

Shelby County Schools
Extended Learning Guide



Algebra I

9-3 Study Guide and Intervention

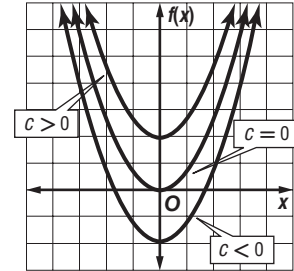
Transformations of Quadratic Functions

Translations A **translation** is a change in the position of a figure either up, down, or diagonal. When a constant c is added to or subtracted from the parent function, the resulting function $f(x) \pm c$ is a translation of the graph up or down.

The graph of $f(x) = x^2 + c$ translates the graph of $f(x) = x^2$ vertically.

If $c > 0$, the graph of $f(x) = x^2$ is translated $|c|$ units up.

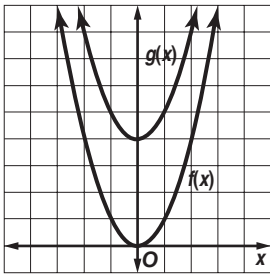
If $c < 0$, the graph of $f(x) = x^2$ is translated $|c|$ units down.



Example Describe how the graph of each function is related to the graph of $f(x) = x^2$.

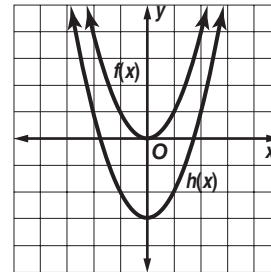
a. $g(x) = x^2 + 4$

The function can be written as $f(x) = x^2 + c$. The value of c is 4, and $4 > 0$. Therefore, the graph of $g(x) = x^2 + 4$ is a translation of the graph of $f(x) = x^2$ up 4 units.



b. $h(x) = x^2 - 3$

The function can be written as $f(x) = x^2 + c$. The value of c is -3 , and $-3 < 0$. Therefore, the graph of $g(x) = x^2 - 3$ is a translation of the graph of $f(x) = x^2$ down 3 units.



Exercises

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

1. $g(x) = x^2 + 1$

2. $h(x) = x^2 - 6$

3. $g(x) = x^2 - 1$

4. $h(x) = 20 + x^2$

5. $g(x) = -2 + x^2$

6. $h(x) = -\frac{1}{2} + x^2$

7. $g(x) = x^2 + \frac{8}{9}$

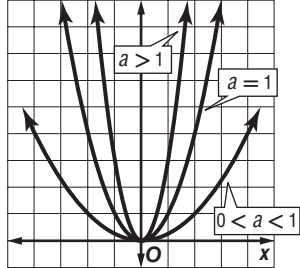
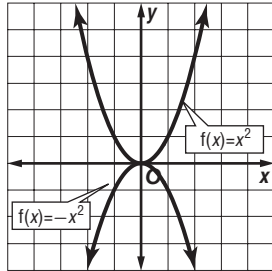
8. $h(x) = x^2 - 0.3$

9. $g(x) = x^2 - 4$

9-3 Study Guide and Intervention *(continued)*

Transformations of Quadratic Functions

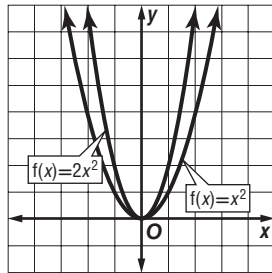
Dilations and Reflections A **dilation** is a transformation that makes the graph narrower or wider than the parent graph. A **reflection** flips a figure over the x - or y -axis.

<p>The graph of $f(x) = ax^2$ stretches or vertically compresses the graph of $f(x) = x^2$.</p> <p>If $a > 1$, the graph of $f(x) = x^2$ is stretched vertically.</p> <p>If $0 < a < 1$, the graph of $f(x) = x^2$ is compressed vertically.</p>	
<p>The graph of the function $-f(x)$ flips the graph of $f(x) = x^2$ across the x-axis.</p> <p>The graph of the function $f(-x)$ flips the graph of $f(x) = x^2$ across the y-axis.</p>	

Example Describe how the graph of each function is related to the graph of $f(x) = x^2$.

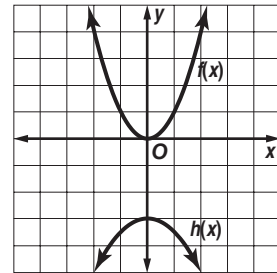
a. $g(x) = 2x^2$

The function can be written as $f(x) = ax^2$ where $a = 2$. Because $|a| > 1$, the graph of $y = 2x^2$ is the graph of $y = x^2$ that is stretched vertically.



b. $h(x) = -\frac{1}{2}x^2 - 3$

The negative sign causes a reflection across the x -axis. Then a dilation occurs in which $a = \frac{1}{2}$ and a translation in which $c = -3$. So the graph of $y = -\frac{1}{2}x^2 - 3$ is reflected across the x -axis, dilated wider than the graph of $f(x) = x^2$, and translated down 3 units.



Exercises

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

1. $h(x) = -5x^2$

2. $g(x) = -x^2 + 1$

3. $g(x) = -\frac{1}{4}x^2 - 1$

9-3 Skills Practice

Transformations of Quadratic Functions

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

1. $g(x) = x^2 + 2$

2. $h(x) = -1 + x^2$

3. $g(x) = x^2 - 8$

4. $h(x) = 7x^2$

5. $g(x) = \frac{1}{5}x^2$

6. $h(x) = -6x^2$

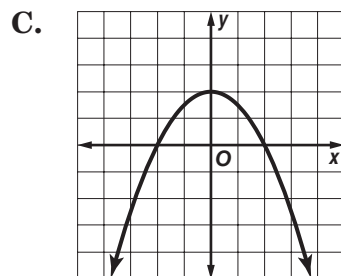
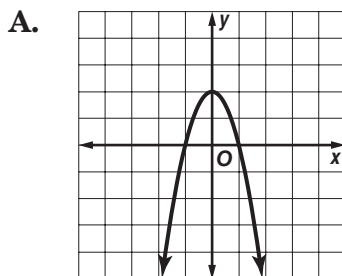
7. $g(x) = -x^2 + 3$

8. $h(x) = 5 - \frac{1}{2}x^2$

9. $g(x) = 4x^2 + 1$

Match each equation to its graph.

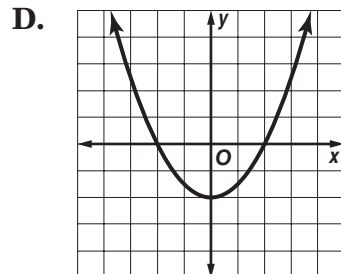
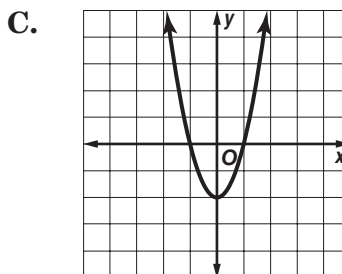
10. $y = 2x^2 - 2$



11. $y = \frac{1}{2}x^2 - 2$

12. $y = -\frac{1}{2}x^2 + 2$

13. $y = -2x^2 + 2$



9-3 Practice

Transformations of Quadratic Functions

Describe how the graph of each function is related to the graph of $f(x) = x^2$.

1. $g(x) = 10 + x^2$

2. $h(x) = -\frac{2}{5} + x^2$

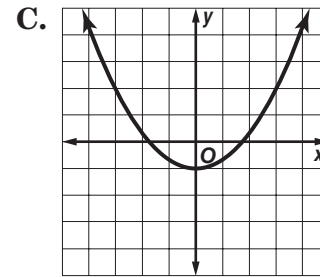
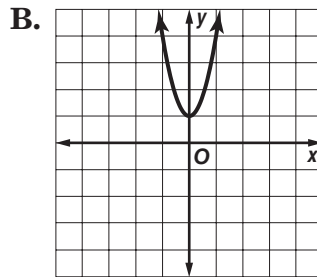
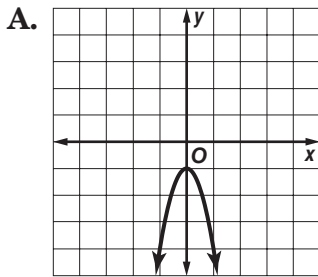
3. $g(x) = 9 - x^2$

4. $h(x) = 2x^2 + 2$

5. $g(x) = -\frac{3}{4}x^2 - \frac{1}{2}$

6. $h(x) = 4 - 3x^2$

Match each equation to its graph.



7. $y = -3x^2 - 1$

8. $y = -\frac{1}{3}x^2 + 1$

9. $y = 3x^2 + 1$

List the functions in order from the most vertically stretched to the least vertically stretched graph.

10. $f(x) = 3x^2, g(x) = \frac{1}{2}x^2, h(x) = -2x^2$

11. $f(x) = \frac{1}{2}x^2, g(x) = -\frac{1}{6}, h(x) = 4x^2$

12. PARACHUTING Two parachutists jump from two different planes as part of an aerial show. The height h_1 of the first parachutist in feet after t seconds is modeled by the function $h_1 = -16t^2 + 5000$. The height h_2 of the second parachutist in feet after t seconds is modeled by the function $h_2 = -16t^2 + 4000$.

- What is the parent function of the two functions given?
- Describe the transformations needed to obtain the graph of h_1 from the parent function.
- Which parachutist will reach the ground first?

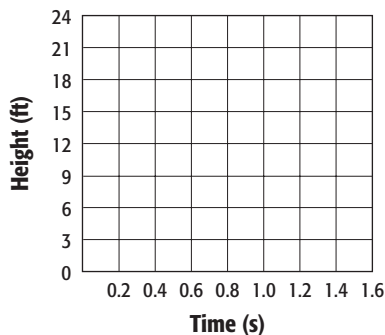
9-3 Word Problem Practice

Transformations of Quadratic Functions

1. SPRINGS The potential energy stored in a spring is given by the function $U_s = \frac{1}{2}kx^2$ where k is a constant known as the spring constant, and x is the distance the spring is stretched or compressed from its initial position. Explain how the graph of the function for a spring where $k = 2$ newtons/meter differs from the graph of the function for a spring where $k = 10$ newtons/meter.

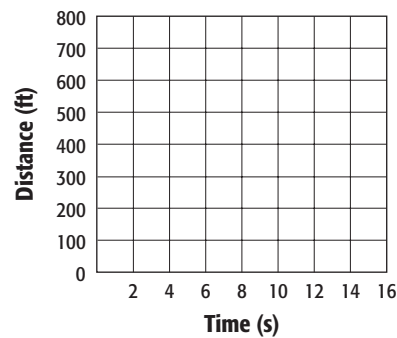
2. CYLINDERS The volume of a cylinder is given by the equation $V = \pi r^2 \ell$, where r is the radius and ℓ is the length. A poster company wants to increase the volume of its 1-foot long shipping tube by 2-cubic feet without increasing the length. Explain how the graph of the original tube differs from the graph of the newly redesigned tube.

3. PHYSICS A ball is dropped from a height of 20 feet. The function $h = -16t^2 + 20$ models the height of the ball in feet after t seconds. Graph the function and compare this graph to the graph of its parent function.



4. ACCELERATION The distance d in feet a car accelerating at 6 ft/s^2 travels after t seconds is modeled by the function $d = 3t^2$. Suppose that at the same time the first car begins accelerating, a second car begins accelerating at 4 ft/s^2 exactly 100 feet down the road from the first car. The distance traveled by second car is modeled by the function $d = 2t^2 + 100$.

a. Graph and label each function on the same coordinate plane.



b. Explain how each graph is related to the graph of $f(x) = x^2$.

c. After how many seconds will the first car pass the second car?

