents the values of p on the graph?

A (1, 9)   **C** (10, 36)

B (3, 12)   **D** (6, 18)

3 Describe a situation with two variables that you can represent with an equation that uses two operations. Write the equation. Explain the relationship between the variables.

How does knowing which variable is the dependent variable help you to write an equation?

_____

_____

_____

_____

Solve.

4 Use a graph to compare a pattern with the rule "add 4" to a pattern with the rule "add 2." Start both patterns at 0. Describe the relationship between the corresponding terms.
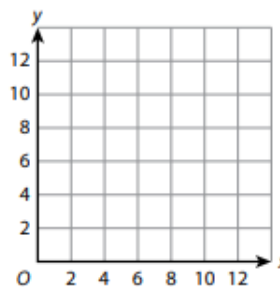
How do you use two patterns to locate points on a graph?

Show your work.

Solution: _____

_____

5 Some students volunteer to plant trees in a new park. They can plant 8 trees per hour. The table shows the relationship between the total number of trees they plant, *s*, and the number of hours, *h*. Tell whether each statement is *True* or *False*.

What is the relationship between h and s?

Number of Hours, *h*	2	4	6	8	10
Total Number of Trees Planted, *s*	16	32	48	64	80

a. In 7 hours, the students can plant 49 trees. ☐ True ☐ False

b. The equation $s = 8h$ represents the relationship between *s* and *h*. ☐ True ☐ False

c. The students need 3 hours to plant 24 trees. ☐ True ☐ False

d. The number of trees is the dependent variable. ☐ True ☐ False

Unit 3 Game

Twenty-Four

What you need: Recording Sheet, Number and Variable Cards

Directions

- Your goal is to write an expression and evaluate it to get as close as possible to 24.

- Mix up the cards and make two piles, one for numbers and one for variables. Players take turns picking two number cards and two variable cards.

- Write an expression using all of the numbers and variables on your cards. You can use any operation, as well as parentheses and exponents.

- Write an equivalent expression.

- Try to find numbers to substitute for the variables in either expression so that you get 24 when you evaluate it. Then record the value of the expression for the numbers you chose.

- Earn 1 point for making an equivalent expression and 2 more points if the evaluated expression equals 24. After 5 rounds, the player with the most points wins.
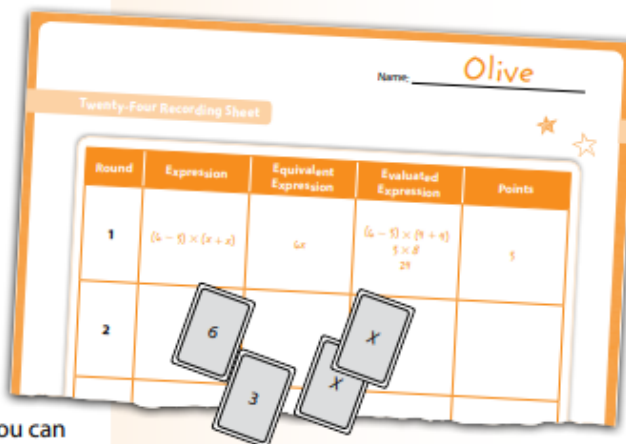
> I try writing different expressions. I think about ways to make 24. And I try different ways to evaluate the expression before I write it on my recording sheet.

Twenty-Four Recording Sheet

Round	Expression	Equivalent Expression	Evaluated Expression	Points
1				
2				
3				
4				
5				

Total Points: _____

Number and Variable Cards

✂

1	**1**	**1**	**2**	**2**
2	**3**	**3**	**3**	**4**
4	**4**	**6**	**6**	**6**
8	**8**	**8**	**x**	**x**
x	**x**	**y**	**y**	**y**

Polygons in the Coordinate Plane

Name: _____

Study the example showing how to solve a measurement problem using a shape on a coordinate plane. Then solve problems 1–9.

Example

Mr. Hiroshi plans to tile the floor of his family room. He draws a rectangle on the coordinate plane to represent the floor. What is the area of the floor in square units?

The area of a rectangle is length × width. You can count units to find the length and the width.

The length of $\overline{AB}$ is 5 units. The length of $\overline{BC}$ is 6 units. The area of rectangle *ABCD* is 5 × 6, or 30 square units.

You can also use ordered pairs to find the horizontal distance and the vertical distance between points on the coordinate plane.
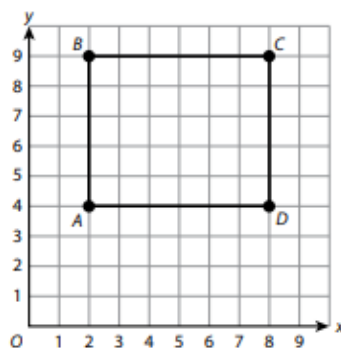
1 Write the ordered pair for each point.

A (_____) B(_____) C(_____) D(_____)

2 Explain how to use the *x*-coordinates of point *A* and point *D* to find the distance between the two points.

_____

3 Explain how to use the *y*-coordinates of point *C* and point *D* to find the distance between the two points.

_____

4 Find the lengths of these sides using the coordinates of their endpoints.

$\overline{AD}$ _____ $\overline{CD}$ _____

5 What is the perimeter of rectangle *ABCD*? Explain how you found the perimeter.

_____

_____

Solve.

Use the shape on the coordinate plane to solve problems 6–8.

6 What are the coordinates of each point on the shape?

A(_____) B(_____) C(_____) D(_____)

E(_____) F(_____) G(_____) H(_____)

7 Find the area of the shape. Explain how you found your answer.

Show your work.
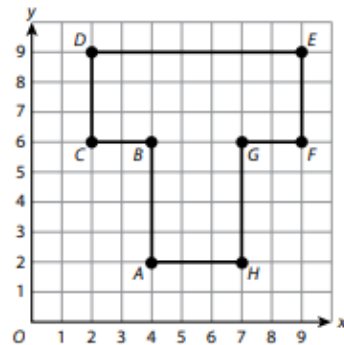
Solution: _____

8 Find the perimeter of the shape.

Show your work.

Solution: _____

9 Use the coordinate plane to draw a rectangle with an area of 24 square units. Label the corners of the rectangle W, X, Y, and Z. Explain how you know that the area of the rectangle is 24 square units.

Name: _____
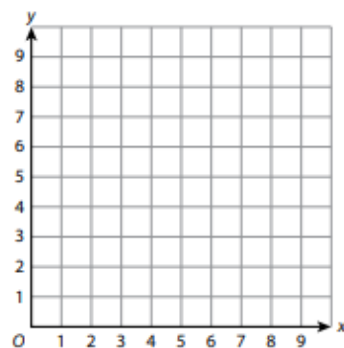
Find Missing Coordinates and Dimensions

Study the example problem showing how to find missing coordinates and dimensions of a rectangle. Then solve problems 1–9.

Example

Ms. Issa is planning to build a rectangular fishpond in her garden. A drawing shows three corners of the pond with coordinates (4, −2), (−2, −2), and (−2, 5). Where is the fourth corner?

You can graph the information given and then sketch the rectangle.
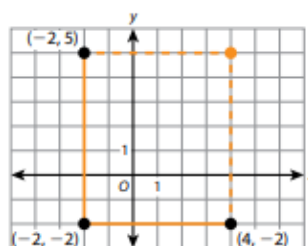
1 What are the coordinates of the fourth corner?

_____

2 How did you locate the fourth corner to sketch the rectangle?

_____

_____

_____

3 Explain how to use counting to find the distance between (−2, −2) and (4, −2).

_____

4 Explain how to use absolute value to find the distance between (−2, −2) and (−2, 5).

_____

_____

5 Explain how to find the area of the pond.

_____

_____

Solve.

Use the following situation to solve problems 6–8.

Mrs. Rockwell is buying a rectangular lot on which to build a new home. Three corners of the lot are at (5, −3), (−2, −3), and (−2, 2) on the coordinate plane.

6 Graph the three corners on the coordinate plane. What is the ordered pair for the fourth corner of the lot?

_____

7 What is the perimeter of the lot?

Show your work.
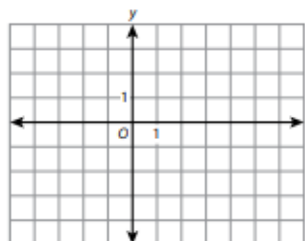
Solution: _____

8 Mr. Brown bought a lot that is half as long and twice as wide as Mrs. Rockwell's lot. How does the area of his lot compare to the area of Mrs. Rockwell's lot? Explain how you know.