9-4 Study Guide and Intervention

Solving Quadratic Equations by Completing the Square

Complete the Square Perfect square trinomials can be solved quickly by taking the square root of both sides of the equation. A quadratic equation that is not in perfect square form can be made into a perfect square by a method called **completing the square**.

Completing the Square

To complete the square for any quadratic equation of the form $x^2 + bx$:

Step 1 Find one-half of b , the coefficient of x .

Step 2 Square the result in Step 1.

Step 3 Add the result of Step 2 to $x^2 + bx$.

$$x^2 + bx + \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2$$

Example Find the value of c that makes $x^2 + 2x + c$ a perfect square trinomial.

Step 1 Find $\frac{1}{2}$ of 2. $\frac{2}{2} = 1$

Step 2 Square the result of Step 1. $1^2 = 1$

Step 3 Add the result of Step 2 to $x^2 + 2x$. $x^2 + 2x + 1$.

Thus, $c = 1$. Notice that $x^2 + 2x + 1$ equals $(x + 1)^2$

Exercises

Find the value of c that makes each trinomial a perfect square.

1. $x^2 + 10x + c$

2. $x^2 + 14x + c$

3. $x^2 - 4x + c$

4. $x^2 - 8x + c$

5. $x^2 + 5x + c$

6. $x^2 + 9x + c$

7. $x^2 - 3x + c$

8. $x^2 - 15x + c$

9. $x^2 + 28x + c$

10. $x + 22x + c$

9-4 Study Guide and Intervention *(continued)***Solving Quadratic Equations by Completing the Square**

Solve by Completing the Square Since few quadratic expressions are perfect square trinomials, the method of **completing the square** can be used to solve some quadratic equations. Use the following steps to complete the square for a quadratic expression of the form $ax^2 + bx$.

Step 1	Find $\frac{b}{2}$.
Step 2	Find $\left(\frac{b}{2}\right)^2$.
Step 3	Add $\left(\frac{b}{2}\right)^2$ to $ax^2 + bx$.

Example Solve $x^2 + 6x + 3 = 10$ by completing the square.

$$x^2 + 6x + 3 = 10 \quad \text{Original equation}$$

$$x^2 + 6x + 3 - 3 = 10 - 3 \quad \text{Subtract 3 from each side.}$$

$$x^2 + 6x = 7 \quad \text{Simplify.}$$

$$x^2 + 6x + 9 = 7 + 9 \quad \text{Since } \left(\frac{6}{2}\right)^2 = 9, \text{ add 9 to each side.}$$

$$(x + 3)^2 = 16 \quad \text{Factor } x^2 + 6x + 9.$$

$$x + 3 = \pm 4 \quad \text{Take the square root of each side.}$$

$$x = -3 + 4 \quad \text{Simplify.}$$

$$x = -3 + 4 \quad \text{or} \quad x = -3 - 4$$

$$= 1 \quad \quad \quad = -7$$

The solution set is $\{-7, 1\}$.

Exercises

Solve each equation by completing the square. Round to the nearest tenth if necessary.

1. $x^2 - 4x + 3 = 0$

2. $x^2 + 10x = -9$

3. $x^2 - 8x - 9 = 0$

4. $x^2 - 6x = 16$

5. $x^2 - 4x - 5 = 0$

6. $x^2 - 12x = 9$

7. $x^2 + 8x = 20$

8. $x^2 = 2x + 1$

9. $x^2 + 20x + 11 = -8$

10. $x^2 - 1 = 5x$

11. $x^2 = 22x + 23$

12. $x^2 - 8x = -7$

13. $x^2 + 10x = 24$

14. $x^2 - 18x = 19$

15. $x^2 + 16x = -16$

16. $4x^2 = 24 + 4x$

17. $2x^2 + 4x + 2 = 8$

18. $4x^2 = 40x + 44$

9-4 Practice**Solving Quadratic Equations by Completing the Square**

Find the value of c that makes each trinomial a perfect square.

1. $x^2 - 24x + c$

2. $x^2 + 28x + c$

3. $x^2 + 40x + c$

4. $x^2 + 3x + c$

5. $x^2 - 9x + c$

6. $x^2 - x + c$

Solve each equation by completing the square. Round to the nearest tenth if necessary.

7. $x^2 - 14x + 24 = 0$

8. $x^2 + 12x = 13$

9. $x^2 - 30x + 56 = -25$

10. $x^2 + 8x + 9 = 0$

11. $x^2 - 10x + 6 = -7$

12. $x^2 + 18x + 50 = 9$

13. $3x^2 + 15x - 3 = 0$

14. $4x^2 - 72 = 24x$

15. $0.9x^2 + 5.4x - 4 = 0$

16. $0.4x^2 + 0.8x = 0.2$

17. $\frac{1}{2}x^2 - x - 10 = 0$

18. $\frac{1}{4}x^2 + x - 2 = 0$

19. NUMBER THEORY The product of two consecutive even integers is 728. Find the integers.

20. BUSINESS Jaime owns a business making decorative boxes to store jewelry, mementos, and other valuables. The function $y = x^2 + 50x + 1800$ models the profit y that Jaime has made in month x for the first two years of his business.

a. Write an equation representing the month in which Jaime's profit is \$2400.

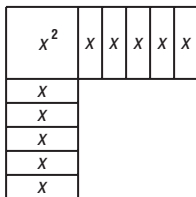
b. Use completing the square to find out in which month Jaime's profit is \$2400.

21. PHYSICS From a height of 256 feet above a lake on a cliff, Mikaela throws a rock out over the lake. The height H of the rock t seconds after Mikaela throws it is represented by the equation $H = -16t^2 + 32t + 256$. To the nearest tenth of a second, how long does it take the rock to reach the lake below? (*Hint:* Replace H with 0.)

9-4 Word Problem Practice

Solving Quadratic Equations by Completing the Square

1. INTERIOR DESIGN Modular carpeting is installed in small pieces rather than as a large roll so that only a few pieces need to be replaced if a small area is damaged. Suppose the room shown in the diagram below is being fitted with modular carpeting. Complete the square to determine the number of 1 ft by 1 ft squares of carpeting needed to finish the room. Fill in the missing terms in the corresponding equation below.



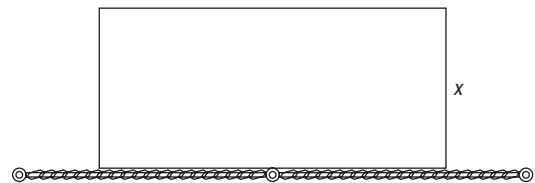
$$x^2 + 10x + \underline{\hspace{2cm}} = (x + \underline{\hspace{2cm}})^2$$

2. FALLING OBJECTS Keisha throws a rock down an old well. The distance d (in feet) the rock falls after t seconds can be represented by the equation $d = 16t^2 + 64t$. If the water in the well is 80 feet below ground, how many seconds will it take for the rock to hit the water?

3. MARS On Mars, the gravity acting on an object is less than that on Earth. On Earth, a golf ball hit with an initial upward velocity of 26 meters per second will reach a maximum height of about 34.5 meters. The height h of an object on Mars that leaves the ground with an initial velocity of 26 meters per second is given by the equation $h = -1.9t^2 + 26t$. Find the maximum height if the same golf ball is hit on Mars. Round your answer to the nearest tenth.

4. FROGS A frog sitting on a stump 3 feet high hops off and lands on the ground. During its leap, its height h (in feet) is given by $h = -0.5d^2 + 2d + 3$, where d is the distance from the base of the stump. How far is the frog from the base of the stump when it landed on the ground?

5. GARDENING Peg is planning a rectangular vegetable garden using 250 feet of fencing material. She only needs to fence three sides of the garden since one side borders an existing fence.



- Let x = the width of the rectangle. Write an expression to represent the area of the garden if she uses all the fencing material.
- Find the vertex of the equation and identify it as a maximum or a minimum.
- Interpret the vertex of the equation in terms of the situation.

9-5 Study Guide and Intervention**Solving Quadratic Equations by Using the Quadratic Formula**

Quadratic Formula To solve the standard form of the quadratic equation, $ax^2 + bx + c = 0$, use the **Quadratic Formula**.

Quadratic Formula	the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ that gives the solutions of $ax^2 + bx + c = 0$, where $a \neq 0$
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Example 1 Solve $x^2 + 2x = 3$ by using the Quadratic Formula.

Rewrite the equation in standard form.

$$x^2 + 2x = 3 \quad \text{Original equation}$$

$$x^2 + 2x - 3 = 3 - 3 \quad \text{Subtract 3 from each side.}$$

$$x^2 + 2x - 3 = 0 \quad \text{Simplify.}$$

Now let $a = 1$, $b = 2$, and $c = -3$ in the Quadratic Formula.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-3)}}{2(1)} \\ &= \frac{-2 \pm \sqrt{16}}{2} \\ x &= \frac{-2 + 4}{2} \quad \text{or} \quad x = \frac{-2 - 4}{2} \\ &= 1 \quad \quad \quad = -3 \end{aligned}$$

The solution set is $\{-3, 1\}$.

Example 2 Solve $x^2 - 6x - 2 = 0$ by using the Quadratic Formula. Round to the nearest tenth if necessary.

For this equation $a = 1$, $b = -6$, and $c = -2$.

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{6 \pm \sqrt{(-6)^2 - 4(1)(-2)}}{2(1)} \\ &= \frac{6 \pm \sqrt{44}}{2} \\ x &= \frac{6 \pm \sqrt{44}}{2} \quad \text{or} \quad x = \frac{6 - \sqrt{44}}{2} \\ &\approx 6.3 \quad \quad \quad \approx -0.3 \end{aligned}$$

The solution set is $\{-0.3, 6.3\}$.

Exercises

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $x^2 - 3x + 2 = 0$

2. $x^2 - 8x = -16$

3. $16x^2 - 8x = -1$

4. $x^2 + 5x = 6$

5. $3x^2 + 2x = 8$

6. $8x^2 - 8x - 5 = 0$

7. $-4x^2 + 19x = 21$

8. $2x^2 + 6x = 5$

9. $48x^2 + 22x - 15 = 0$

10. $8x^2 - 4x = 24$

11. $2x^2 + 5x = 8$

12. $8x^2 + 9x - 4 = 0$

13. $2x^2 + 9x + 4 = 0$

14. $8x^2 + 17x + 2 = 0$

9-5 Study Guide and Intervention *(continued)***Solving Quadratic Equations by Using the Quadratic Formula**

The Discriminant In the Quadratic Formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, the expression under the radical sign, $b^2 - 4ac$, is called the **discriminant**. The discriminant can be used to determine the number of real roots for a quadratic equation.

Case 1: $b^2 - 4ac < 0$	no real roots
Case 2: $b^2 - 4ac = 0$	one real root
Case 3: $b^2 - 4ac > 0$	two real roots

Example State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

a. $12x^2 + 5x = 4$

Write the equation in standard form.

$$12x^2 + 5x = 4 \quad \text{Original equation}$$

$$12x^2 + 5x - 4 = 4 - 4 \quad \text{Subtract 4 from each side.}$$

$$12x^2 + 5x - 4 = 0 \quad \text{Simplify.}$$

Now find the discriminant.

$$\begin{aligned} b^2 - 4ac &= (5)^2 - 4(12)(-4) \\ &= 217 \end{aligned}$$

Since the discriminant is positive, the equation has two real roots.

b. $2x^2 + 3x = -4$

$$2x^2 + 3x = -4 \quad \text{Original equation}$$

$$2x^2 + 3x + 4 = -4 + 4 \quad \text{Add 4 to each side.}$$

$$2x^2 + 3x + 4 = 0 \quad \text{Simplify.}$$

Find the discriminant.

$$\begin{aligned} b^2 - 4ac &= (3)^2 - 4(2)(4) \\ &= -23 \end{aligned}$$

Since the discriminant is negative, the equation has no real roots.