_____

_____

_____

9 Nadim wants to build a square pen for his rabbits. He plots two corners on a coordinate plane at (3, −3) and (−3, 3). Abe says that he should plot another corner at (3, 3). Does this make sense? Explain why or why not.

_____

_____

_____

_____

Name: _____

Find Area on a Coordinate Plane

Study the example problem showing how to find the area of a polygon on a coordinate plane. Then solve problems 1–7.

Example

A floor plan for a building shows corners of the building at (0, 0), (6, 0), (9, 5), and (3, 5). What is the shape of the floor of the building? How can you find the area of the floor?

You can graph the information given and connect the points to find the shape of the floor. The connected points form a parallelogram, so the floor is a parallelogram.

You can find the area of the floor by multiplying its base times its height since it is a parallelogram.
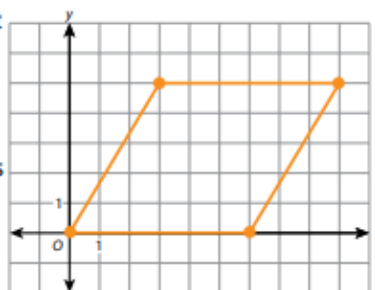
1 What is the base length of the parallelogram in the example? How did you find the base length?

_____

2 What is the height of the parallelogram in the example? How did you find the height?

_____

3 Find the area.

_____

4 Katerine divided the parallelogram into two congruent triangles and a rectangle in order to find its area. Does her method work? If so, show that it works. If not, explain why not.

_____

_____

_____

_____

_____

Solve.

Use the following situation to solve problems 5–6.

Madeline plotted these points to represent the corners of a vegetable garden: (0, 0), (6, 0), (3, 7).

5 Draw the shape on the coordinate plane. What shape is the garden? Find the area of the garden.

   Show your work.
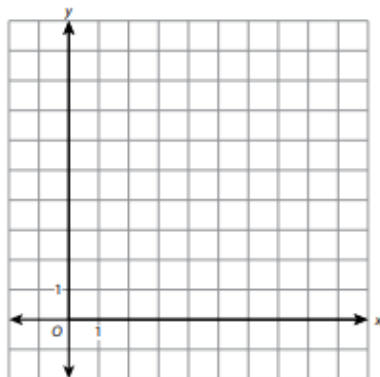
Solution: _____

_____

6 Suppose Madeline uses (6, 7) rather than (3, 7) as the third corner for her garden. How will that change the shape of the garden? How will the areas of the two gardens compare?

_____

_____

7 A flower garden and the lawn around it are shown on the coordinate plane. What is the area of the lawn?

   Show your work.

Solution: _____

Name: _____
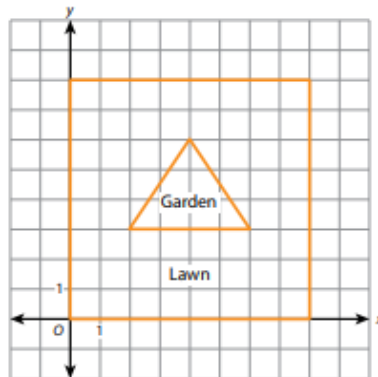
Polygons in the Coordinate Plane

Solve the problems.

1 Find the area of the trapezoid.

Show your work.
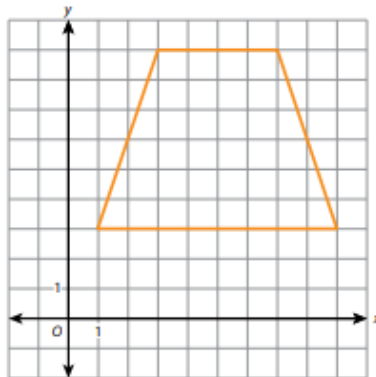
How can you separate this trapezoid into different shapes?

Solution: _____

2 Three corners of a rectangular park are located at $(-3, 1)$, $(4, 1)$, and $(4, -2)$.

How does plotting points for the three corners help you find the point for the fourth corner?

Part A

What are the coordinates of the fourth corner?

_____

Part B

What is the perimeter of the park? Explain how you found your answer.

_____

_____

Solve.

3 Keaton drew a parallelogram on a coordinate plane. Two vertices of the parallelogram were located at (1, 1) and (1, 7). The area of the parallelogram is 18 square units. Tell whether each statement is *True* or *False*.

What is the formula for the area of a parallelogram?

a. The *x*-coordinate of the other two vertices of the parallelogram could be −2. ☐ True ☐ False

b. The *x*-coordinate of the other two vertices of the parallelogram could be 4. ☐ True ☐ False

c. The parallelogram must be a square. ☐ True ☐ False

d. The perimeter of this parallelogram could be 18 units. ☐ True ☐ False

4 Gianna plotted these points and then connected the points in order from *J* to *N* and then back to *J* to show the shape of her room. Draw the room on the coordinate plane. What is the area of Gianna's room?

What shapes do you see when you plot and connect the points?

$J(1, 0)$ $K(1, 6)$ $L(9, 6)$ $M(9, 3)$ $N(6, 3)$

Show your work.

Solution: _____

Nets and Surface Area

Name: _____

Study the example problem showing how to find the area of a polygon. Then solve problems 1–8.

Example

Gary drew a picture of a nameplate that he plans to make. He wants to find the area of the nameplate. How could Gary break apart the figure to find its area?

4 cm

13 cm
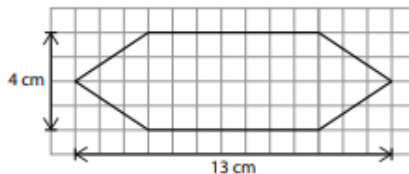
Gary separates the figure he drew into two triangles and a rectangle.
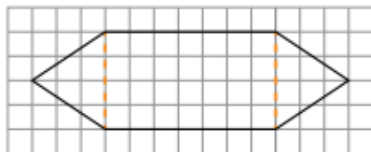
1 Label the dimensions of the rectangle and one of the triangles.

☐ cm

☐ cm   ☐ cm   ☐ cm

☐ cm
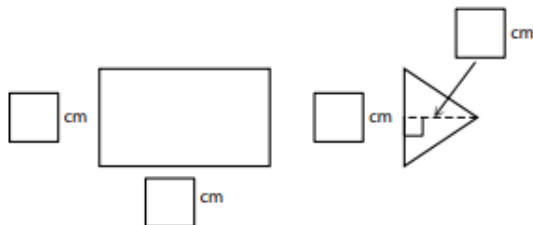
2 What is the area of the rectangle?

_____

3 What is the area of the triangle?

_____

4 What is the area of the nameplate? Write an equation to show your solution.

_____

Solve.

Use the trapezoid to solve problems 5–6.

5 Separate the trapezoid into figures whose areas you can find. Label the dimensions.

6 What is the area of the trapezoid?

Show your work.

5 m

1.5 m

3 m
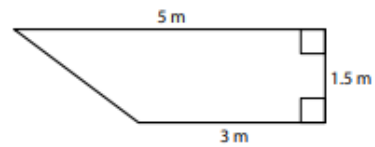
Solution: _____

7 Hector drew three rectangles to show the letter H on his notebook. Use the rectangles to find the area of the letter he drew.

Show your work.

4 cm

4.5 cm

4.5 cm

12 cm

10 cm
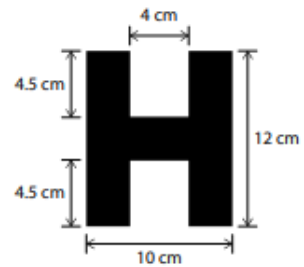
Solution: _____

8 Pat says that the parallelograms below do not have the same area. Is she correct? Explain.

_____

_____

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Name: _____
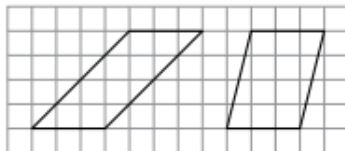
Surface Area of a Rectangular Prism

Study the example showing how to find the surface area of a rectangular prism. Then solve problems 1–8.

Example

Kanene wants to know how much wrapping paper she needs to cover this box. How much wrapping paper does she need?

4 in.
10 in.   5 in.
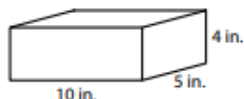
You can use a net to help you solve the problem.