Exercises

State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

1. $3x^2 + 2x - 3 = 0$

2. $3x^2 - 7x - 8 = 0$

3. $2x^2 - 10x - 9 = 0$

4. $4x^2 = x + 4$

5. $3x^2 - 13x = 10$

6. $6x^2 - 10x + 10 = 0$

7. $2x^2 - 20 = -x$

8. $6x^2 = -11x - 40$

9. $9 - 18x + 9x^2 = 0$

10. $12x^2 + 9 = -6x$

11. $9x^2 = 81$

12. $16x^2 + 16x + 4 = 0$

13. $8x^2 + 9x = 2$

14. $4x^2 - 4x + 4 = 3$

15. $3x^2 - 18x = -14$

9-5 Practice**Solving Quadratic Equations by Using the Quadratic Formula**

Solve each equation by using the Quadratic Formula. Round to the nearest tenth if necessary.

1. $x^2 + 2x - 3 = 0$

2. $x^2 + 8x + 7 = 0$

3. $x^2 - 4x + 6 = 0$

4. $x^2 - 6x + 7 = 0$

5. $2x^2 + 9x - 5 = 0$

6. $2x^2 + 12x + 10 = 0$

7. $2x^2 - 9x = -12$

8. $2x^2 - 5x = 12$

9. $3x^2 + x = 4$

10. $3x^2 - 1 = -8x$

11. $4x^2 + 7x = 15$

12. $1.6x^2 + 2x + 2.5 = 0$

13. $4.5x^2 + 4x - 1.5 = 0$

14. $\frac{1}{2}x^2 + 2x + \frac{3}{2} = 0$

15. $3x^2 - \frac{3}{4}x = \frac{1}{2}$

State the value of the discriminant for each equation. Then determine the number of real solutions of the equation.

16. $x^2 + 8x + 16 = 0$

17. $x^2 + 3x + 12 = 0$

18. $2x^2 + 12x = -7$

19. $2x^2 + 15x = -30$

20. $4x^2 + 9 = 12x$

21. $3x^2 - 2x = 3.5$

22. $2.5x^2 + 3x - 0.5 = 0$

23. $\frac{3}{4}x^2 - 3x = -4$

24. $\frac{1}{4}x^2 = -x - 1$

25. CONSTRUCTION A roofer tosses a piece of roofing tile from a roof onto the ground 30 feet below. He tosses the tile with an initial downward velocity of 10 feet per second.

a. Write an equation to find how long it takes the tile to hit the ground. Use the model for vertical motion, $H = -16t^2 + vt + h$, where H is the height of an object after t seconds, v is the initial velocity, and h is the initial height. (*Hint*: Since the object is thrown down, the initial velocity is negative.)

b. How long does it take the tile to hit the ground?

26. PHYSICS Lupe tosses a ball up to Quyen, waiting at a third-story window, with an initial velocity of 30 feet per second. She releases the ball from a height of 6 feet. The equation $h = -16t^2 + 30t + 6$ represents the height h of the ball after t seconds. If the ball must reach a height of 25 feet for Quyen to catch it, does the ball reach Quyen? Explain. (*Hint*: Substitute 25 for h and use the discriminant.)

9-5 Word Problem Practice

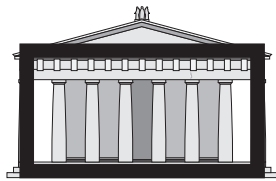
Solving Quadratic Equations by Using the Quadratic Formula

1. BUSINESS Tanya runs a catering business. Based on her records, her weekly profit can be approximated by the function $f(x) = x^2 + 2x - 37$, where x is the number of meals she caters. If $f(x)$ is negative, it means that the business has lost money. What is the least number of meals that Tanya needs to cater in order to have a profit?

2. AERONAUTICS At liftoff, the space shuttle *Discovery* has a constant acceleration of 16.4 feet per second squared and an initial velocity of 1341 feet per second due to the rotation of Earth. The distance *Discovery* has traveled t seconds after liftoff is given by the equation $d(t) = 1341t + 8.2t^2$. How long after liftoff has *Discovery* traveled 40,000 feet? Round your answer to the nearest tenth.

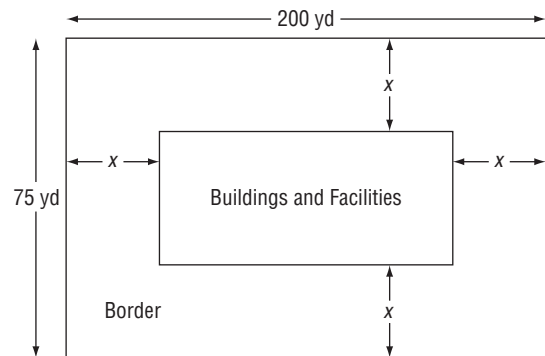
3. ARCHITECTURE

The Golden Ratio appears in the design of the Greek Parthenon because the width and height of the façade are related by the equation $\frac{W + H}{W} = \frac{W}{H}$. If the height of a model of the Parthenon is 16 inches, what is its width? Round your answer to the nearest tenth.



4. CRAFTS Madelyn cut a 60-inch pipe cleaner into two unequal pieces, and then she used each piece to make a square. The sum of the areas of the squares was 117 square inches. Let x = the length of one piece. Write and solve an equation to represent the situation and find the lengths of the two original pieces.

5. SITE DESIGN The town of Smallport plans to build a new water treatment plant on a rectangular piece of land 75 yards wide and 200 yards long. The buildings and facilities need to cover an area of 10,000 square yards. The town's zoning board wants the site designer to allow as much room as possible between each edge of the site and the buildings and facilities. Let x represent the width of the border.



- Use an equation similar to $A = \ell \times w$ to represent the situation.
- Write the equation in standard quadratic form.
- What should be the width of the border? Round your answer to the nearest tenth.

12-2 Reteaching

Frequency and Histograms

Sometimes it is helpful to break data up into intervals and evaluate the frequency of the intervals. The frequency can be displayed graphically in a histogram.

Problem

The ages of people attending the meeting are given below. What would a histogram representing the frequency of the various ages look like?

62, 52, 38, 55, 47, 42, 32, 46, 29, 37, 38, 49, 50, 56, 72, 68, 36, 28, 51, 49, 71

a. How do you know what the intervals should be?

Step 1: Find the difference between the greatest value, 72, and the least value, 28: $72 - 28 = 44$.

Step 2: Use this difference to determine the number of intervals.

The Interval		
If the number of years in the interval is:	Divide	Intervals
2	$\frac{44}{2} = 22$	22—too many
10	$\frac{44}{10} = 4.4$	4—too few
8	$\frac{44}{8} = 5.5$	6—good

There should be about 6 intervals of 8 years each.

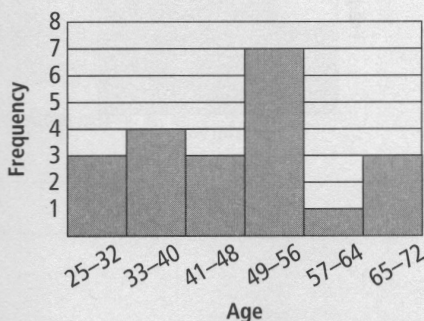
b. How can you make a frequency table for the data?

Since there should be 6 intervals of 8 years each, the table should reflect this. To find the frequency of each age range, simply count the number of people in the data set that are in that age range.

Age of Attendees	
Age	Frequency
25–32	3
33–40	4
41–48	3
49–56	7
57–64	1
65–72	3

c. How can you represent the data in a histogram?

Use the frequency table to make the histogram. Each bar should be drawn to correspond with the frequency.



12-2 **Reteaching** (continued)

Frequency and Histograms

Exercises

Use the data to make a frequency table.

1. height (in.): 78 56 99 82 108 65 76
82 95 100 85 73 99

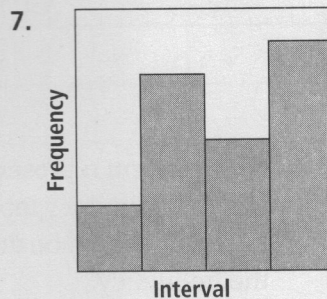
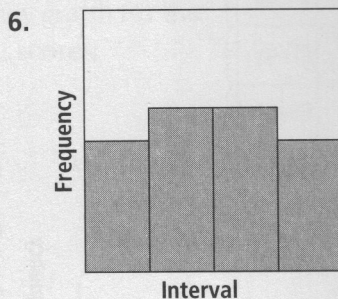
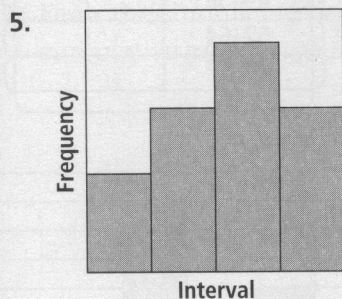
2. distance (mi): 12 21 19 25 8 17 16
29 31 20 5 13

Use the data to make a histogram.

3. test scores: 99 72 65 83 87 76 94
80 67 59 73 91 70 82

4. goals per game: 2 1 4 2 2 1 1 3
1 3 2 2 1 3 1 1 2

Tell whether each histogram is *uniform*, *symmetric*, or *skewed*.



12-2

Practice

Form K

Frequency and Histograms

Use the data to make a frequency table.

1. strikeouts per game: 10 5 1 7 6 7 5 3 9 8 4 6 6 8 2

2. weight (kg): 27 19 21 31 29 24 18 19 30 25 26 20 18 27

Use the data to make a histogram.

3. number of lawns mowed:

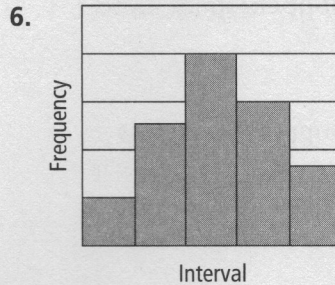
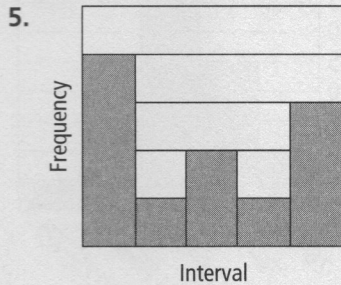
12 15 10 22 7 12 18 14 9 11 5 14 19

4. price per foot of rope:

\$3 \$7 \$5 \$9 \$10 \$7 \$12

\$8 \$4 \$6 \$5 \$7 \$7 \$12

Tell whether each histogram is *uniform*, *symmetric*, or *skewed*.



12-2

Practice (continued)

Form K

Frequency and Histograms

Use the data to make a cumulative frequency table.

7. minutes used per month:

675 815 747 508 642 588 821 818 689 590 777

8. weight (lb):

3.5 2.75 1.1 5.25 6.1 2.4 1.8 4.2 3.15 5.25 3.9 2.5

9. Rainfall amounts, in centimeters, are listed below. Make a histogram of the data that uses intervals of 2.

12 6 9 6 2 10 7 12 3

3 4 5 2 11 8 6 3 7

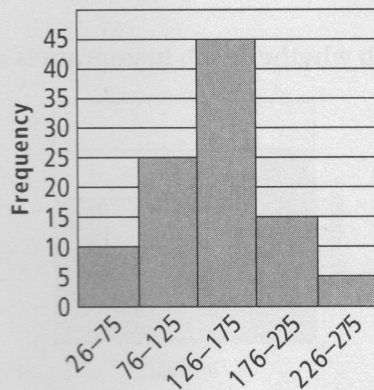
The histogram below shows the amount of money that 100 shoppers spent on groceries this week.

10. Which interval represents the greatest number of shoppers?

11. How many shoppers spent more than \$175?

12. How many shoppers spent less than \$126?

Money Spent on Groceries



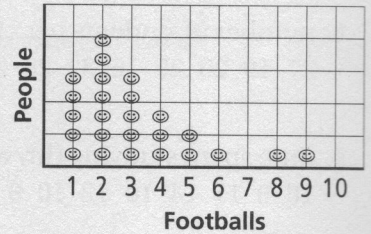
12-3 Reteaching

Measures of Central Tendency and Dispersion

Mean, median, and mode are all measures of central tendency which are different ways of describing a set of data.

Problem

The chart at the right shows the number of people in a small town who own a certain number of footballs. So, for example, there are 5 people who own 1 football and 7 people who own 2 footballs. Find the mean, median, and mode.



Using the data presented in the chart, you can determine different measures of central tendency.

The mode is the number that appears most often. For this data, the mode is 2.

The median is the middle number. Since there are 25 people, the middle is the thirteenth person. The thirteenth person has 3 footballs. Therefore, the median is 3.

To find the mean, calculate the total number of footballs

$$1 \times 5 + 2 \times 7 + 3 \times 5 + 4 \times 3 + 5 \times 2 + 1 \times 6 + 1 \times 8 + 1 \times 9 = 79$$

Now divide that by the number of people, $\frac{79}{25} = 3.16$.

You can use what you know about mean to find a data value.

Problem

Sheralee has scored 15, 23, and 17 points in the first three basketball games of the season. How many points does she need to score in the fourth game to bring her points per game average up to 20 points per game?

First, how many points total does she need to score in the 4 games?

$$20 \times 4 = 80 \text{ points}$$

Now how many points has she scored through 3 games?

$$15 + 23 + 17 = 55 \text{ points}$$

So, she needs to score $80 - 55 = 25$ points in the fourth game.

You can check your answer by finding the average number points she scores if she scored 25 points in the fourth game.

$$\frac{15 + 23 + 17 + 25}{4} = \frac{80}{4} = 20 \checkmark$$

The answer checks.

12-3

Reteaching (continued)

Measures of Central Tendency and Dispersion

Exercises

Find the mean, median, and mode of each data set. Which measures of central tendency best describes the data?

- number of students per class:
27 19 20 25 16 32 28 20
- temperatures ($^{\circ}\text{F}$):
 67° 58° 67° 70° 69° 61° 65°
- time spent studying (hr/week):
10 8 11 14 10 12 10 9 8
- salaries (\$):
35,000 32,000 41,000 28,000 35,000

Find the value of x so that the data set has the given mean.

- 32, 48, 56, 40, x ; mean 42.6
- 1.2, 6.5, 3.3, 4.9, x ; mean 3.34
- 2.85, 12.6, 8.57, 10.1, x ; mean 9.024
- 112.5, 68.9, 45.2, 85.4, x ; mean 82.4

9. The line plot at the right shows test scores Cheryl has received so far in the semester. Her goal is to have a 91% test average at the end of the semester. What does she need to score on her final test in order to achieve her goal?

Test Scores									
X									X
X	X								X
88	89	90	91	92	93	94			

Find the range and mean of each data set. Use your results to compare the two data sets.

- Set M: 25 36 31 28 30
Set N: 15 22 34 18 25
- Set O: 2.6 5.1 3.7 4.8 3.2
Set P: 4.8 1.3 6.7 5 4.5

Find the mean, median, mode, and range of each data set if you perform the given operation on each data value.

- 11, 14, 9, 7, 11; multiply by 2
- 4.6, 7.3, 5.8, 6.5, 5.8; add 7
- 127, 115, 135, 115, 142; divide by 5
- 22.3, 18, 13.6, 15.2, 22.3; subtract 3.5

12-3

Practice

Form K

Measures of Central Tendency and Dispersion

Find the mean, median, and mode of each data set. Which measure of central tendency best describes the data?

- price per item:
\$12 \$8 \$15 \$20 \$15
- average rate (rev/sec):
75 81 79 68 79
- distance from the park (km):
2.2 3 4.1 3.5 3 2.5
- extra points kicked:
3 5 3 2 4

Find the value of x so that the data set has the given mean.

- 14, 10, 17, 9, x ; mean 14
- 101, 92, 76, 88, x ; mean 93
- 2.5, 6.1, 7.8, 3.7, x ; mean 5.04
- 22.6, 32.9, 29.7, 19.8, x ; mean 26.5
- 0.9, 1.6, 3.4, 0.5, x ; mean 1.4
- 77, 100, 92, 84, x ; mean 88
- One runner's times in the first six races of the year were 18.5, 18.2, 19, 18.75, 19.1, and 19 minutes. Another runner's times were 17.2, 18, 17.5, 18.75, 19, and 18.2 minutes. What are the range and mean of each runner's scores? Use your results to compare the runners' skills.

Find the range and mean of each data set. Use your results to compare the two data sets.

- Set A: 17 13 21 10 14
Set B: 12 16 15 11 13
- Set C: 5.6 4.8 3.7 7.1 9.2
Set D: 10 5.2 3.8 2.9 6.8
- The lengths of some pieces of lumber are 6 ft, 12 ft, 9 ft, 11 ft, and 8 ft. What are the mean, median, mode, and range of the lengths?

12-3**Practice** (continued)

Form K

Measures of Central Tendency and Dispersion

Find the mean, median, mode, and range of each data set after you perform the given operation on each data value. Round your answer to the nearest tenth.

15. 8, 12, 10, 7, 4, 7; subtract 1
16. 37, 32, 35, 41, 48, 36; add 3
17. 16.2, 16.7, 16.1, 16, 16.9, 16; add 3.5
18. 19, 12, 15, 14, 21, 9; divide by 3
19. 7.5, 14.2, 11.8, 19.6, 4.8, 11.8; multiply by 2
20. 2.1, 3.2, 4.5, 4.2, 3.2, 5.6; add -2
21. The lengths of George's last five road trips were 6 hr, 4 hr, 8 hr, 12 hr, and 10 hr. Jenny's last five trips were 6 hr, 9 hr, 2 hr, 15 hr, and 4 hr. Find the mean, median, mode, and range of George's trips and Jenny's trips. Use your results to compare each person's travels.
22. The goalkeeper had 7 saves, 4 saves, 9 saves, 12 saves, 2 saves, 7 saves, and 5 saves in the first seven games of the season. How many saves must the goalkeeper make in the next game to achieve an average of 8 saves per game?
23. Over six months, a phone bill averaged \$35 per month. The bills for the first five months were \$32, \$35, \$48, \$29, and \$31. What was the phone bill in the sixth month? Find the median, mode, and range of the six electric bills.

12-4 Reteaching

Box-and-Whisker Plots

Box-and-whisker plots are a visual representation of data that is divided into four parts or quartiles. The four divisions of the data are shown using the median, the upper and lower quartiles, and the greatest and least values of the data.

Problem

How is a box-and-whisker plot created for the data set shown below?

1, 2, 3, 3, 4, 4, 5, 6, 7, 7, 7, 8, 8, 9, 10

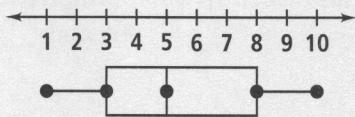
Step 1: Find the median of the data. 1, 2, 3, 3, 4, 4, 5, 5, 6, 7, 7, 8, 8, 9, 10

Step 2: To find the upper and the lower quartiles, find the median of the lower half and the upper half. 1, 2, 3, 3, 4, 4, 5, 5, 6, 7, 7, 8, 8, 9, 10
Q1 median Q3

Step 3: List the values for the minimum, Q_1 , median, Q_3 , and the maximum.

minimum = 1, $Q_1 = 3$, median = 5, $Q_3 = 8$, maximum = 10

Step 4: Draw a number line which includes all of the numbers in step 3. Below the number line, draw a box that runs from Q_1 to Q_3 . Next, draw a vertical line at the median. Draw a line from the minimum to Q_1 , the maximum to Q_3 , and bullets at all of the values listed in step 3.



Often standardized tests divide scores into percentile rank in order to compare the individual test-taker with everyone else who took the test.

Problem

When Stefan received his score for the college readiness exam, his score placed him in the 80th percentile rank. If there were 850 test takers, how many scored lower than Stefan?

This means that 80% of the test takers scored lower than Stefan. The percent proportion can be used to determine the number who scored lower.

$$\frac{80}{100} = \frac{x}{850}$$

Percent Proportion

$$100x = (80)(850)$$

Cross Products Property

$$100x = 68,000$$

Multiply.

$$x = 680$$

Divide each side by 100.

680 test takers scored lower than Stefan.

12-4 Reteaching (continued)

Box-and-Whisker Plots

Exercises

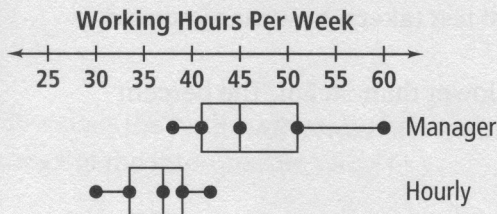
Find the minimum, first quartile, median, third quartile, and maximum of each data set.

1. 72, 78, 61, 48, 59, 76, 65
2. 11, 12, 8, 19, 16, 10, 14
3. 3.6, 5.7, 8.3, 6.5, 2.9, 4.3, 5.1
4. 155, 151, 158, 156, 155, 153, 158

Make a box-and-whisker plot to represent each set of data.

5. daily fair visitors: 2576 3255 1876 2285 3589 4277 996
6. computer prices: \$1499 \$699 \$999 \$2999 \$499 \$4499 \$3299
7. lengths (ft): 15 21 10 17 12 14 18

8. Use the box-and-whisker plot below. What does it tell you about the number of hours each type of employee works for the company per week? Explain.



9. In a certain city with a working population of 10,500, 8925 people earn less than \$75,000 per year. What is the percentile rank of someone who earns \$75,000 per year?

12-4**Practice**

Form K

Box-and-Whisker Plots

Find the minimum, first quartile, median, third quartile, and maximum of each data set.

1. 99 88 77 85 62 93 71

2. 22 19 32 35 28 25 33 24 27

3. 45.5 52.7 41 49.8 59 72.1 51.7 53.2

4. 7 7 11 14 15 13 19 17

5. 175 198 225 179 182 185 201 215

6. 42 58 18 37 51 49 32 61 45

Make a box-and-whisker plot to represent each set of data.

7. fair attendance: 2515 2725 2972 3125 3195 3250 3555

8. fundraiser revenue: \$195 \$275 \$295 \$185 \$210 \$115 \$340 \$285 \$195

9. swimmers practicing: 17 22 19 17 25 9 14

10. games won: 97 55 78 86 77 68 92 81

11. admission prices: \$14 \$17 \$10 \$12.50 \$19.50 \$25 \$15 \$9 \$11.50

12. height (in.): 66 58 80 72 65 70 62 66 59 60

12-4**Practice** (continued)

Form K

Box-and-Whisker Plots

13. Of 350 runners competing in a race, 50 run the race in less than or equal to 20 minutes. What is the percentile rank of the runners who finish in under 20 minutes?
14. Of 50 babies born, 5 weigh more than 10 pounds. What is the percentile rank of a baby that weighs 10 pounds?
15. Ten students earned the following scores on a test: 92, 73, 81, 90, 79, 66, 94, 83, 61, and 99. Which score has a percentile rank of 90? Which score has a percentile rank of 10?