10 in.
4 in.   4 in.   10 in.
5 in.   5 in.
4 in.
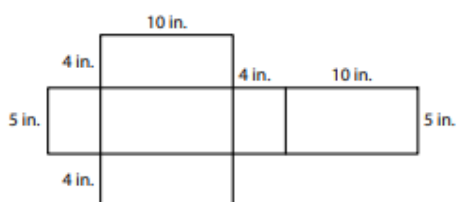
1 Complete the table to find the area of each face.

Face	Length (in.)	Width (in.)	Area (sq in.)
Top	10	5	
Bottom			
Front			
Back			
Right side			
Left side			

2 Which pairs of faces have the same areas?

_____

3 What is the surface area of the box? Use your answer to problem 2 to write an equation.

_____

4 What is the relationship between the surface area of a rectangular prism and the area of each face?

_____

Lesson 24 Nets and Surface Area   **269**

Solve.

5 Carl drew this net for a wooden shed that he will build. He wants to protect the wood against the weather by using a sealant on all of the outside surfaces, including the bottom. Will a container of sealant that covers 200 square feet be enough to protect the outside surfaces?

Show your work.
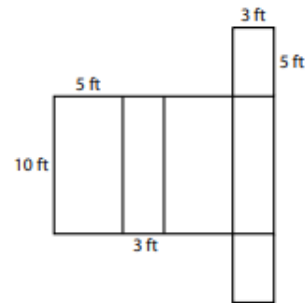
Solution: _____

6 Susana is making a small box. The 20-cm by 20-cm front of the box will be glass. The other faces will be wood. How much wood does Susana need to make the box?

Show your work.
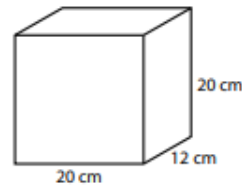
Solution: _____

7 The surface area of a cube is 216 square meters. What is the height of the cube? Explain.

_____

_____

8 Mike says that if he doubles each dimension of any rectangular prism, the surface area also doubles. Is Mike correct? Give an example to support your answer.

_____

_____

_____

_____

Name: _____

Surface Area of a Triangular Prism

Study the example showing how to find the surface area of a triangular prism. Then solve problems 1–7.

Example

What is the surface area of the triangular prism shown?

You can draw and label a net of the prism to help you.
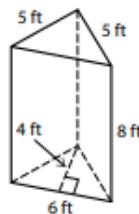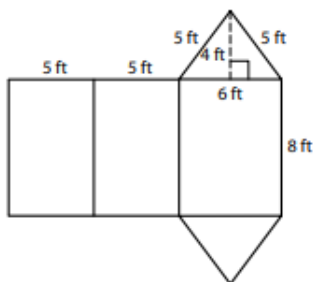
1 Complete the table to find the area of each face.

Face	Base (ft)	Height (ft)	Area (sq ft)
Triangle	6	4	
Triangle			
Rectangle			
Rectangle			
Rectangle			

2 Why do the rectangular faces have different areas?

_____

_____

_____

3 What is the surface area of the triangular prism? Write two equations to represent the surface area.

_____

Solve.

Use the following situation to solve problems 4–6.

Jane is decorating a paperweight in the shape of a triangular prism. The diagram shows its dimensions.
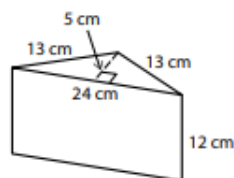
4 Label the net of the triangular prism to show the dimensions of the faces.

5 What is the surface area of the paperweight?

Show your work.
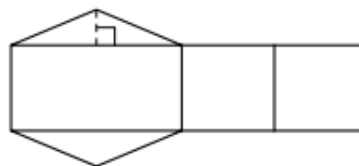
Solution: _____

6 Amad used the expression $2\left(\frac{1}{2} \cdot 24 \cdot 5\right) + 3(13 \cdot 12)$ to find the surface area of the paperweight. What is wrong with his expression? Correct Amad's mistake.

_____

_____

_____

7 The picture shows the dimensions of one base of a triangular prism. The height of the prism is 2 meters. What is the surface area of the triangular prism? Explain how to find the answer.

_____

_____

_____

_____

_____

_____

Name: _____
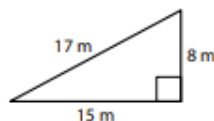
Surface Area of a Pyramid

Study the example problem showing how to find the surface area of a pyramid. Then solve problems 1–8.

Example

What is the surface area of the pyramid?