Make box-and-whisker plots to compare the data sets.

16. Earned commission:

Dale's: \$150 \$125 \$145 \$175 \$105 \$100 \$200 \$180

Juanita's: \$155 \$185 \$215 \$205 \$170 \$165 \$195 \$200

17. Weekly cars sold:

Kathy's: 5 8 3 12 7 11 9 4 8

Samuel's: 9 2 5 10 9 7 7 6 10

18. Video length (min):

Training 1: 78 62 45 65 50 59 67 62 51 70

Training 2: 60 67 50 58 62 71 69 54 60 64

12-2 Study Guide and Intervention**Statistical Analysis****Measures of Central Tendency**

Term	Definition	Best Used When
Mean	sum of the data divided by number of items in the data	the data set has no outliers
Median	the middle number or mean of two middle numbers of the ordered data	the data set has outliers but no big gaps in the middle of the data
Mode	the number or numbers that occur most often	the data has many repeated numbers
Margin of Sampling Error	$\pm \frac{1}{\sqrt{n}}$, for a random sample of n items	estimating how well a sample represents the whole population

Example 1 Which measure of central tendency best represents the data and why?

{2.1, 21.5, 22.3, 22.8, 23.1, 159.4}

There are outliers, but no large gaps in the middle, the median best represents this data.

Example 2 What is the margin of sampling error and the likely interval that contains the percentage of the population?

Of 400 people surveyed in a national poll, 51% say they will vote for candidate González.

Since 400 people are surveyed, the margin of sampling error is $\pm \frac{1}{\sqrt{400}}$ or $\pm 5\%$. The percentage of people who are likely to vote for candidate González is the percentage found in the survey plus or minus 5%, so the likely interval is from 46% to 56%.

Exercises

Which measure of central tendency best represents the data, and why?

- {45, 16, 30, 45, 29, 45}
- {100, 92, 105, 496, 77, 121}
- {2.5, 99.5, 110.5, 76}
- {60, 50, 55, 62, 44, 65, 51}

- BOOKS** A survey of 28 random people found that 40% read at least three books each month. What is the margin of sampling error? What is the likely interval that contains the percentage of the population that reads at least three books each month?

12-2 Study Guide and Intervention (continued)

Statistical Analysis

Measures of Variation

Standard Deviation Formulas		
	Variable	Formula
For Samples	s	$\sqrt{\frac{\sum_{k=1}^n (x_n - \bar{x})^2}{n - 1}}$
For Populations	σ	$\sqrt{\frac{\sum_{k=1}^n (x_n - \mu)^2}{n}}$

Example For the following data, determine whether it is a sample or a population. Then find the standard deviation of the data. Round to the nearest hundredth.

The test scores of twelve students in a college mathematics course are displayed below.

Test Scores of Twelve Students Enrolled in a College Mathematics Course					
61	99	75	83	92	69
77	94	73	65	98	91

Because the scores of all 12 students enrolled are given, this is a population. Find the mean.

$$\mu = \frac{\sum_{k=1}^{12} x_k}{12} = \frac{977}{12} = 81.42$$

Next, take the sum of the squares of the differences between each score and the mean.

$$\Sigma [(61 - 81.42)^2 + (99 - 81.42)^2 + (75 - 81.42)^2 + \dots + (91 - 81.42)^2] = 2,095.55$$

Putting this into the standard deviation formula, $\sqrt{\frac{2095.55}{12}} \approx 13.21$.

Exercises

1. Determine whether each is a sample or a population. Then find the standard deviation of the data. Round to the nearest hundredth.

a.

The Test Scores of Some of the Females in a College History Course					
88	91	82	95	76	88
75	94	92	85	82	90

b.

The Age of All Students in the Chess Club					
14	17	15	14	15	16

12-2 Skills Practice**Statistical Analysis****Which measure of central tendency best represents the data, and why?**

1. {10.2, 11.5, 299.7, 15.5, 20}
2. {75, 60, 60, 71, 74.5, 60, 67, 72.5}
3. {200, 250, 225, 25, 268, 250, 7}
4. {410, 405, 397, 450, 376, 422, 401}

Determine whether the following represents a *population* or a *sample*.

5. a school lunch survey that asks every fifth student that enters the lunch room
6. Tenth graders at a high school are surveyed about school athletics.
7. a list of the test scores of all the students in a class
8. a list of the scores of 1000 students on an SAT test
9. **MOVIES** A survey of 728 random people found that 72% prefer comedies over romantic movies. What is the margin of sampling error and the likely interval that contains the percent of the population?

10. SPORTS A survey of 3441 random people in one U.S. state found that 80% watched College football games every weekend in the Fall. What is the margin of sampling error and the likely interval that contains the percent of the population?

11. Determine whether each is a sample or a population. Then find the standard deviation of the data. Round to the nearest hundredth.

a.

The Shoe Sizes of 12 Students at a High School					
4	8	5	6	6	5
9	7	10	7	9	8

b.

The Number of Sit Ups Completed by All Students in a Gym Class					
50	28	41	61	54	28
47	33	45	50	50	61
23	41	31	38	42	

12-2 Practice

Statistical Analysis

Which measure of central tendency best represents the data, and why?

1. {12.1, 14.9, 6.7, 10, 12.8, 14, 18}
2. {77.9, 101, 78.9, 105, 4.2, 110, 87.9}
3. {10, 14.7, 14.7, 21, 7.4, 14.7, 8, 14.7}
4. {29, 36, 14, 99, 16, 15, 12, 30}

Determine whether the following represents a *population* or a *sample*.

5. a list of the times every student in gym class took to run a mile
6. the test scores of seven students in a chemistry class are compared
7. friends compare the batting averages of players who are listed in their collections of baseball cards
8. every student in a high school votes in a class president election

9. CARS A survey of 56 random people in a small town found that 14% drive convertibles year-round. What is the margin of sampling error? What is the likely interval that contains the percentage of the population that drives convertibles year-round?

10. BEACHES A survey of 812 random people in Hawaii found that 57% went to the beach at least four times last July. What is the margin of sampling error? What is the likely interval that contains the percentage of the population that went to the beach at least four times last July?

11. Determine whether each is a sample or a population. Then find the standard deviation of the data. Round to the nearest hundredth.

a.

The Number of Wins for Each Player on a Tennis Team Last Season					
10	2	9	17	4	8
9	9	10	15	19	5

b.

The Number of Medals Earned by 18 High School Debaters					
7	10	4	5	10	9
11	5	6	4	4	3
12	7	8	5	3	9

12-2 Word Problem Practice

Statistical Analysis

- 1. BASEBALL** The following shows the number of wins for Major League Baseball's American League East teams at the end of the 2007 regular season.

Team	Number of Wins
Boston	96
New York	94
Toronto	83
Baltimore	69
Tampa Bay	66

The American League also has Central and Western divisions. If the data is used to study the number of wins for all American League baseball teams, is it a sample or a population?

- 2. VOTING** A poll was taken of registered voters to see if they planned to vote in the next presidential election. What is the margin of sampling error if the likely interval of those who plan on voting is between 46.5% and 49.5%?

- 3. APPLES** The following data set depicts the number of apples in 20 different bushel baskets: {80, 75, 68, 82, 77, 74, 81, 85, 73, 79, 75, 73, 80, 71, 82, 81, 77, 80, 78, 84}. What is the median number of apples in a bushel basket?

- 4. CARS** A police officer clocked the following speed of cars (in miles per hour) on the highway: {65, 61, 72, 54, 78, 61, 74, 75, 61, 55, 64, 66, 70}.

What is the mode of the data set?

- 5. SOLAR SYSTEM** For much of the 20th century, astronomers considered the solar system to have nine planets. The table below lists the masses of these planets.

Planet	Mercury	Venus	Earth	Mars	Jupiter
Mass (10 ²¹ tons)	0.364	5.37	6.58	0.708	2093

Planet	Saturn	Uranus	Neptune	Pluto
Mass (10 ²¹ tons)	627	95.7	113	0.0138

- Which measure of central tendency best represents the data, and why?
- What is the correct number for the measurement that best represents the data?
- What is the standard deviation of the data?
- How many of the planets have masses within the range of the central measure of the data plus or minus one standard deviation?

12-3 Study Guide and Intervention

Statistics and Parameters

Statistics and Parameters A **statistic** is a quantity calculated from a sample. A **parameter** is a characteristic of the population of a whole. Statistics can change from sample to sample while parameters do not.

Example Identify the sample and the population for each situation. Then describe the sample statistic and the population parameter.

- a. **At a local supermarket, a random sample of 50 shoppers is selected. The median amount spent at the supermarket is calculated for the sample.**

Sample: the group of 50 shoppers

Population: all shoppers at the supermarket

Sample statistic: median amount spent of customers in the sample

Population parameter: median amount spent at the supermarket

- b. **Every 20 minutes at a furniture factory, a finished sofa is pulled from the assembly line and checked for defects. The mean number of defective sofas of a day's sample is then calculated.**

Sample: the sofas checked for defects

Population: all sofas manufactured

Sample statistic: mean number of defective sofas in the sample

Population parameter: mean number of defective sofas manufactured

Exercises

Identify the sample and the population for each situation. Then describe the sample statistic and the population parameter.

- 1. WEATHER** A meteorologist places ten weather stations in a county to measure rainfall. The median annual rainfall is calculated for the sample.
- 2. BOTANY** A scientist randomly selects 20 trees in a forest. The mean height of the 20 trees is then calculated.
- 3. POLITICS** A political reporter randomly selects 25 congressional districts across the country. The mean number of votes cast in the 25 congressional districts is calculated.

12-3 Study Guide and Intervention *(continued)***Statistics and Parameters**

Statistical Analysis The **mean absolute deviation** is the average of the absolute values of the differences between the mean and each value in the data set. It is used to predict errors and judge equality. The **standard deviation** is the calculated value that shows how data deviate from the mean of the set of data. The **variance** of data is the square of the standard deviation.

Example

EMPLOYMENT Employees at a law firm keep track of how many hours they work each week: {44, 48, 44, 40, 59}.

a. Find the mean absolute deviation.

Step 1 Find the mean. For this set of data, the mean is 47.

Step 2 Find the sum of the absolute values of the differences between each value in the set of data and the mean.

$$|44 - 47| + |48 - 47| + |44 - 47| + |40 - 47| + |59 - 47| = 3 + 1 + 3 + 7 + 12 = 26$$

Step 3 Divide the sum by the number of values in the set of data:
 $26 \div 5 \neq 5.2$

The mean absolute deviation is 5.2.

b. Find the variance and standard deviation.

Step 1 To find the variance, square the difference between each number and the mean. Then divide by the number of values.

$$\begin{aligned} \sigma^2 &= \frac{(44 - 47)^2 + (48 - 47)^2 + (44 - 47)^2 + (40 - 47)^2 + (59 - 47)^2}{5} \\ &= \frac{(-3)^2 + (1)^2 + (-3)^2 + (-7)^2 + (12)^2}{5} = \frac{9 + 1 + 9 + 49 + 144}{5} = \frac{188}{5} \end{aligned}$$

Step 2 The standard deviation is the square root of the variance.

$$\begin{aligned} \sqrt{\sigma^2} &= \sqrt{\frac{188}{5}} \\ \sigma &\approx 6.13 \end{aligned}$$

The variance of the data set is $\frac{188}{5}$ and the standard deviation is approximately 6.13.

Exercises

Find the mean, mean absolute deviation, variance, and standard deviation for each set of data.

1. {2, 4, 9, 5}

2. {13, 17, 17, 22, 16}

12-3 Skills Practice**Statistics and Parameters**

Identify the sample and the population for each situation. Then describe the sample statistic and the population parameter.

- 1. RESTAURANTS** A restaurant randomly selects 10 patrons on Saturday night. The median amount spent on beverages is then calculated for the sample.
- 2. KITTENS** A veterinarian randomly selects 3 kittens from a litter. The mean weight of the 3 kittens is calculated.
- 3. PRODUCE** A produce clerk randomly selects 20 bags of apples from each week's shipment and counts the total number of apples in each bag. The mode number of apples is calculated for the sample.

Find the mean absolute deviation.

- 4. WILDLIFE** A researcher counts the number of river otters observed on each acre of land in a state park: {0, 10, 14, 6, 0, 8, 4}.
- 5. FISHING** A fisherman records the weight of each black bass he catches during a fishing trip: {12, 7, 8, 13, 6, 14}.
- 6. BUDGETING** Xavier keeps track of how much money he spends on gasoline each week: {20, 13, 26, 0, 33, 16, 18}.

Find the mean, variance, and standard deviation of each set of data.

- 7.** {2, 0, 10, 4}
- 8.** {6, 7, 6, 9}
- 9.** {10, 9, 13, 6, 7}
- 10.** {6, 8, 2, 3, 2, 9}
- 11.** {23, 18, 28, 26, 15}
- 12.** {44, 35, 50, 37, 43, 38, 40}

- 13. PARKING** A city councilor wants to know how much revenue the city would earn by installing parking meters on Main Street. He counts the number of cars parked on Main Street each weekday: {64, 79, 81, 53, 63}. Find the standard deviation.

12-3 Practice**Statistics and Parameters**

Identify the sample and the population for each situation. Then describe the sample statistic and the population parameter.

- MARINE BIOLOGY** A marine biologist randomly selects 30 oysters from a research tank. The mean weight of the 30 oysters is calculated.
- CIVIL ENGINEERING** A civic engineer randomly selects 5 city intersections with traffic lights. The median length of a red light is calculated for the sample.
- BASEBALL** A baseball commissioner randomly selects 10 home games played by a major league team. The median attendance is calculated for the games in the sample.

Find the mean absolute deviation.

- INVESTING** A stock broker keeps a record of the daily closing price of a share of stock in Bicsomm Corporation: {45.20, 46.10, 46.85, 42.55, 40.80}.
- GOLF** A golfer keeps track of his scores for each round: {78, 81, 86, 77, 75}.
- WEATHER** A meteorologist keeps track of the number of thunderstorms occurring each month in Sussex County: {0, 4, 7, 1, 3, 5, 2}.

Find the mean, variance, and standard deviation of each set of data.

- {6, 11, 16, 9}
- {2, 5, 8, 11, 4}
- {23.4, 16.8, 9.7, 22.1}
- $\{1, \frac{5}{2}, 4, \frac{11}{2}, \frac{1}{2}, 3\}$
- {145, 166, 171, 150, 88}
- {13, 24, 22, 17, 14, 29, 15, 22}

- QUALITY CONTROL** An inspector checks each automobile that comes off of the assembly line. He keeps a record of the number of defective cars each day: {3, 1, 2, 0, 0, 4, 3, 6, 1, 2}. Find the standard deviation.

12-3 Word Problem Practice

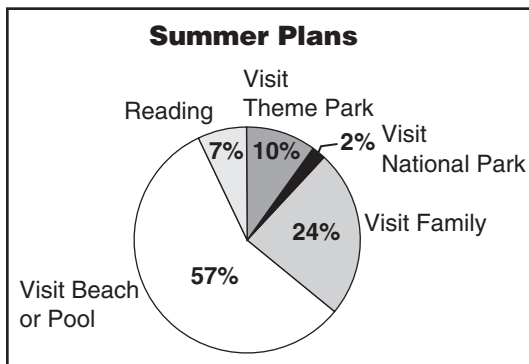
Statistics and Parameters

1. GAS PRICES Renee is planning a road trip to her aunt’s house. To estimate how much the trip will cost, she goes online and finds the price of a gallon of gasoline for 5 randomly selected gas stations along the route. She then calculates the median price per gallon for the 5 selected gas stations.

- a. Identify the sample and the population.

- b. Describe the sample statistic and the population parameter.

2. SUMMER VACATION Mr. Siipola’s algebra class took a survey to find students’ plans for the summer. The results are shown in the circle graph.



Find the mean absolute variation for the poll results.

3. HOCKEY A hockey team keeps track of how many goals it scores each game: {2, 4, 0, 3, 7, 2}. Find the mean, variance, and standard deviation of the data.

4. ERROR ANALYSIS Myau and Alice are studying for their test on statistics. Myau hypothesizes that the variance of a set of data will always be a greater number than the standard deviation. Alice believes that Myau is incorrect and that the standard variation can be a greater number than the variance. Who is correct? Explain.

5. HEALTH CLUBS To plan their future equipment purchases, the Northville Health Club randomly chooses 8 patrons and tracks how many minutes they spend on the treadmill: {30, 30, 45, 20, 60, 30, 30, 15}. The mean treadmill time is then calculated.

- a. Identify the sample and the population. Then describe the sample statistic and the population parameter.

- b. Find the sample statistic specified in part a.

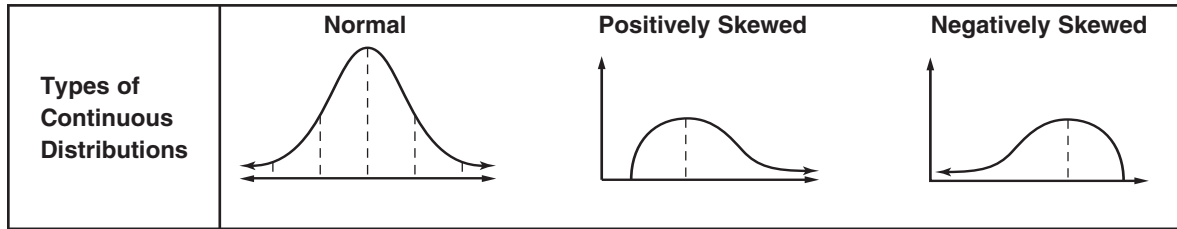
- c. Find the mean absolute deviation for the sample.

- d. Find the variance and standard deviation for the sample.

12-5 Study Guide and Intervention

The Normal Distribution

Normal and Skewed Distributions A continuous probability distribution is represented by a curve.

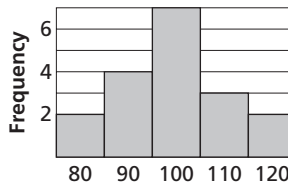


Example Determine whether the data below appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

{100, 120, 110, 100, 110, 80, 100, 90, 100, 120, 100, 90, 110, 100, 90, 80, 100, 90}

Make a frequency table for the data.

Value	80	90	100	110	120
Frequency	2	4	7	3	2



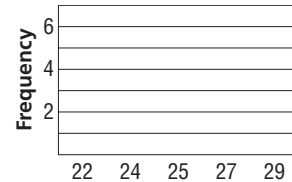
Then use the data to make a histogram.

Since the histogram is roughly symmetric, the data appear to be normally distributed.

Exercises

Determine whether the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*. Make a histogram of the data.

1. {27, 24, 29, 25, 27, 22, 24, 25, 29, 24, 25, 22, 27, 24, 22, 25, 24, 22}



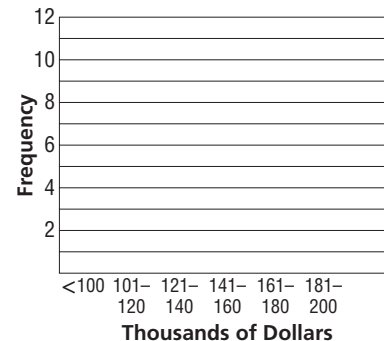
2.

Shoe Size	4	5	6	7	8	9	10
No. of Students	1	2	4	8	5	1	2



3.

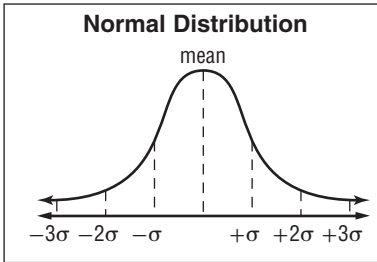
Housing Price	No. of Houses Sold
less than \$100,000	0
\$100,00–\$120,000	1
\$121,00–\$140,000	3
\$141,00–\$160,000	7
\$161,00–\$180,000	8
\$181,00–\$200,000	6
over \$200,000	12



12-5 Study Guide and Intervention *(continued)*

The Normal Distribution

The Empirical Rule



Normal distributions have these properties.

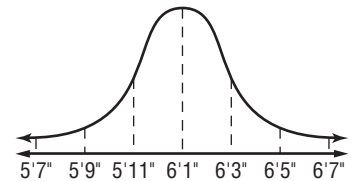
- The graph is maximized at the mean.
- The mean, median, and mode are about equal.
- About 68% of the values are within one standard deviation of the mean.
- About 95% of the values are within two standard deviations of the mean.
- About 99% of the values are within three standard deviations of the mean.

Example The heights of players in a basketball league are normally distributed with a mean of 6 feet 1 inch and a standard deviation of 2 inches.

- a. What is the probability that a player selected at random will be shorter than 5 feet 9 inches?

Draw a normal curve. Label the mean and the mean plus or minus multiples of the standard deviation.

The value of 5 feet 9 inches is 2 standard deviations below the mean, so approximately 2.5% of the players will be shorter than 5 feet 9 inches.



- b. If there are 240 players in the league, about how many players are taller than 6 feet 3 inches?

The value of 6 feet 3 inches is one standard deviation above the mean. Approximately 16% of the players will be taller than this height.

$$240 \times 0.16 \approx 38$$

About 38 of the players are taller than 6 feet 3 inches.

Exercises

1. **EGG PRODUCTION** The number of eggs laid per year by a particular breed of chicken is normally distributed with a mean of 225 and a standard deviation of 10 eggs.
 - a. About what percent of the chickens will lay between 215 and 235 eggs per year?
 - b. In a flock of 400 chickens, about how many would you expect to lay more than 245 eggs per year?
2. **MANUFACTURING** The diameter of bolts produced by a manufacturing plant is normally distributed with a mean of 18 mm and a standard deviation of 0.2 mm.
 - a. What percent of bolts coming off of the assembly line have a diameter greater than 18.4 mm?
 - b. What percent have a diameter between 17.8 and 18.2 mm?

12-5 Skills Practice

The Normal Distribution

Determine whether the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

1.

Miles Run	Track Team Members
0–4	3
5–9	4
10–14	7
15–19	5
20–23	2

2.

Speeches Given	Political Candidates
0–5	1
6–11	2
12–17	3
18–23	8
24–29	8

3. **PATIENTS** The frequency table to the right shows the average number of days patients spent on the surgical ward of a hospital last year.

Days	Number of Patients
0–3	5
4–7	18
8–11	11
12–15	9
16+	6

- What percentage of the patients stayed between 4 and 7 days?
- Does the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*? Explain.

4. **DELIVERY** The time it takes a bicycle courier to deliver a parcel to his farthest customer is normally distributed with a mean of 40 minutes and a standard deviation of 4 minutes.