You can draw and label a net to help you.
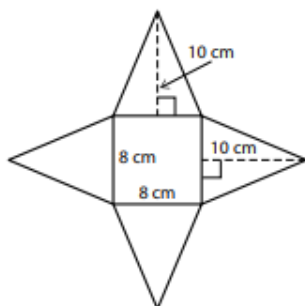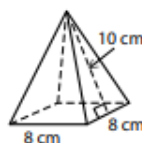
1 Complete the table to find the area of each face.

Face	Base (cm)	Height (cm)	Area (sq cm)
Triangle	8	10	
Triangle			
Triangle			
Triangle			
Square			

2 Describe the number of faces and their shapes.

_____

3 Use formulas to explain how to find the area of each face.

_____

_____

4 What is the surface area of the pyramid? Write an equation to represent the surface area.

_____

Solve.

Use the following situation to solve problems 5–7.

Marcos is making a pyramid in his wood shop class. The base of the pyramid is a rectangle.

10 cm   12 cm

12 cm   8 cm
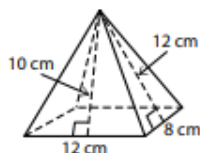
5 Label the net of the pyramid with the dimensions of the faces.

6 What is the surface area of the pyramid?

Show your work.
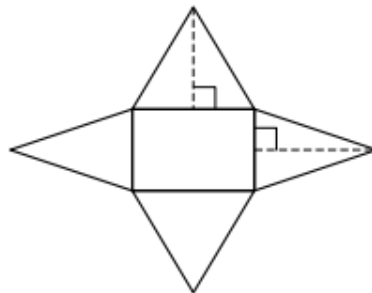
Solution: _____

7 Yolanda used the expression $\left(\frac{1}{2} \cdot 12 \cdot 10\right) + \left(\frac{1}{2} \cdot 8 \cdot 12\right) + (12 \cdot 8)$ to find the surface area of the pyramid. What is wrong with the expression? Correct Yolanda's mistake.

_____

_____

8 The surface area of a pyramid is 540 square inches. Its base is a square with a side length of 10 inches. What is the height of one of the triangular faces of the pyramid? Explain how to find the answer.
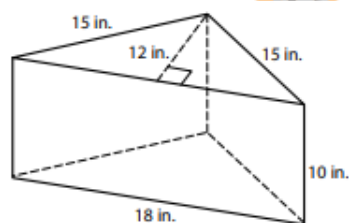
_____

_____

_____

_____

_____

Name: _____

Nets and Surface Area

Solve the problems.

1 Rita keeps her craft supplies in a container without a top. The container is a triangular prism. Rita plans to cover the outside of the container with decorative paper. How much paper does she need?
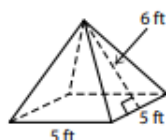
How many faces should you include in your calculations?

Show your work.

15 in.
12 in.
15 in.
10 in.
18 in.

Solution: _____

2 Look at the pyramid below.

What do you need to know to find the surface area of a pyramid?

6 ft
5 ft
5 ft

Tell whether each statement about the pyramid is *True* or *False*.

a. The area of each triangular face is 30 square feet. ☐ True ☐ False

b. The surface area of the pyramid is 85 square feet. ☐ True ☐ False

c. A net of the pyramid would have three triangular faces. ☐ True ☐ False

d. The area of the base is 25 square feet. ☐ True ☐ False

Solve.

3 The net represents a rectangular prism. Which expression represents the surface area? Select all that apply.

3 cm

5 cm

2 cm

5 cm

How do you find the surface area of a rectangular prism?

A $(3 \cdot 5) + (5 \cdot 2) + (2 \cdot 3)$

B $15 + 15 + 6 + 6 + 10$

C $2(3 \cdot 5) + 2(3 \cdot 2) + 2(2 \cdot 5)$
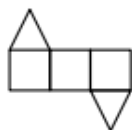
D $2(10) + 2(6) + 2(15)$

Horus chose **A** as the correct answer. How did he get that answer?

_____

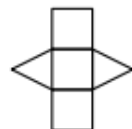**4** Does the diagram represent the net of a triangular prism? Choose *Yes* or *No*.

How many faces on a triangular prism are triangles?

a.

☐ Yes ☐ No

b.

☐ Yes ☐ No

c.

☐ Yes ☐ No

5 Design your own pyramid. Describe your pyramid, and then choose its dimensions and find its surface area.

_____

_____