- About what percent of the courier’s trips to this customer take between 36 and 44 minutes?
- About what percent of the courier’s trips to this customer take between 40 and 48 minutes?
- About what percent of the courier’s trips to this customer take less than 32 minutes?

5. **TESTING** The average time it takes sophomores to complete a math test is normally distributed with a mean of 63.3 minutes and a standard deviation of 12.3 minutes.

- About what percent of the sophomores take more than 75.6 minutes to complete the test?
- About what percent of the sophomores take between 51 and 63.3 minutes?
- About what percent of the sophomores take less than 63.3 minutes to complete the test?

12-5 Practice

The Normal Distribution

Determine whether the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*.

1. **Time Spent at a Museum Exhibit**

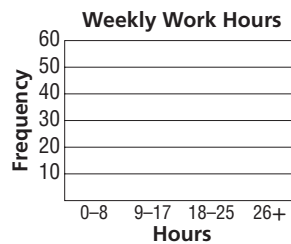
Minutes	Frequency
0–25	27
26–50	46
51–75	89
75–100	57
100+	24

2. **Average Age of High School Principals**

Age in Years	Number
31–35	3
36–40	8
41–45	15
46–50	32
51–55	40
56–60	38
60+	4

3. **STUDENTS** The frequency table to the right shows the number of hours worked per week by 100 high school students.

- What percentage of the students worked between 9 and 17 days?
- Do the data appear to be *positively skewed*, *negatively skewed*, or *normally distributed*? Explain.



Hours	Number of Students
0–8	30
9–17	45
18–25	20
26+	5

4. **TESTING** The scores on a test administered to prospective employees are normally distributed with a mean of 100 and a standard deviation of 15.

- About what percent of the scores are between 70 and 130?
- About what percent of the scores are between 85 and 130?
- About what percent of the scores are over 115?
- About what percent of the scores are lower than 85 or higher than 115?
- If 80 people take the test, how many would you expect to score higher than 130?
- If 75 people take the test, how many would you expect to score lower than 85?

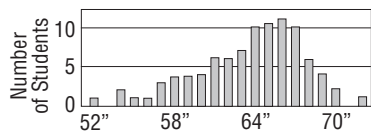
5. **TEMPERATURE** The daily July surface temperature of a lake at a resort has a mean of 82° and a standard deviation of 4.2° . If you prefer to swim when the temperature is at least 77.8° , about what percent of the days does the temperature meet your preference?

12-5 Word Problem Practice

The Normal Distribution

1. PARKING Over several years, Bertram conducted a study of how far into parking spaces people tend to park by measuring the distance from the end of a parking space to the front fender of a car parked in the space. He discovered that the distribution of the data closely approximated a normal distribution with mean 8.5 inches. He found that about 5% of cars parked more than 11.5 inches away from the end of the parking space. What percentage of cars would you expect parked less than 5.5 inches away from the end of the parking space?

2. HEIGHT Chandra's graph of the number of tenth grade students of different heights is shown below.



Is the data positively skewed, negatively skewed, or normally distributed?

3. OVENS An oven manufacturer tries to make the temperature setting on its ovens as accurate as possible. However, if one measures the actual temperatures in the ovens when the temperature setting is 350°F, they will differ slightly from 350°F. The set of actual temperatures for all the ovens is normally distributed around 350°F with a standard deviation of 0.5°F. About what percentage of ovens will be between 350°F and 351°F when their temperature setting is 350°F?

4. LIGHT BULBS The time that a certain brand of light bulb will last before burning out is normally distributed. About 2.5% of the bulbs last longer than 6800 hours and about 16% of the bulbs last longer than 6500 hours. How long does the average bulb last?

5. DOGS The weights of adult male greyhound dogs are normally distributed. The mean weight is about 68 pounds and the standard deviation is about 10 pounds.

a. Approximately what percentage of adult male greyhound dogs would you expect weigh between 58 and 78 pounds?

b. Approximately what percentage of adult male greyhound dogs would you expect weigh more than 98 pounds?

c. Approximately what percentage of adult male greyhound dogs would you expect weigh less than 48 pounds?

d. What would you expect an adult male greyhound dog to weigh if it weighed less than 0.5% of an average adult greyhound?

12-6 Study Guide and Intervention

Probability Distributions

Random Variables and Probability A random variable X is a variable whose value is the numerical outcome of a random event.

Example A teacher asked her students how many siblings they have. The results are shown in the table at the right.

Number of Siblings	Number of Students
0	1
1	15
2	8
3	2
4	1

a. Find the probability that a randomly selected student has 2 siblings.

The random variable X can equal 0, 1, 2, 3, or 4. In the table, the value $X = 2$ is paired with 8 outcomes, and there are 27 students surveyed.

$$P(X = 2) = \frac{2 \text{ siblings}}{27 \text{ students surveyed}}$$

$$= \frac{8}{27}$$

The probability that a randomly selected student has 2 siblings is $\frac{8}{27}$, or 29.6%.

b. Find the probability that a randomly selected student has at least three siblings.

$$P(X \geq 3) = \frac{2 + 1}{27}$$

The probability that a randomly selected student has at least 3 siblings is $\frac{1}{9}$, or 11.1%.

Exercises

For Exercises 1–3, use the grade distribution shown at the right. A grade of A = 5, B = 4, C = 3, D = 2, F = 1.

X = Grade	5	4	3	2	1
Number of students	6	9	5	4	1

- Find the probability that a randomly selected student in this class received a grade of C.
- Find the probability that a randomly selected student in this class received a grade lower than a C.
- What is the probability that a randomly selected student in this class passes the course, that is, gets at least a D?
- The table shows the results of tossing 3 coins 50 times. What is the probability of getting 2 or 3 heads?

X = Number of Heads	0	1	2	3
Number of Times	6	20	19	5

12-6 Study Guide and Intervention *(continued)*

Probability Distributions

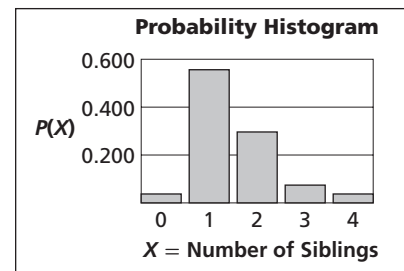
Probability Distributions The probabilities associated with every possible value of the random variable X make up what are called the **probability distribution** for that variable. A probability distribution has the following properties.

Properties of a Probability Distribution	<ol style="list-style-type: none"> The probability of each value of X is greater than or equal to 0. The probabilities for all values of X add up to 1.
---	---

The probability distribution for a random variable can be given in a table or in a **probability graph** and used to obtain other information.

Example The data from the example on the previous page can be used to determine a probability distribution and to make a probability graph.

$X = \text{Number of Siblings}$	$P(X)$
0	0.037
1	0.556
2	0.296
3	0.074
4	0.037



a. Show that the distribution is valid.

For each value of X , the probability is greater than or equal to 0 and less than or equal to 1. Also, the sum of the probabilities is 1.

b. What is the probability that a student chosen at random has fewer than 2 siblings?

Because the events are independent, the probability of fewer than 2 siblings is the sum of the probability of 0 siblings and the probability of 1 sibling, or $0.037 + 0.556 = 0.593$.

Exercises

The table at the right shows the probability distribution for students by school enrollment in the United States in 2000. Use the table for Exercises 1–3.

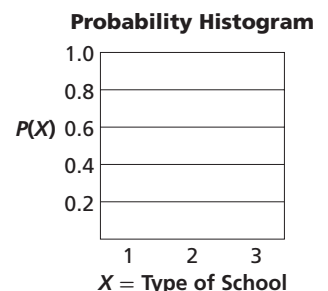
$X = \text{Type of School}$	$P(X)$
Elementary = 1	0.562
Secondary = 2	0.215
Higher Education = 3	0.223

Source: U.S. Census Bureau

1. Show that the distribution is valid.

2. If a student is chosen at random, what is the probability that the student is in elementary or secondary school?

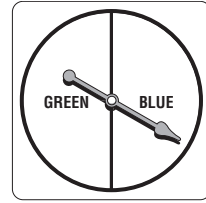
3. Make a probability graph of the data.



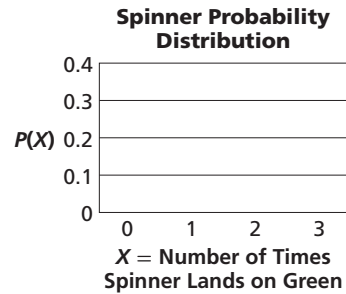
12-6 Skills Practice

Probability Distributions

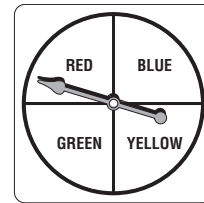
For Exercises 1–3, the spinner shown is spun three times.



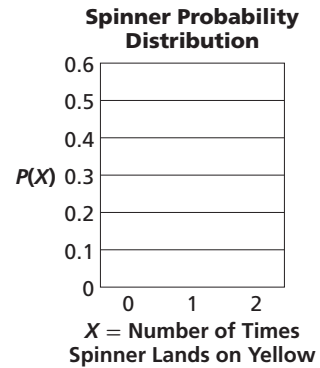
1. Write the sample space with all possible outcomes.
2. Find the probability distribution X , where X represents the number of times the spinner lands on green for $X = 0$, $X = 1$, $X = 2$, and $X = 3$.
3. Make a probability graph of the data.



For Exercises 4–6, the spinner shown is spun two times.



4. Write the sample space with all possible outcomes.
5. Find the probability distribution X , where X represents the number of times the spinner lands on yellow for $X = 0$, $X = 1$, and $X = 2$.
6. Make a probability graph of the data.



7. **BUSINESS** Use the table that shows the probability distribution of the number of minutes a customer spends at the express checkout at a supermarket.

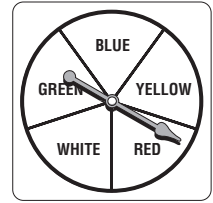
X = Minutes	1	2	3	4	5 +
Probability	0.09	0.13	0.28	0.32	0.18

- a. Show that the distribution is valid.
- b. What is the probability that a customer spends less than 3 minutes at the checkout?
- c. What is the probability that the customer spends at least 4 minutes at the checkout?

12-6 Practice

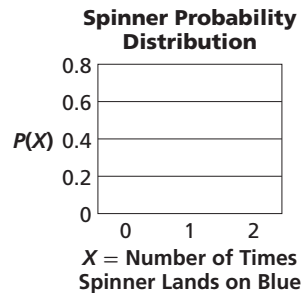
Probability Distributions

For Exercises 1–3, the spinner shown is spun two times.



- Write the sample space with all possible outcomes.
- Find the probability distribution X , where X represents the number of times the spinner lands on blue for $X = 0$, $X = 1$, and $X = 2$.

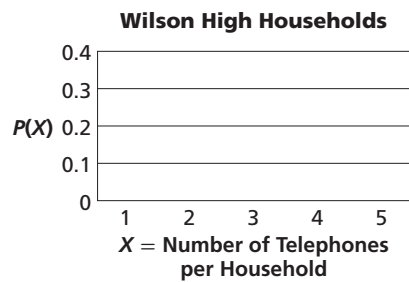
3. Make a probability graph of the data.



4. **TELECOMMUNICATIONS** Use the table that shows the probability distribution of the number of telephones per student's household at Wilson High.

X = Number of Telephones	1	2	3	4	5+
Probability	0.01	0.16	0.34	0.39	0.10

- Show that the distribution is valid.
- If a student is chosen at random, what is the probability that there are more than 3 telephones at the student's home?
- Make a probability graph of the data.



5. **LANDSCAPING** Use the table that shows the probability distribution of the number of shrubs (rounded to the nearest 50) ordered by corporate clients of a landscaping company over the past five years.

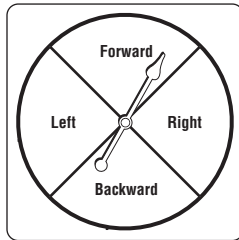
X = Number of Shrubs	50	100	150	200	250
Probability	0.11	0.24	0.45	0.16	0.04

- Define a random variable and list its values.
- Show that the distribution is valid.
- What is the probability that a client's (rounded) order was at least 150 shrubs?

12-6 Word Problem Practice

Probability Distributions

1. **GAMES** A spinner for an adventure game decides how a player will move. Write the sample space with all possible outcomes if the spinner is spun twice.



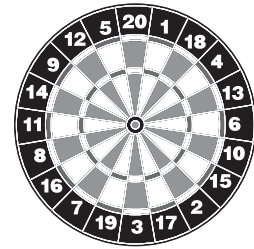
2. **JOBS** The probability distribution table shows the results of a newspaper survey that asked babysitters how much they make per hour.

Hourly Pay	\$3	\$4	\$5	\$6	\$7+
Probability	0.1	0.2	0.5	0.15	0.05

What is the probability that a babysitter chosen at random from the survey makes more than \$5 per hour?

3. **TIME** James looks at his 12-hour digital clock when he wakes up in the middle of the night. Make a probability graph for all of the possible outcomes if you let X equal the first digit of the time. For instance, if it is 3:41, the first digit is a 3.

4. **DARTS** Suppose two darts are thrown at a traditional dart board and land near the center, but not on a bullseye. What is the probability that the sum of their scores is less than or equal to 5?



5. **TENNIS** All season Oliver's tennis coach has kept track of where each of his serves has landed during practice. The results are shown in the probability distribution table. A serve in region 2 is "in", while a serve in any other region is a fault. A serve in region 1 represents hitting the net.

Region	1	2	3	4	5
Probability	0.19	0.52	0.15	0.09	0.05

- What is the probability that any one of Oliver's serves does not hit the net and is a fault?
- Make a probability graph that represents the data in the table.

10-2 Study Guide and Intervention**Simplifying Radical Expressions**

Product Property of Square Roots The **Product Property of Square Roots** and prime factorization can be used to simplify expressions involving irrational square roots. When you simplify radical expressions with variables, use absolute value to ensure nonnegative results.

Product Property of Square Roots	For any numbers a and b , where $a \geq 0$ and $b \geq 0$, $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$.
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Example 1 Simplify $\sqrt{180}$.

$$\begin{aligned} \sqrt{180} &= \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 5} && \text{Prime factorization of 180} \\ &= \sqrt{2^2 \cdot 3^2 \cdot 5} && \text{Product Property of Square Roots} \\ &= 2 \cdot 3 \cdot \sqrt{5} && \text{Simplify.} \\ &= 6\sqrt{5} && \text{Simplify.} \end{aligned}$$

Example 2 Simplify $\sqrt{120a^2 \cdot b^5 \cdot c^4}$.

$$\begin{aligned} &\sqrt{120a^2 \cdot b^5 \cdot c^4} \\ &= \sqrt{2^3 \cdot 3 \cdot 5 \cdot a^2 \cdot b^5 \cdot c^4} \\ &= \sqrt{2^2} \cdot \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5} \cdot \sqrt{a^2} \cdot \sqrt{b^4 \cdot b} \cdot \sqrt{c^4} \\ &= 2 \cdot \sqrt{2} \cdot \sqrt{3} \cdot \sqrt{5} \cdot |a| \cdot b^2 \cdot \sqrt{b} \cdot c^2 \\ &= 2|a|b^2c^2\sqrt{30b} \end{aligned}$$

Exercises

Simplify each expression.

- | | | | |
|----------------------------------|--------------------------------------|------------------------------|-------------------------------|
| 1. $\sqrt{28}$ | 2. $\sqrt{68}$ | 3. $\sqrt{60}$ | 4. $\sqrt{75}$ |
| 5. $\sqrt{162}$ | 6. $\sqrt{3} \cdot \sqrt{6}$ | 7. $\sqrt{2} \cdot \sqrt{5}$ | 8. $\sqrt{5} \cdot \sqrt{10}$ |
| 9. $\sqrt{4a^2}$ | 10. $\sqrt{9x^4}$ | 11. $\sqrt{300a^4}$ | 12. $\sqrt{128c^6}$ |
| 13. $4\sqrt{10} \cdot 3\sqrt{6}$ | 14. $\sqrt{3x^2} \cdot 3\sqrt{3x^4}$ | 15. $\sqrt{20a^2b^4}$ | 16. $\sqrt{100x^3y}$ |
| 17. $\sqrt{24a^4b^2}$ | 18. $\sqrt{81x^4y^2}$ | 19. $\sqrt{150a^2b^2c}$ | |
| 20. $\sqrt{72a^6b^3c^2}$ | 21. $\sqrt{45x^2y^5z^8}$ | 22. $\sqrt{98x^4y^6z^2}$ | |

10-2 Study Guide and Intervention *(continued)***Simplifying Radical Expressions**

Quotient Property of Square Roots A fraction containing radicals is in simplest form if no radicals are left in the denominator. The **Quotient Property of Square Roots** and **rationalizing the denominator** can be used to simplify radical expressions that involve division. When you rationalize the denominator, you multiply the numerator and denominator by a radical expression that gives a rational number in the denominator.

Quotient Property of Square RootsFor any numbers a and b , where $a \geq 0$ and $b > 0$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$.**Example**Simplify $\sqrt{\frac{56}{45}}$.

$$\begin{aligned}\sqrt{\frac{56}{45}} &= \sqrt{\frac{4 \cdot 14}{9 \cdot 5}} \\ &= \frac{2 \cdot \sqrt{14}}{3 \cdot \sqrt{15}} \\ &= \frac{2\sqrt{14}}{3\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} \\ &= \frac{2\sqrt{70}}{15}\end{aligned}$$

Simplify the numerator and denominator.

Multiply by $\frac{\sqrt{5}}{\sqrt{5}}$ to rationalize the denominator.

Product Property of Square Roots

Exercises

Simplify each expression.

1. $\frac{\sqrt{9}}{\sqrt{18}}$

2. $\frac{\sqrt{8}}{\sqrt{24}}$

3. $\frac{\sqrt{100}}{\sqrt{121}}$

4. $\frac{\sqrt{75}}{\sqrt{3}}$

5. $\frac{8\sqrt{2}}{2\sqrt{8}}$

6. $\sqrt{\frac{2}{5}} \cdot \sqrt{\frac{6}{5}}$

7. $\sqrt{\frac{3}{4}} \cdot \sqrt{\frac{5}{2}}$

8. $\sqrt{\frac{5}{7}} \cdot \sqrt{\frac{2}{5}}$

9. $\sqrt{\frac{3a^2}{10b^6}}$

10. $\sqrt{\frac{x^6}{y^4}}$

11. $\sqrt{\frac{100a^4}{144b^8}}$

12. $\sqrt{\frac{75b^3c^6}{a^2}}$

13. $\frac{\sqrt{4}}{3 - \sqrt{5}}$

14. $\frac{\sqrt{8}}{2 + \sqrt{3}}$

15. $\frac{\sqrt{5}}{5 + \sqrt{5}}$

16. $\frac{\sqrt{8}}{2\sqrt{7} + 4\sqrt{10}}$

10-2 Practice**Simplifying Radical Expressions**

Simplify.

1. $\sqrt{24}$

2. $\sqrt{60}$

3. $\sqrt{108}$

4. $\sqrt{8} \cdot \sqrt{6}$

5. $\sqrt{7} \cdot \sqrt{14}$

6. $3\sqrt{12} \cdot 5\sqrt{6}$

7. $4\sqrt{3} \cdot 3\sqrt{18}$

8. $\sqrt{27tu^3}$

9. $\sqrt{50p^5}$

10. $\sqrt{108x^6y^4z^5}$

11. $\sqrt{56m^2n^4p^5}$

12. $\frac{\sqrt{8}}{\sqrt{6}}$

13. $\sqrt{\frac{2}{10}}$

14. $\sqrt{\frac{5}{32}}$

15. $\sqrt{\frac{3}{4}} \cdot \sqrt{\frac{4}{5}}$

16. $\sqrt{\frac{1}{7}} \cdot \sqrt{\frac{7}{11}}$

17. $\frac{\sqrt{3k}}{\sqrt{8}}$

18. $\sqrt{\frac{18}{x^3}}$

19. $\sqrt{\frac{4y}{3y^2}}$

20. $\sqrt{\frac{9ab}{4ab^4}}$

21. $\frac{3}{5 - \sqrt{2}}$

22. $\frac{8}{3 + \sqrt{3}}$

23. $\frac{5}{\sqrt{7} + \sqrt{3}}$

24. $\frac{3\sqrt{7}}{-1 - \sqrt{27}}$

25. SKY DIVING When a skydiver jumps from an airplane, the time t it takes to free fall a given distance can be estimated by the formula $t = \sqrt{\frac{2s}{9.8}}$, where t is in seconds and s is in meters. If Julie jumps from an airplane, how long will it take her to free fall 750 meters?

26. METEOROLOGY To estimate how long a thunderstorm will last, meteorologists can use the formula $t = \sqrt{\frac{d^3}{216}}$, where t is the time in hours and d is the diameter of the storm in miles.

a. A thunderstorm is 8 miles in diameter. Estimate how long the storm will last. Give your answer in simplified form and as a decimal.

b. Will a thunderstorm twice this diameter last twice as long? Explain.

10-2 Word Problem Practice**Simplifying Radical Expressions**

- 1. SPORTS** Jasmine calculated the height of her team's soccer goal to be $\frac{15}{\sqrt{3}}$ feet. Simplify the expression.

- 2. NATURE** In 2004, an earthquake below the ocean floor initiated a devastating tsunami in the Indian Ocean. Scientists can approximate the velocity (in feet per second) of a tsunami in water of depth d (in feet) with the formula $V = \sqrt{16d}$. Determine the velocity of a tsunami in 300 feet of water. Write your answer in simplified radical form.

- 3. AUTOMOBILES** The following formula can be used to find the “zero to sixty” time for a car, or the time it takes for a car to accelerate from a stop to sixty miles per hour.

$$V = \sqrt{\frac{2PT}{M}}$$

V is the velocity (in meters per second).
 P is the car's average power (in watts).
 M is the mass of the car (in kilograms).
 T is the time (in seconds).

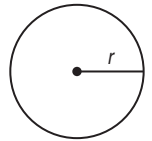
Find the time it takes for a 900-kilogram car with an average 60,000 watts of power to accelerate from stop to 26.82 meters per second (60 miles per hour). Round your answer to the nearest tenth.

- 4. PHYSICAL SCIENCE** When a substance such as water vapor is in its gaseous state, the volume and the velocity of its molecules increase as temperature increases. The average velocity V of a molecule with mass m at temperature T

is given by the formula $V = \sqrt{\frac{3kT}{m}}$.

Solve the equation for k .

- 5. GEOMETRY** Suppose Emeryville Hospital wants to build a new helipad on which medic rescue helicopters can land. The helipad will be circular and made of fire resistant rubber.



- a.** If the area of the helipad is A , write an equation for the radius r .
- b.** Write an expression in simplified radical form for the radius of a helipad with an area of 288 square meters.
- c.** Using your calculator, find a decimal approximation for the radius. Round your answer to the nearest hundredth.